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Par : **Matthieu Dufourd**

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Membres présents du jury de la filière
Nom : Pierre Picard

Membres présents du jury de l'Institut des Actuaires

Entreprise :

Nom : *AXA Group Ceded Reinsurance*
Signature :

Directeur de mémoire en entreprise :

Nom : Nicolas Richard
Signature :

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Résumé

Au sein d'AXA Group Ceded Reinsurance (AGCRe), les traités de réassurance sont modélisés et appréciés via la production de différents indicateurs pour estimer leur rentabilité en vue des comités de souscription. Cependant, AGCRe ne dispose pas d'un contrôle direct sur le processus de modélisation, complexifiant l'adaptation des modèles aux besoins spécifiques des entités d'AXA et l'évaluation de nouveaux traités. En outre, certains traités de réassurance ne sont pas modélisés et nécessitent des études ponctuelles au cas par cas. S'inscrivant dans ce contexte, ce mémoire a pour objectif de développer une méthodologie globale et centralisée d'évaluation du risque cédé au travers des traités de réassurance non-vie (souscrits ou prospectifs). Cette méthodologie sera déployée de manière automatisée sous la forme d'un outil opérationnel ayant pour objectif d'offrir une vision interne propre à l'entité lui permettant d'accroitre sa capacité à tarifer un plus grand nombre de traités.

Ce mémoire présente donc les étapes et la réflexion autour de la méthodologie développée dans le cadre de la conception de cet outil de tarification. Ce dernier s'articule autour de trois modules, chacun correspondant à un axe principal de la méthodologie. Le premier module est dédié à la transformation des sinistres historiques, dont l'objectif principal est d'obtenir la charge à l'ultime. Le deuxième module, s'appuyant sur les résultats du premier, se concentre sur la modélisation des sinistres sous-jacents à travers diverses approches, telles que les courbes d'exposition ou les modèles de type fréquence-coût, adaptés à certains types de traités. Enfin, le troisième module utilise la méthode de Monte Carlo pour générer les indicateurs de risque et de profitabilité relatifs à chaque traité en intégrant des aspects liés au coût en capital du traité.

Mots clés : *Tarification en réassurance non-vie, Tarification par expérience, Tarification par exposition, Modèle collectif, Théorie des valeurs extrêmes, Provisionnement des sinistres, Mise en as-if des sinistres historiques, Ajustement de distributions type fréquence/sévérité, Simulations de Monte Carlo, Recouvrements modélisés, Indicateurs de risque et de rentabilité de la réassurance proportionnelle et non proportionnelle.*

Abstract

Within AXA Group Ceded Reinsurance (AGCRe), reinsurance treaties are modeled and assessed by producing various risk and profitability indicators for underwriting committees. However, AGCRe does not have direct control over the modeling process, making it difficult to adapt models for the specific needs of AXA entities and to evaluate new treaties. In addition, around 20% of reinsurance treaties are not modeled, requiring case-by-case studies. Against this backdrop, the aim of this thesis is to develop a global, centralized methodology for assessing the risk ceded through non-life reinsurance treaties (underwritten or prospective). This methodology will be deployed in the form of an operational tool, with the aim of providing an internal view specific to the entity, enabling it to increase its capacity to price a greater number of treaties.

This actuarial thesis presents the steps and considerations that guided the development of the pricing tool's methodology. The tool itself is organized around three main modules, each corresponding to a core aspect of the methodology. The first module focuses on transforming historical claims, with the primary purpose of determining the ultimate loss. The second module, built on the results of the first one, aims to model underlying claims through various approaches, such as exposure curves or frequency-severity models. Finally, the third module employs the Monte Carlo method to generate indicators related to each treaty, incorporating aspects linked to the treaty's cost of capital.

Keywords: *P&C reinsurance pricing, Experience rating, Exposure rating, Collective risk model, Extreme value theory, Chain-Ladder claims reserving, As-if Ultimate historical claims adjustment, Frequency & severity distribution fitting, Monte Carlo simulations, Modeled recoveries, Proportional and non-proportional reinsurance risk & profitability indicators*

Note de synthèse

Souvent qualifiée d'« assurance des assureurs », la réassurance est un mécanisme de répartition des risques par lequel un assureur transfère tout ou une partie de ses risques à une société d'assurance tierce, le réassureur. Elle est matérialisée par différents types de contrats appelés traités de réassurance. Ces traités peuvent être classés en quatre grandes catégories : les Excess of Loss (XL) par risque ou par événement, les Aggregate (AGG), les Quota-Share (QS) et les Surplus (SP). Ces traités permettent de transférer les risques de manière proportionnelle ou non proportionnelle.

En tant que réassureur interne du groupe AXA, Axa Group Ceded Reinsurance (AGCRe) est responsable de la gestion de l'ensemble des opérations de réassurance du groupe. Pour ce faire, AGCRe dispose de plusieurs stratégies concernant la gestion des traités : elle peut choisir d'accepter entièrement les traités venant des entités et agir en tant que seul réassureur, de les céder à d'autres réassureurs, ou de n'en retenir qu'une partie qu'elle place sein d'un pool de réassurance. La figure ci-dessous illustre le fonctionnement de la gestion des traités de réassurance au sein d'AGCRe.

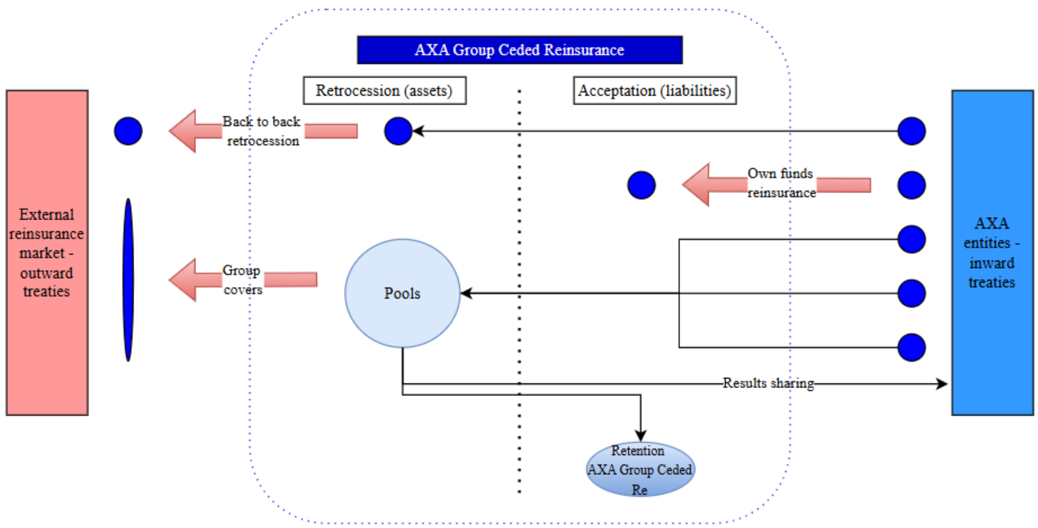


Figure 1 : Les traités de réassurance au sein d'AGCRe

En vue des comités de souscription, l'équipe Analytics and Pricing est chargée de fournir divers indicateurs relatifs aux traités afin d'évaluer leur risque. À cet effet, même si l'équipe est aujourd'hui capable d'évaluer la majorité des traités souscrits par AGCRe, environ 20% des traités ne sont pas modélisés à l'aide du modèle interne, qui concerne principalement les traités XL placés au sein de pools de réassurance. Les traités QS, SP, AGG et certains traités XL par risque font donc actuellement l'objet d'études au cas par cas en fonction des demandes des cédantes, ce qui tend à être fastidieux. En outre, les modèles actuels sont basés sur des directives établies à l'échelle du groupe AXA et reposent sur des logiciels externes, ce qui restreint le contrôle de l'équipe sur leur processus de modélisation. Cette dépendance réduit par ailleurs la capacité de l'équipe à exploiter pleinement ses propres bases de données et empêche d'offrir une vision propre à l'entité sur les traités de réassurance souscrits. Cette situation soulève donc la problématique suivante et pose le cadre de ce mémoire d'actuariat : Comment AGCRe peut-elle, grâce à une approche de modélisation centralisée, robuste, flexible et entièrement contrôlée, développer une vision interne lui permettant d'évaluer les risques associés au plus grand nombre de traités de réassurance ?

A l'aide du développement d'une méthodologie propre à l'entité matérialisée par le développement d'un outil de tarification sur R shiny, nous visons à augmenter les capacités d'évaluation des traités en modélisant de manière automatique les différents traités qui ne sont pas intégrés au modèle interne. Cet outil a également pour objectif de fournir à l'équipe Analytics & Pricing une opinion supplémentaire leur permettant de produire les indicateurs d'intérêt en vue des comités de souscription via un processus entièrement contrôlé et transparent. L'étude se limitera cependant aux traités de réassurance non-vie couvrant un horizon temporel d'une durée d'un an et concernera quatre types de traités de réassurance souscrits par AXA : les XL par risque, les QS, les AGG et les SP[1], qui sont des traités actuellement partiellement ou non modélisés via le modèle interne. La Line of Business (LoB) CAT n'est à ce jour pas prise en charge au sein de l'outil même si ce dernier est conçu pour intégrer à l'avenir les données issues de modèles CAT, et permettre ainsi la modélisation et la tarification de traités CAT.

L'outil ainsi développé et son architecture reposent sur trois modules principaux, correspondant à des axes majeurs de la méthodologie de tarification ici développée : un module de sélection et de transformation des données, un module de modélisation des pertes couvertes par le traité de rassurance et enfin le calcul des différents indicateurs retenus pour les comités de souscription.

Lors de la tarification et de l'estimation des indicateurs relatifs à un traité de réassurance, la première étape consiste à sélectionner les bases de données nécessaires à son évaluation. En fonction du type de traité à tarifer, ces dernières vont différer en raison des modèles utilisés qui ne nécessitent pas les mêmes besoins en termes de données. Nous disposons dans un premier temps de l'historique des sinistres (montants payés et provisionnés) déclarés au-delà de certains seuils par toutes les entités d'AXA, sur l'ensemble des LoBs et atteignant une profondeur historique pouvant aller jusqu'à 30 ans. Dans le cadre d'un traité XL par risque, la sinistralité individuelle est considérée tandis que la sinistralité agrégée est retenue pour la tarification des AGG et QS. Afin d'estimer la sinistralité sous-jacente de la manière la plus fine possible, il est nécessaire d'évaluer les perte s à l'ultime, qu'elles soient annuelles ou agrégés. Pour ce faire, la méthode de Chain-Ladder a été retenue afin d'estimer les IBNR (Incurred But Not Reported) et donc le montant ultime associée à chaque sinistre. Afin d'évaluer les traités AGG et QS, les loss ratio à l'ultime seront modélisés en utilisant les montants de primes sous-jacents au traité. Concernant les traités XL par risque, la sinistralité individuelle étant considérée, le nombre de sinistres par année est une variable d'intérêt. Étant donné les limites du caractère purement multiplicatif de Chain-Ladder, le modèle de Schnieper a été retenu pour estimer la fréquence des sinistres. Une mise en as-if visant à prendre en compte des facteurs tel que l'inflation a également été réalisée afin d'ajuster les montants des sinistres individuelles. Enfin, un payment pattern est calibré pour chaque traité afin d'estimer la cadence de paiement des sinistres et ce pour des considérations d'actualisation dans le calcul des paiements réalisés par le réassureur, qui peuvent s'étendre sur plusieurs années.

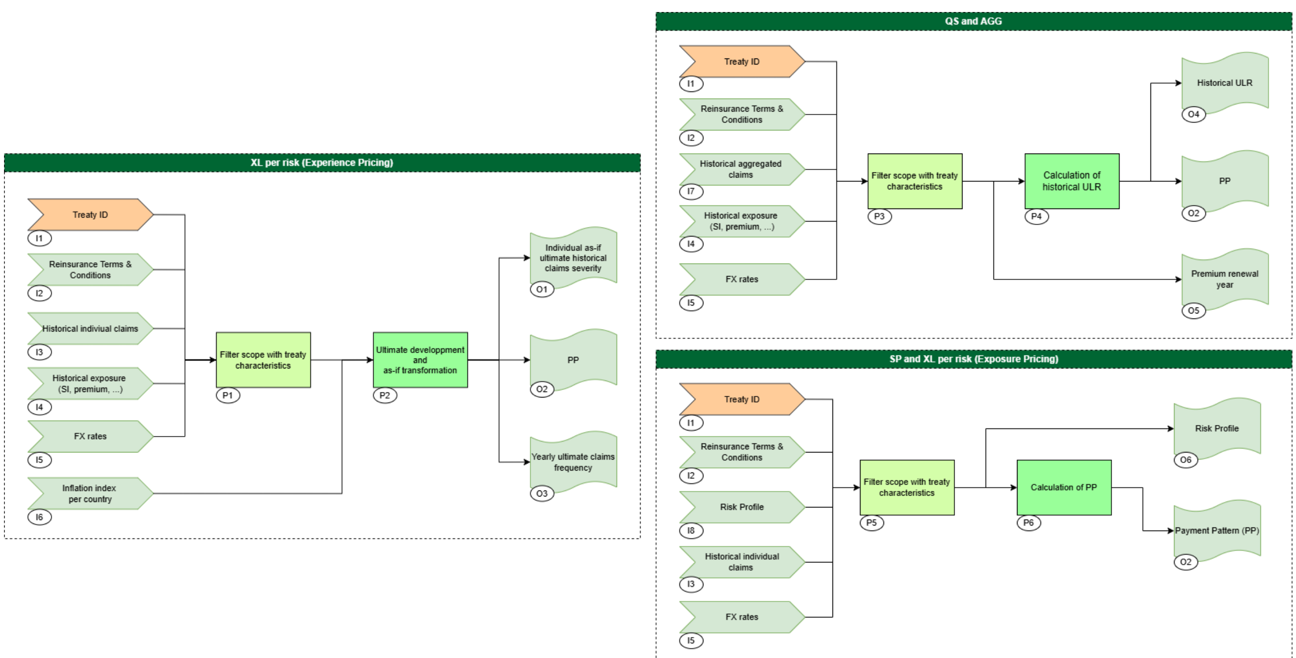


Figure 2 : Module de sélection et transformation des données

[1] Seulement pour les surplus couvrant la LoB Property

La méthode de Monte Carlo a été retenue pour modéliser la sinistralité sous-jacente en raison de sa flexibilité. En permettant d'obtenir une distribution de recouvrements intégrant l'impact des conditions des traités, cette méthode permet de calculer un nombre d'indicateurs plus conséquent. Concernant la modélisation des pertes sous-jacentes, la méthodologie retenue est similaire pour les traités XL per risk, AGG et QS. Elle se résume en une estimation des paramètres des distributions envisageables pour modéliser les loss ratios ou les sinistres individuels, une sélection des distributions appropriées, une simulation des quantités d'intérêt et enfin une application des conditions du traité pour obtenir les pertes cédées à la réassurance. Pour les traités XL, un modèle de type cout-fréquence va ainsi être calibré pour chaque traité. Les distributions choisies sont basées sur la théorie des valeurs extrêmes, avec notamment des distributions de Pareto, et d'autres types comme les Mixed Erlang. Des distributions par morceaux ont également été utilisées afin de modéliser les queues de destruction et donc les événements extrêmes qui impactent fortement les pertes liées aux traités XL. Les distributions sont ensuite évaluées et classées afin de sélectionner la distribution la plus appropriée. Concernant les lois de fréquence, ces dernières sont sélectionnées en fonction de la valeur du coefficient de dispersion du nombre de sinistres à l'ultime parmi la loi Binomiale, la loi de Poisson, et la loi Binomiale Négative. Une méthode alternative a également été envisagée pour les traités XL couvrant la LoB property, à savoir une méthode basée sur les courbes d'exposition. Cette dernière permet d'obtenir une distribution des récupérations dans les cas où le modèle collectif n'est pas satisfaisant, notamment en raison d'un nombre trop faible de sinistres ou de rétentions trop élevées. Cette méthodologie est également utilisée pour évaluer les pertes couvertes par les traités SP qui n'étaient pas modélisées auparavant. Concernant les traités AGG et QS, les loss ratios à l'ultime seront directement modélisés à l'aide de distributions statistiques. Les conditions des traités sont ensuite appliquées aux pertes simulées en tenant compte de potentielles clauses d'indexation ainsi que d'autres particularités, comme les loss corridors pour les traités QS.

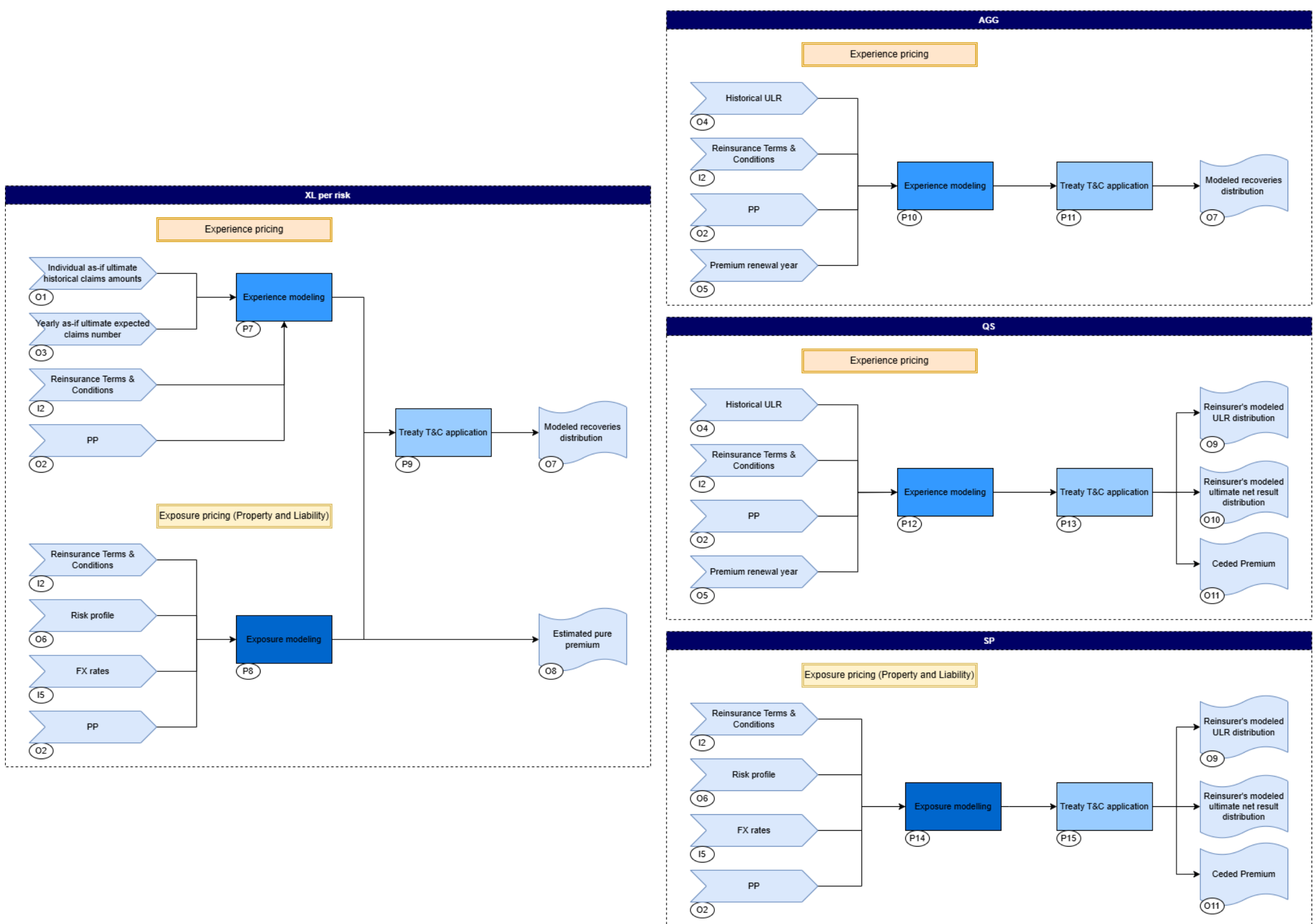


Figure 3 : Module de modélisation des pertes

Enfin, les indicateurs de risque et de profitabilité sont directement dérivés des distributions de recouvrements ou de loss ratios résultant du processus de modélisation décrit précédemment. Pour les traités non proportionnels, tels que les traités AGG et XL, cela inclut notamment les probabilités de dépassement, les périodes de retour, ainsi que divers indicateurs statistiques relatifs aux distributions de pertes. Pour ces types de traités, la prime technique est également un indicateur d'intérêt dans l'évaluation du traité. Cette dernière nécessite le calcul de chargements visant à couvrir le réassureur en cas de pertes extrêmes. Une approche basée sur le coût en capital du traité a donc été considérée pour évaluer ces chargements. La méthode de calcul diffère selon que le traité soit inclus dans un pool ou non. Si le traité est placé dans un pool, les chargements sont calculés en fonction de la part du traité au sein du pool de réassurance selon des métriques telles que la variance, l'écart type ou le quantile. Dans le cas contraire, le chargement est basé sur l'impact marginal du traité sur le Solvency Capital Required (SCR) de la branche non-vie d'AXA SA. Concernant les traités QS et SP, qui ne requièrent pas le calcul d'une prime technique, nous renvoyons une estimation des pertes cédées, du résultat technique net, du coût du capital, des commissions et d'autres variables dépendant des pertes cédées.

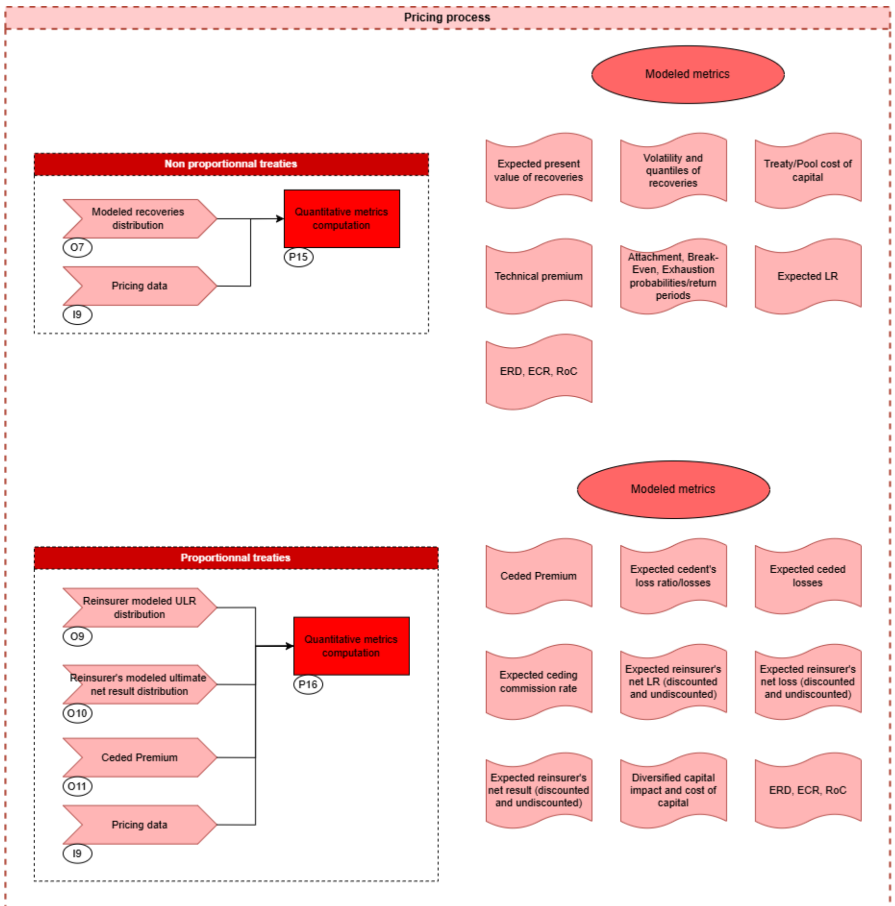


Figure 4 : Module de calcul des indicateurs

Afin d'évaluer la performance de l'outil développé, une comparaison avec les primes de marché a été réalisée pour les traités XL. Cette analyse visait à estimer si la méthodologie proposée est fiable et permet d'estimer correctement la prime technique. Les conclusions, basées sur un ensemble de 90 traités, montrent que la méthodologie fournit des estimations relativement cohérentes. Pour les traités poolés, les écarts constatés sont de l'ordre de 5 %, ce qui reflète une bonne concordance avec le marché. En revanche, pour les traités non poolés, la prime technique est sous-estimée d'environ 13 % par rapport aux primes de marché.

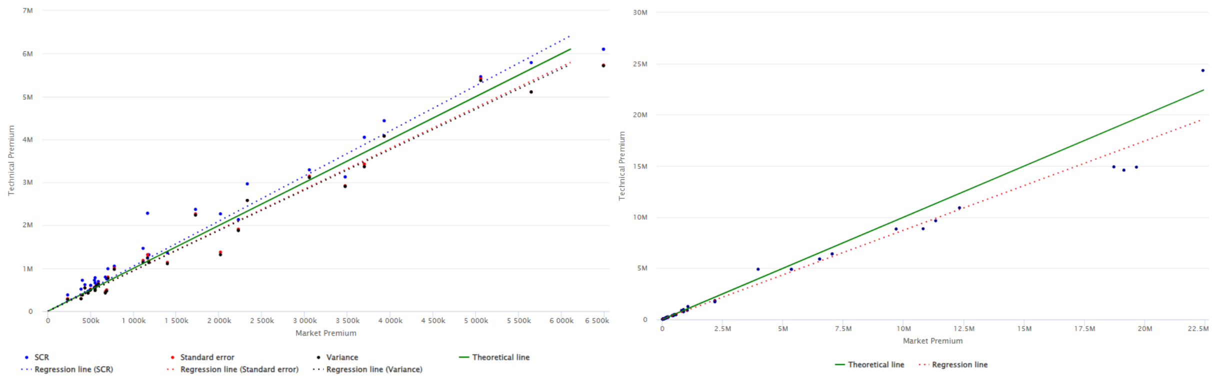
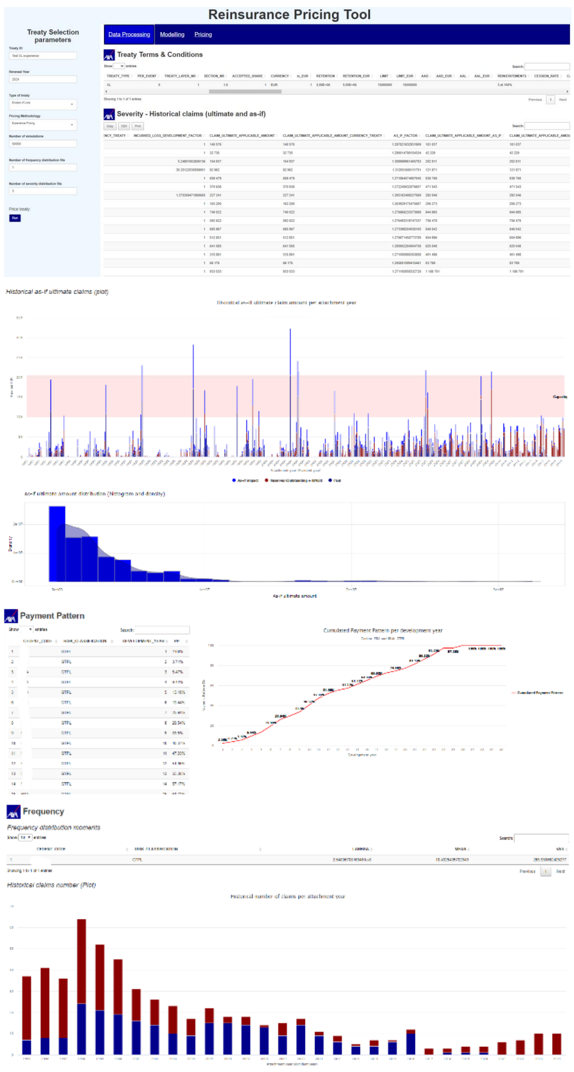
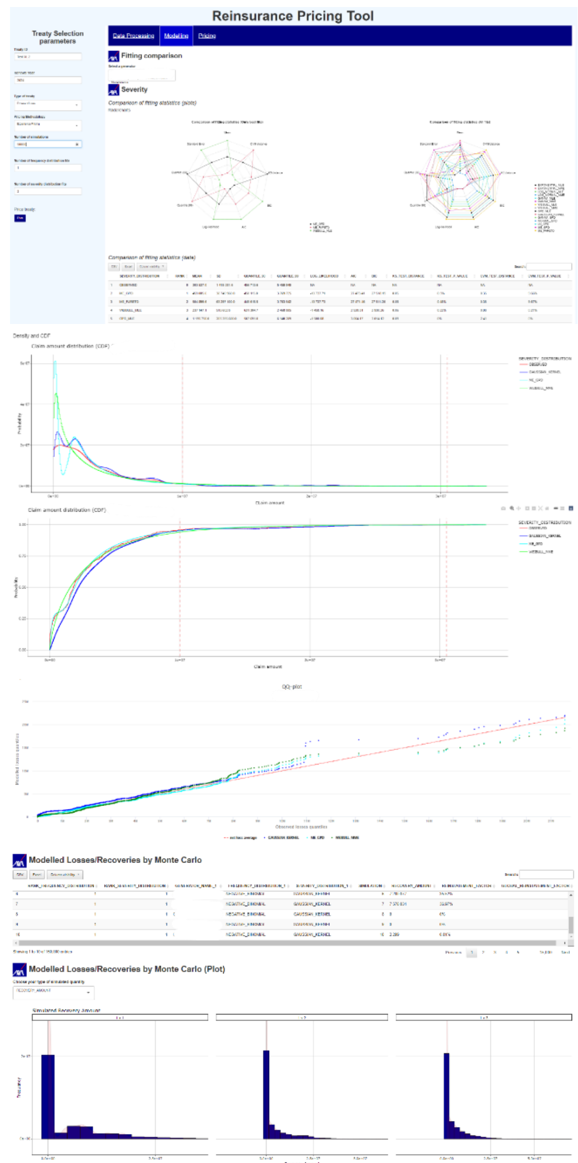


Figure 5 : Comparaison des primes techniques et primes marchés pour les traités non poolés (droite) et poolés (gauche)

In fine, cette étude aura permis de proposer une méthodologie de tarification s’appuyant sur les données disponibles applicable à l’échelle de l’équipe en proposant une première réponse à la problématique étudiée. En réunissant de manière centralisée un ensemble de méthodes et de processus, elle aura ainsi visé à élargir le spectre des traités pouvant être tarifés en fournissant un outil opérationnel à cet effet. Cet outil, composé de trois onglets, facilite l'automatisation et la transparence des processus impliqués dans l'évaluation des risques liés aux traités, dont la sélection des données, la projection des sinistres à l’ultime, l'ajustement des distributions et le calcul des indicateurs relatifs aux traités de réassurance. Conçu pour être interactif, il permet aux utilisateurs d'exporter les quantités estimées au cours du processus de modélisation, telles que les paramètres des distributions, les variables simulées, et de visualiser les différents indicateurs en fonction des choix de l'utilisateur. En outre, l'outil permet aux utilisateurs de spécifier le nombre de simulations et de distributions qu’il souhaite ajuster.

Table 1 : Aperçu de l'outil de tarification

Data Processing panel	Modeling panel	Pricing panel
		

Executive Summary

Often referred to as “insurance of insurers,” reinsurance is a risk distribution mechanism where a "ceding company" (an insurer) transfers all or part of its insurance liabilities risks to a third-party insurance company, the reinsurer. It takes the form of various types of contracts, known as reinsurance treaties. These treaties fall into four main categories: Excess of Loss (XL) per risk or per event, Aggregate (AGG), Quota-Share (QS) and Surplus (SP). These treaties enable risks to be transferred on a proportional or non-proportional basis.

As AXA Group's internal reinsurer, Axa Group Ceded Reinsurance (AGCRe) is responsible for managing all the group's reinsurance operations. To this end, AGCRe has several treaty management strategies at its disposal: it can choose to accept all treaties from entities and act as sole reinsurer, cede them to other reinsurers, or retain only a portion, which it places in a reinsurance pool. The figure below illustrates how reinsurance treaties are managed within AGCRe.

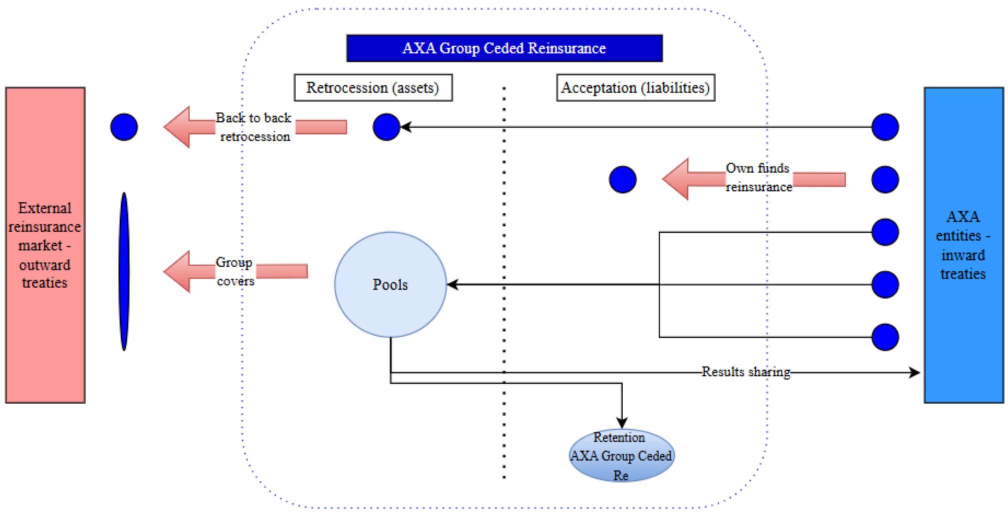


Figure 6: Diagram of AXA Group Ceded Re reinsurance commitments

In preparation for underwriting committees, the Analytics and Pricing team is responsible for providing various indicators relating to treaties in order to assess their profitability and risk. To this end, although the team is currently able to evaluate most treaties underwritten by AGCRe, around 20% of treaties are not modeled using the internal model, which mainly concerns XL treaties placed within reinsurance pools. QS, SP, AGG and certain XL treaties by risk are therefore currently subject to case-by-case studies, which tend to be tedious. Furthermore, the modeled treaties are based on AXA Group-wide guidelines and rely on external software which restricts the team's control over their modeling process. This dependence also reduces the team's ability to make full use of its own databases and prevents it from offering an internal view of the reinsurance treaties underwritten. This situation therefore raises the following question and sets the framework for this actuarial thesis: How can AGCRe, through a centralized, reliable, flexible, and fully controlled modeling approach, evaluate the risks associated with the largest number of reinsurance treaties using internal data available across all cedents and risk classifications?

By developing a methodology and a tool on R shiny for treaty’s risk assessment, we aim to increase treaty valuation capabilities by modeling treaties not included in the internal model scope in an automatic way, while also providing an additional view for underwriting committees. However, the study will be limited to non-life reinsurance treaties covering a one-year time horizon and will concern four types of reinsurance treaties underwritten by AXA: XL per risk, QS, AGG and SP[1]. The CAT Line of Business

[1] Only for the property LoB

(LoB) has not been considered for now, even though the tool is designed to integrate in the future event loss tables issued from any CAT models and thus allow per event reinsurance modeling and rating.

The tool developed and its architecture are based on three main modules, each one corresponding to major aspects of the underwriting methodology: a data selection and transformation module, a module for modeling the losses covered by the reinsurance treaty, and finally a module for the calculation of the indicators used for underwriting committees.

When estimating indicators, the first step is to select the databases required for treaty evaluation. Depending on the type of treaty to be priced, these databases will differ according to the models employed, which do not bear the same data requirements. We first have at our disposal all historical claims developments (paid and case reserves amounts) reported above certain thresholds by all AXA entities, related to all kinds of LoBs, and sometimes reaching an historical depth up to 30 years. For XL treaties, the individual claims experience is taken into account, while the aggregate claims experience is considered for AGG and QS treaties. To estimate the underlying loss experience as accurately as possible, it is necessary to evaluate the ultimate losses, whether individually or aggregated. To this end, the Chain Ladder method has been used to estimate the IBNR (Incurred But Not Reported) and therefore the ultimate loss amount associated with each claim. For AGG and QS treaties, the ultimate loss ratios will be modeled using the premium amounts that falls under the treaty perimeter. Concerning XL treaties, since individual claims experience is considered, the number of claims per year is a variable of interest. Given the limits of the purely multiplicative nature of Chain Ladder, the Schnieper model was chosen to estimate ultimate claims frequency. An as-if adjustment to take account of factors such as inflation was also carried out to adjust individual claim amounts. Finally, the structure of claims payments is estimated through a payment pattern for discounting purposes in the calculation of reinsurer's payments, which may extend over several years.

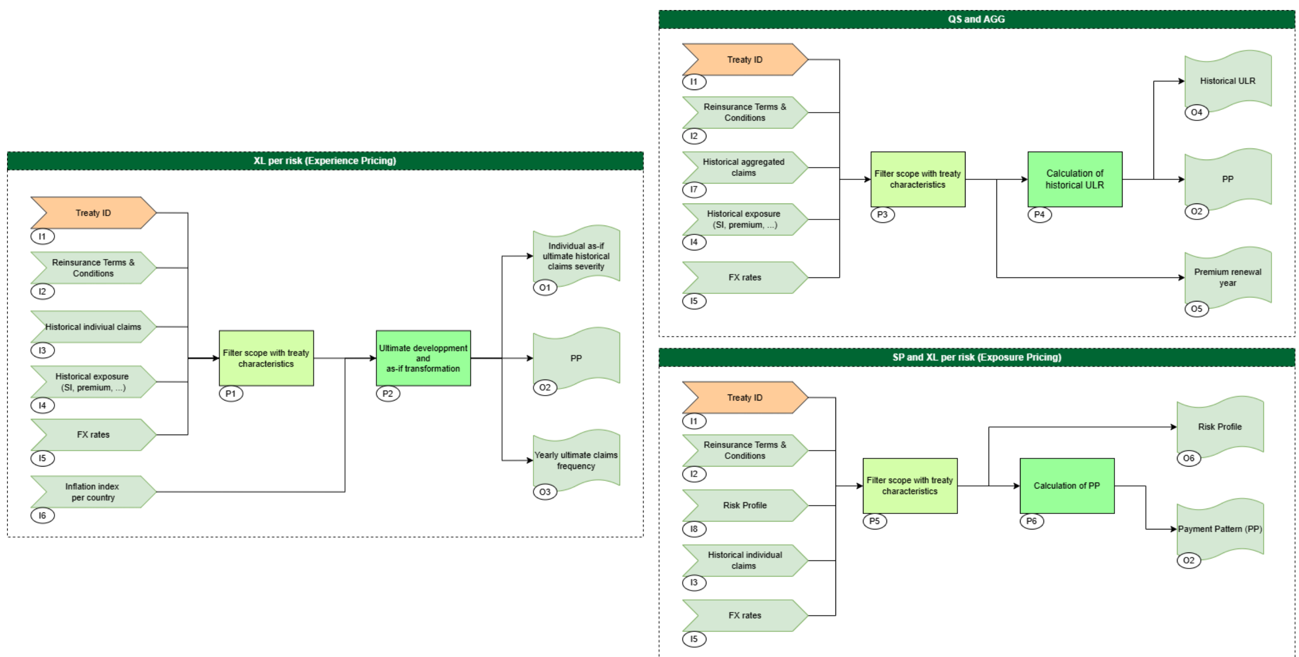


Figure 7: Data selection and transformation modules

The Monte Carlo method was chosen to model the underlying loss experience because of its flexibility. By providing a loss distribution that incorporates the impact of treaty conditions on recoveries, this method makes it possible to calculate a larger number of indicators. The methodology considered to model underlying losses is similar for XL per risk, AGG and QS treaties. It consists of estimating the parameters of possible distributions to model loss ratios or individual claims, selecting the most appropriate distribution, simulating the quantities of interest and, finally, applying the treaty conditions to obtain the losses ceded to reinsurance. For XL per risk treaties, a cost-frequency model is calibrated for each treaty. The distributions chosen are based on extreme value theory, including Pareto distributions, and others such as Mixed Erlang distributions. Splitted distributions have also been implemented to model distribution tails, and therefore extreme events which have a major impact on losses linked to XL treaties. Distributions are then ranked and evaluated to select the most appropriate. Frequency distributions is selected based on the dispersion coefficient of the ultimate number of claims among the Binomial distribution, the Poisson distribution and the Negative Binomial distribution. An alternative method has also been considered for XL property treaties based on exposure curves. The latter makes it possible to obtain a distribution of recoveries in cases where the collective model is unsatisfactory, notably due to too few claims or too high retentions. This methodology is also used to estimate ceded losses under a SP treaty, which were not previously modeled. For AGG and QS treaties, ultimate loss ratios are directly modeled using statistical distributions. Treaty conditions are then applied to the simulated losses, considering potential indexation clauses among other clauses, such as loss corridors for QS treaties.

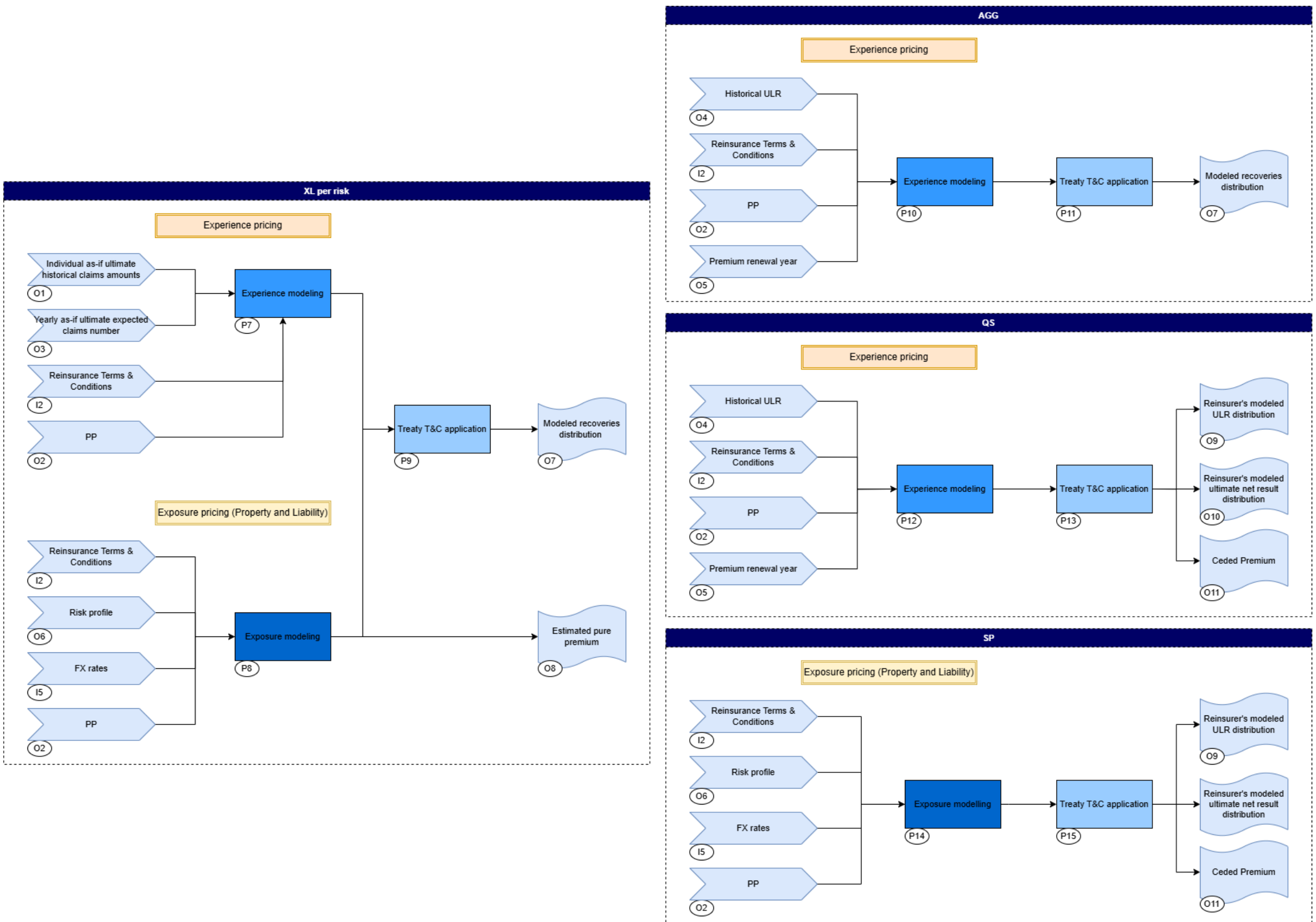


Figure 8: Modeling modules

Finally, quantitative metrics are directly derived from the recoveries or loss ratios distributions resulting from the previously described modeling process. For non-proportional treaties, such as AGG and XL treaties, this includes exceedance probabilities, return periods and various statistical indicators related to loss distributions. For these types of treaties, the technical premium is also an indicator of interest in treaty valuation. The latter requires the calculation of safety loadings to cover the reinsurer in the event of extreme losses. An approach based on the treaty's cost of capital has therefore been considered to evaluate these safety loadings. The calculation method differs according to whether the treaty is pooled or not. If the treaty is placed within a pool, safety loadings are calculated using the treaty's share within the pool in terms of variance, standard deviation or quantile. Otherwise, the safety loading is related to the treaty's marginal impact on the Solvency Capital Required (SCR) of AXA SA's non-life branch. Considering QS and SP treaties, which do not require the calculation of a technical premium, we carried out the simulation of ceded losses, net underwriting result, cost of capital, commissions, and other potential loss dependent features.

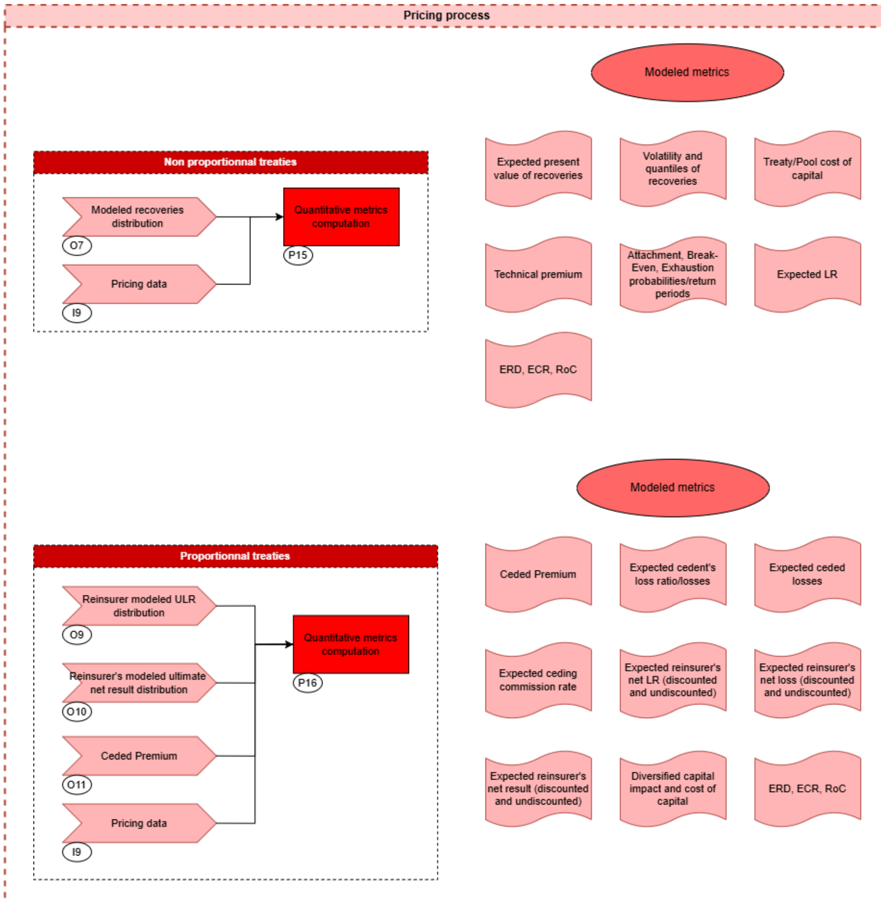


Figure 9: Pricing modules

To assess the performance of the developed tool, a comparison was made with market premiums for XL treaties. The aim of this analysis was to assess whether the proposed methodology is reliable and enables the technical premium to be estimated appropriately. This study, based on a set of 90 treaties, shows that the methodology provides relatively consistent values. Regarding pooled treaties, the deviations observed are of the order of 5%, reflecting a good match with the market. On the other hand, for non-pooled treaties, the technical premium is underestimated by around 13% compared with market premiums.

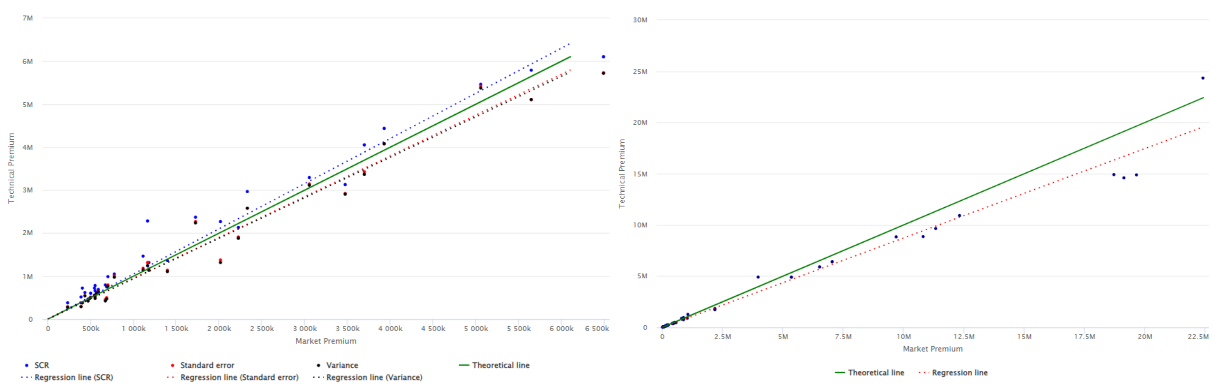
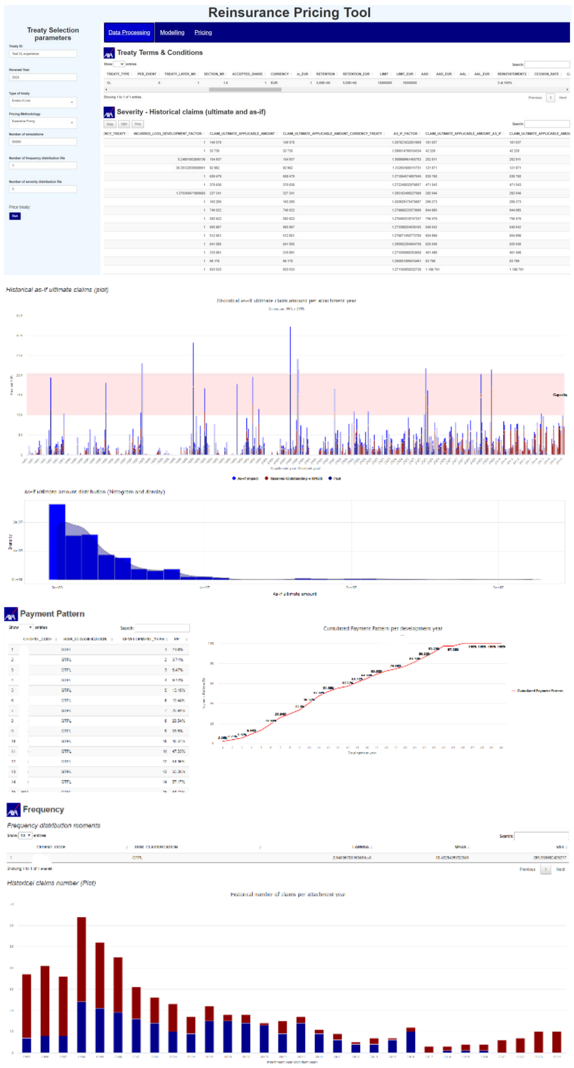


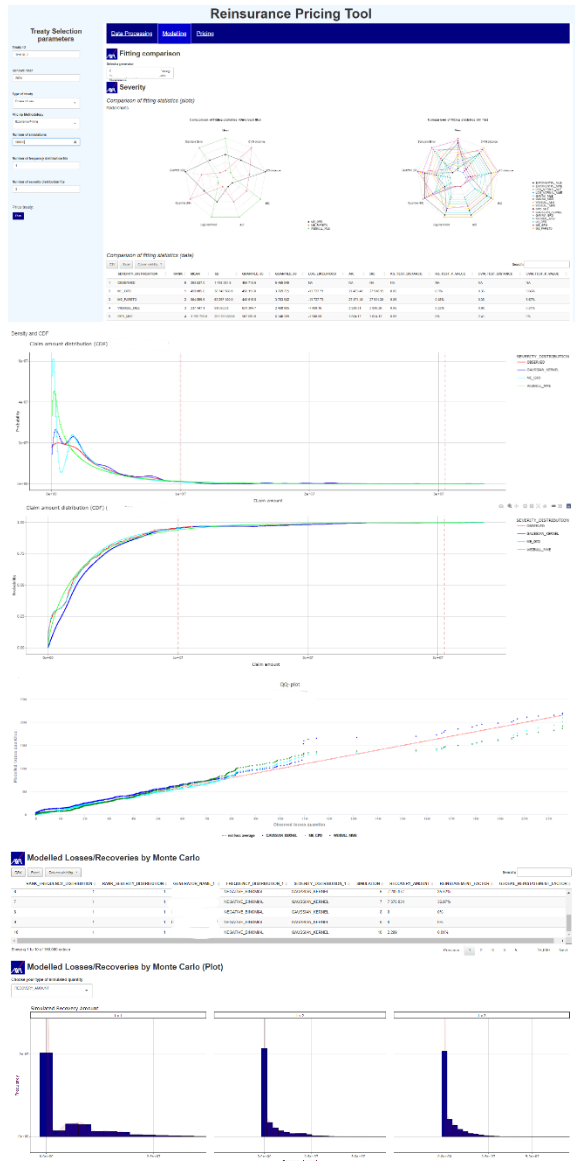
Figure 10: Comparison of technical and market premiums for non-pooled (right) and pooled (left) treaties

Ultimately, this study will have enabled us to propose a pricing methodology based on available data, applicable across the entire team and thus offering an initial response to the problem studied. By bringing together a range of methods and processes in a centralized manner, we broaden the spectrum of treatments that can be priced, by providing an operational tool for this purpose. This tool, composed of three panels, facilitates the automation of processes involved in treaty risk assessment, including data selection, ultimate projection, fitting of distributions and treaty's indicators calculations. Designed to be interactive, it allows users to export estimated quantities during the modeling process, such as distribution fitting parameters, simulated variables, and to plot treaty indicators based on the user's selections. Additionally, the tool enables users to specify the number of simulations and the number of fitted distributions.

Table 2: Overview of the reinsurance pricing tool

Data Processing panel	Modeling panel	Pricing panel





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Glossary

AMSE: Average Mean Squared Error
AGCRe: AXA Group Ceded Reinsurance
AGG: Aggregate
CDF: Cumulative Distribution Function
CoC: Cost of Capital
CPI: Consumer Price Index
CPPI: Construction Producer Prices Index
CVM: Cramer Von Misses
ERD: Expected Reinsurer's deficit
GEP: Gross Earned Premium
GEV: Generalized Extreme Value
GLR: Gross Loss Ratio
GPD: Generalized Pareto Distribution
GWP: Gross Written Premium
HTE: Heavier Than Exponential
KS: Kolmogorov-Smirnov
LoB: Line of Business
LR: Loss Ratio
LTE: Lighter Than Exponential
ME: Mixed Erlang
MLE: Maximum Likelihood Estimation
MME: Method of Moments Estimation
OOP: Object Oriented Programming
PP: Payment Pattern
QS: Quota-Share
RoC: Return on Capital
SCR: Solvency Capital Required
SI: Sum Insured
SP: Surplus
T&C: Terms and Conditions
TP: Technical Premium
ULR: Ultimate Loss Ratio
VaR: Value at Risk
XL: Excess of Loss

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Introduction

Reinsurance serves as a strategic cornerstone in the insurance industry, enabling companies to access higher levels of risk diversification pools and mitigate their exposure to risks while ensuring financial stability. Obligatory reinsurance takes the form of various types of contracts, known as reinsurance treaties. These treaties fall into four main categories: Excess of Loss (XL) per risk or per event, Aggregate (AGG), Quota-Share (QS) and Surplus (SP). These treaties enable risks to be transferred on a proportional or non-proportional basis.

Following the recent merger between AXA Global Re and AXA SA, AXA Group Ceded Reinsurance (AGCRe) has taken on a significant role in managing reinsurance for the entire AXA Group. Acting both as an internal reinsurer and as a purchaser of reinsurance on the international market, it plays a vital part in optimizing reinsurance programs and managing the Group's overall risk portfolio through reinsurance. Given the growing imperative to retain profitability while arbitrating with risk appetite, a detailed and comprehensive approach to modeling reinsurance treaties is essential. The ability to deliver accurate pricing by incorporating a wide range of relevant indicators and tailoring evaluations to optimize reinsurance structures is a critical asset for meeting AXA's strategic objectives under the constraints of reducing the net result volatility and complying with AXA risk appetite limits. In this context, the Analytics & Pricing team is tasked with modeling reinsurance coverages to evaluate the ceded risks under reinsurance treaties through the generation of several risk and profitability indicators.

While existing pricing models within the organization provide some capability in this regard, they are largely based on directives established at the group level. As a result, they do not allow AGCRe to have its own perspective on the ceded risks associated with underwritten reinsurance treaties. Developing a pricing methodology tailored to the entity's available data would allow AGCRe to diversify its approach and offer a complementary view of ceded risks through reinsurance treaties in addition to market and AXA Group perceptions. Moreover, the current models give the entity only limited control over the modeling processes. This lack of transparency and flexibility can become problematic when it comes to incorporating modifications, such as new treaty clauses, the addition of newly subscribed treaties, or the integration of alternative modeling methods. This issue highlights a key challenge explored in this actuarial thesis: the establishment of an own pricing methodology that relies on the entity's data and grants the team full ownership of the modeling process, from end to end. Furthermore, the models currently employed within AGCRe primarily focus on XL treaties and do not encompass the entirety of Lines of Business (LoBs) or cedents. As a result, approximately 20% of the treaties underwritten by AXA are not assessed by the internal model and are instead handled on a case-by-case basis on demand, which tends to be tedious. This limitation raises a critical second challenge: the need to establish a centralized modeling approach for treaties currently excluded from the internal model.

These issues lead to the formulation of a broader problem statement: How can AGCRe, through a centralized, automated, flexible, and fully controlled modeling approach, evaluate the risks associated with the largest number of reinsurance treaties using internal data available across all cedents and risk classifications?

Given the vast possibilities for development, we chose to be pragmatic in choosing the treaties to be studied given operational considerations, as well as the choice of models to be implemented. To this end, the study will be limited to non-life reinsurance treaties, as life reinsurance treaties constitute less than 3% of AXA's total reinsurance portfolio. In this work, we will try to give a first answer to this problem by assessing four types of reinsurance treaties that are partially or not modeled with the internal model such as XL per Risk, QS, AGG and SP[1] treaties. Treaties within the CAT LoB will not be examined in this paper, as their modeling is significantly more complex and falls under the purview of other specialized teams.

To meet this challenge, the process involved standardizing, centralizing and refining the data needed to evaluate different types of reinsurance treaties. It also involved examining feasible existing pricing methods based on the data collected and the specific risk and profitability indicators to be calculated in order to identify the most appropriate approach. Depending on the type of treaty

[1]Only those covering the property LoB

and pricing method chosen, some methodologies require risk profiles, while others are based on historical claims, either aggregated or individual for instance. Furthermore, the data format had to strike a balance between sufficient granularity to ensure both modeling reliability and compatibility between all entities. Determining the necessary transformations to be applied to the data to meet the modeling requirements such as ultimate loss calculations, computing moments for the number of claims, or calibrating payment patterns as part of the methodology process, represents one of the added values of the project. Additionally, proposing a broader variety of modeling methods, including several probability distributions displayed with an automated ranking, as well as distributions derived from exposure curves enabling an alternative modeling approach for XL per risk treaties and an initial modeling method for previously unmodeled SP treaties, along with corresponding graphs, intermediary datasets and fitting statistics to assess model validity, provides the team with improved control over the modeling process. Furthermore, uniformizing the calculation of risk and profitability indicators, as well as incorporating additional risk metrics beyond those currently used by the team, are integral components of the study.

This actuarial work will thus address the problem at hand by presenting a forward-looking pricing method from end to end based on statistical models. This methodology has been incorporated and centralized through the entire development of an automated and user interactive R shiny app for assessing reinsurance contracts, which is one of the main added values of this work, and part of the answer to the question at hand. It was designed for the Analytics & Pricing team and integrates all the necessary data, pricing methodologies and processes into a coherent framework, supported by a clear and structured code architecture. The proposed tool aims to expand the application scope of pricing methods, to include a wider array of reinsurance treaties while ensuring greater flexibility, transparency and the ability to integrate new data from multiple sources, both internal and cedent-provided. It thus intends to broaden and deepen the analysis of reinsurance treaties within the group while increasing AGCRe's internal pricing capabilities and operational efficiency in treaty evaluation. It also provides an alternative view of the risks underwritten, compared with the view based on the internal model and that based on market prices.

This paper will start with a description of the diverse types of reinsurance treaties and their distinct characteristics. It will then detail the necessary data requirements, and the transformations applied to prepare the data for modeling. Subsequently, the methodology and steps for modeling the underlying claims covered by reinsurance treaties will be presented. Risk and profitability indicators will then be described, followed by a comparison with market premiums for a certain type of treaty to evaluate the performance of the pricing tool.

1. Reinsurance

1.1 Definition

Reinsurance was initiated in France by Colbert in 1668 with the creation of the *Compagnie générale des assurances et grosses aventures de France*, a joint association of insurers and insureds covering the value of goods in the event of loss or damage during maritime transport. However, reinsurance was mainly developed in Germany towards the end of the 19th century, fueled by the industrial revolution and the growing need for insurance. The founding in 1846 of *Kölnische Rück*, now known as Gen Re, marked the growth of specialized reinsurance firms. Over time, other reinsurance companies emerged, extending their coverage across various lines of insurance on a global scale. Today, reinsurance predominantly operates in developed countries, with most reinsurance companies based in nations such as Germany, Switzerland, the UK, the USA and France.

Often referred to as "insurance of insurers," reinsurance is a risk distribution mechanism where a ceding company (an insurer) transfers all or part of its insurance liabilities risks to a third-party insurance company, the reinsurer. This operation is formalized through a reinsurance contract or treaty, under which the reinsurer, in exchange for a reinsurance premium, undertakes to indemnify the ceding company for covered losses. Reinsurance contracts can vary in format and may have fixed or open-ended durations, though most are valid for one year.

Reinsurance now occupies a significant place in the insurance industry, and its influence continues to grow. Insurance companies are progressively leveraging this mechanism due to the numerous benefits it offers. The main advantages for the ceding company are:

- Risk Transfer: By transferring a portion of its risks - particularly those that are highly volatile or less predictable -to a reinsurer, the cedent reduces its probability of financial failure.
- Reduced Capital Requirements: Reinsurance allows insurers to lower their retained risk exposure, resulting in reduced regulatory capital requirements for the ceding company.
- Income Stabilization: Reinsurance helps insurers smooth their financial results by absorbing significant and unexpected losses. This reduces the variability of claims payouts and can stabilize cash flows.
- Enhanced Underwriting Capacity: Some reinsurance treaties such as Quota-Share enable insurers to expand their underwriting capacity by compensating for insufficient equity capital, thereby allowing them to take on additional policies.

Reinsurance can take various forms and have specific characteristics. The following section describes these features.

1.2 Reinsurance characteristics

1.2.1 Reinsurance types

Reinsurance can be categorized into three main types:

- **Obligatory reinsurance** takes the form of a reinsurance treaty under which the reinsurer must cover all insurance policies issued by the ceding company that fall within the scope of the treaty.
- **Facultative reinsurance**, in the form of a contract, is negotiated separately for each insurance policy. Ceding companies generally purchase facultative reinsurance for individual risks that are not sufficiently covered by their compulsory reinsurance treaties. It is called facultative because neither the ceding company nor the reinsurer is forced to accept the contract.

- **Facultative/Obligatory reinsurance** is a type of reinsurance contract in which the insurer is free to choose which risk will be ceded to the reinsurer, as in facultative reinsurance. However, the reinsurer must accept the risk if ceded.

It is then necessary to determine which risks will be covered by the reinsurance contracts, particularly from a temporal perspective, leading to the definition of the coverage period.

1.2.2 Coverage period

The period of coverage is the period during which the reinsurer is responsible for claims arising from policies ceded during the treaty period of effect. Two types of coverage periods are considered in reinsurance:

- Under a **"Loss Occurring"** treaty, the reinsurer covers any claims that occur during the validity period of the treaty, regardless of when the underlying insurance contract was issued. For instance, if a Loss Occurring reinsurance treaty covers the year 2024, it will cover any claim occurring in 2024, even if the underlying insurance policy was underwritten in 2023 or earlier.
- On the other hand, in a **"Risk Attaching"** treaty, the reinsurer covers claims associated with policies that are underwritten or renewed during the treaty's validity period, regardless of when the claim itself occurs. For example, if a Risk Attaching treaty covers the year 2024, the reinsurer will assume all claims related to policies issued or renewed in 2024, even if those claims arise in 2025 or beyond.

1.2.3 Reinsurance treaties

To meet the diverse coverage needs of insurance companies, reinsurers offer two primary forms of reinsurance: **proportional reinsurance** and **non-proportional reinsurance**.

Proportional reinsurance is based on a proportional sharing of the loss between the insurer and the reinsurer. There are two types of proportional reinsurance treaties:

- Quota-Share (QS)
- Surplus (SP)

Non-proportional reinsurance, unlike proportional reinsurance, is based on a different mechanism of risk-sharing between the reinsurer and the insurer. In non-proportional reinsurance, the reinsurer only intervenes when the insurer's losses exceed a predefined threshold, known as the retention or priority, up to a certain limit. In exchange for this coverage, the insurer pays a fixed annual premium. There are two types of proportional reinsurance treaties:

- Excess of Loss (XL)
- Stop Loss or Aggregate (AGG)

The figure below summarizes the different reinsurance types:

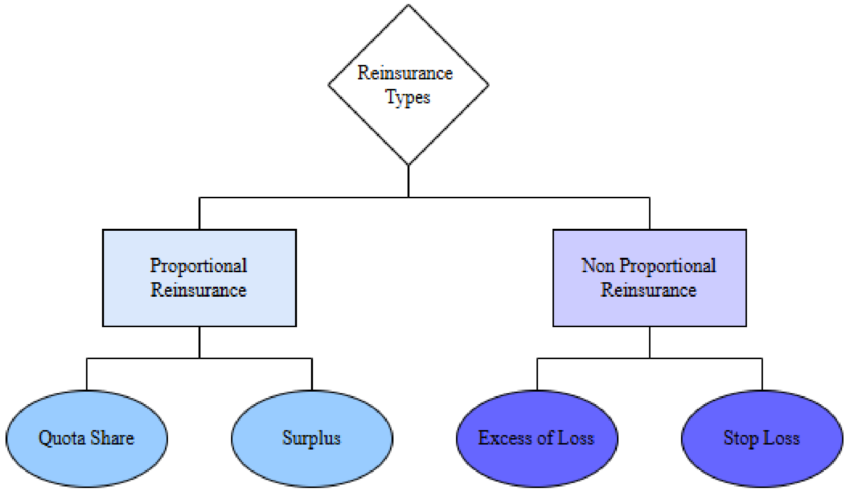

Figure 11: Classification of reinsurance treaties

Let us examine the different types of treaties.

1.2.3.1 Proportional treaties

1.2.3.1.1 Quota-Share (QS)

Under a QS treaty, the reinsurer agrees to cover a percentage of the ceding company's claims portfolio, based on a specified cession rate. In exchange, the reinsurer receives a proportional share of the premiums collected by the ceding company, along with a commission fee. This type of reinsurance applies to all risks within the cedent's portfolio, regardless of the sum insured. QS treaties are often used when companies are setting up to increase their underwriting capacity and gain experience in a new market with limited risk.

Mathematically, if one considers a QS reinsurance treaty with:

- A cession rate c
- The annual premium amount collected by the insurer P
- The annual claim amount attached to the treaty S

Then:

- $(1 - c)P$ corresponds to premium retained by the ceding company while the reinsurer receives an amount of premiums equal to $c * P$
- $(1 - c)S$ corresponds to the claim amount retained by the ceding company while the reinsurer is granted with an amount of claims equal to $c * S$

The following figure presents the amounts ceded to reinsurance in the case of a QS treaty with a 40% cession rate.

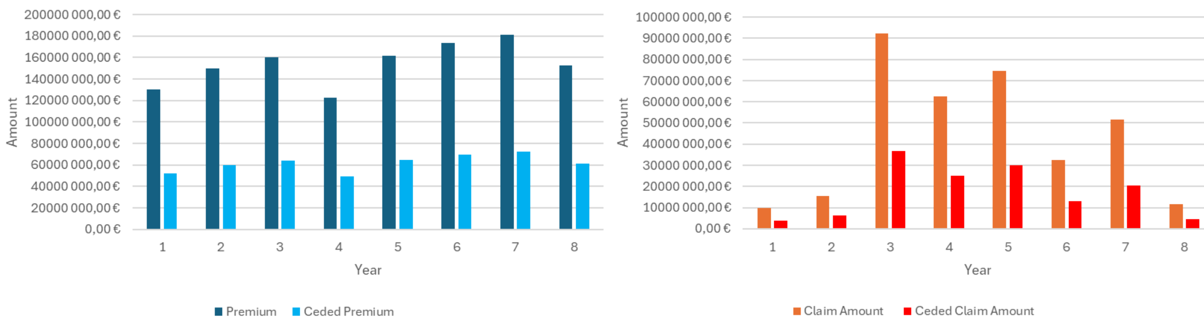

Figure 12: Comparison of retained/ceded premium and claim amounts for a QS 40%

A major advantage of quota-share reinsurance is that it simplifies the insurer's management and offers the opportunity to diversify its activities. This form of reinsurance also plays a key role in protecting the insurer's equity by (virtually) increasing it. Indeed, in a simplified way, the probability that the insurer will not go bankrupt can be written as $\mathbb{P}(w + P - S > 0)$ where w corresponds to the insurer's equity. If the ceding company has subscribed a QS treaty with a cession rate c, this probability becomes:

$$\mathbb{P}(w + (1-c)P - (1-c)S > 0) = \mathbb{P}\left(\frac{w}{1-c} + P - S > 0\right) \geq \mathbb{P}(w + P - S > 0)$$

However, this reinsurance type has its drawbacks, particularly in terms of ceded premiums, which can sometimes be significant. This can lead to a loss of potential profitability for the ceding company, as it may be transferring risks that could have been assumed without external help. Additionally, because the reinsurer's share of premiums and claims is proportional, it does not impact the ceding company's loss ratio. The ceding company remains exposed to fluctuations in claims frequency and severity, which can still affect its financial stability and profitability.

1.2.3.1.2 Surplus (SP)

A second type of proportional reinsurance contract is the surplus treaty. It is a proportional reinsurance agreement in which the ceding company and the reinsurer share both claims and premiums based on a cession rate. However, the key difference between SP and QS lies in the definition of the cession rate, which is not explicitly fixed in SP treaties. Furthermore, surplus reinsurance is generally applied on a certain risk category, or group of risks basis within a particular branch.

In this type of contract, the ceding company defines a retention amount R and a capacity C, which is the maximum amount that the reinsurer is willing to insure. The reinsurer assumes claims and collects premiums from the cedent based on an implicit cession rate c, which is determined by the sum insured, the retention, and the capacity.

Mathematically, by noting SI_i the sum insured of the risk i, R the retention, c_i the cession rate and C capacity, the cession rate c_i is:

$$c_i = \frac{\min\left(\max(0, SI_i - R), C\right)}{SI_i}$$

The premiums and claims associated with the risk will then be shared between the ceding company and the reinsurer according to this cession rate.

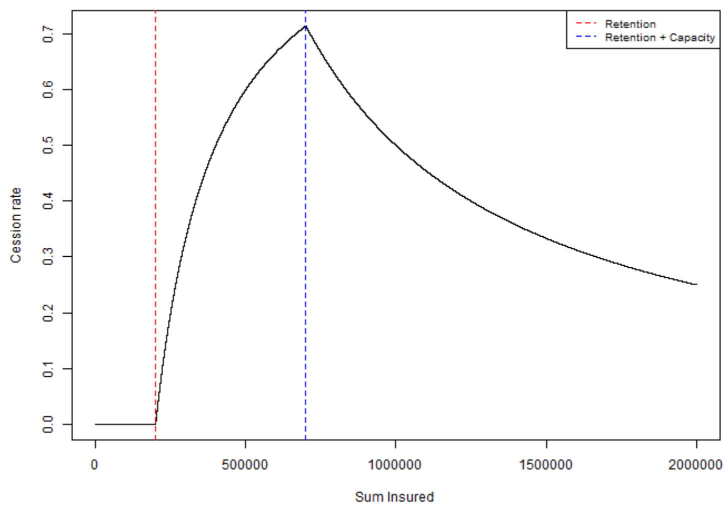

Figure 13: Cession rate evolution of a SP treaty

Figure 13 exhibits the evolution of the cession rate for a surplus treaty with 200 000 € retention and 500 000 € capacity. As the sum insured rises from 200 000 € towards 700 000 €, the cession rate increases sharply. For a sum insured of 700 000 €, the cession rate reaches its peak at 70%. After 700 000 €, the cession rate begins to decrease slowly: the ceding company retains a higher portion of the risk, and the reinsurer's share of the risk becomes proportionally smaller.

1.2.3.2 Non-proportional treaties

1.2.3.2.1 Excess of Loss (XL)

Excess of Loss treaties trigger the reinsurer's intervention when losses surpass a predetermined amount, known as the retention, and continue up to a specified limit, which is the maximum commitment of the reinsurer in the event of a claim. There are two main types of Excess of Loss treaties:

- **Excess of Loss per risk:** the reinsurer intervenes when a claim exceeds the retention threshold. This type of coverage is typically used for losses tied to a single risk, such as a fire affecting one insured property.
- **Excess of Loss per event:** this kind of treaty allows for the aggregation of a generally large number of claims caused by the same event. For example, in the case of a natural disaster, multiple claims can be accumulated. The reinsurer intervenes when the total amount of these claims exceeds the retention level.

XL treaties are particularly useful for protecting against catastrophic losses, as only significant claims are likely to exceed the retention and trigger the reinsurer's coverage.

By noting L the limit, R the retention, X the amount declared by the cedent, the ceded amount C_{reins} to the reinsurer for an XL treaty $L\ xs\ R$ will be:

$$C_{reins} = \min((X - R)_+, L)$$

For example, the *Figure 14* points out the allocation of losses between the cedent and the reinsurer for an XL 100 € M xs 50 € M. Regarding losses below 50 € M, the cedent bears the entire loss amount, as this falls within the retention. Concerning losses between 50 € M and 150 € M, the cedent covers the first 50 € M, while the reinsurer assumes responsibility for the portion above 50 € M, up to a maximum of 100 € M. In cases where losses exceed 150 € M, the reinsurer still only covers 100 € M (its maximum liability under the treaty), and the cedent is responsible for the retention (50 € M) and any amount exceeding 150 € M. For example, a 200 € M loss would result in the cedent paying 50 € M (retention) plus 50 € M (excess beyond the reinsured layer), while the reinsurer pays 100 € M.

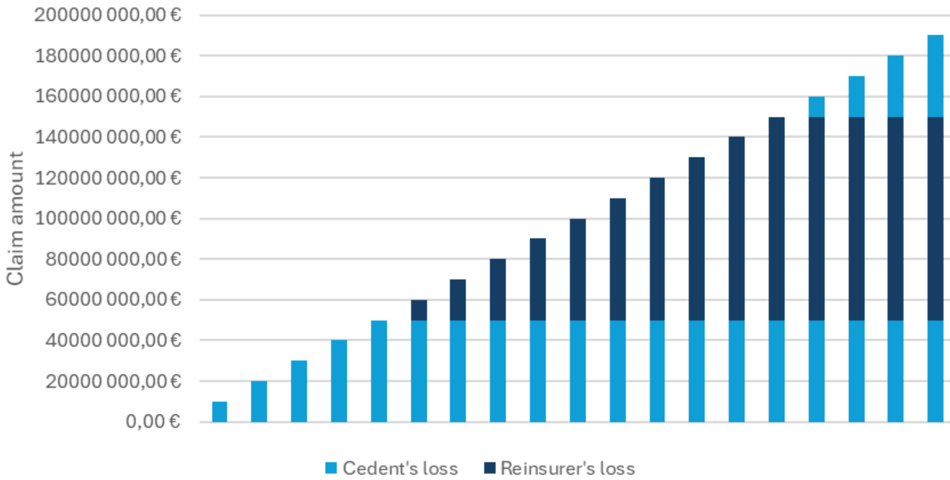

Figure 14: Example of an XL treaty 100 € M xs 50 € M

This type of treaty often includes additional clauses. The main ones are the Annual Aggregate Limit (AAL) and the Annual Aggregate Deductible (AAD). The AAL is the reinsurer's annual maximum coverage obligation towards the cedent. The AAL thus offers financial protection to the reinsurer by limiting its maximum exposure. The AAD is the total aggregate amount of losses within the layer that the ceding company must retain before the reinsurer intervenes. Once the insurer's total losses exceed the AAD, the reinsurer begins to pay for the claims, up to the AAL.

Another component frequently found in XL treaties is the reinstatement premium: when the amount of claims borne by the reinsurer consume the capacity, it is possible to "reinstate" the capacity through the payment of a so-called reinstatement premium. The number of reinstatements is defined contractually and is noted as "number of reinstatement@rate", where the "rate" designates the reinstatement premium price. The reinstatement premium is paid as soon as a claim consumes part of the scope, in proportion to the amount of the scope consumed, sometimes including a time factor based on the number of days remaining until the end of the contract, which reduces the reinstatement premium. For instance, let us consider a 100 € M xs 50 € M treaty with an upfront premium of 50 € M and the following reinstatement conditions: 1@50%, 2@100%. Let us assume the annual recovery amount is 175 M €. Since the treaty capacity has been entirely consumed, the cost of the first reinstatement is 50% of the treaty premium, amounting to 25 € M. The second reinstatement costs 100% of the treaty premium, prorated to the remaining capital to be reinstated (75/100), which results in 75 €M. Consequently, the cedent will pay an upfront premium of 50 € M and a total of 100 € M in reinstatement premiums.

XL treaties may also contain indexation clauses. An indexation clause manages the effect of inflation by adjusting the retention and limit so that the inflation effect is equally shared between the cedent and the reinsurer. This clause helps maintain the relevance of retention and limit during periods of economic fluctuation. The indexation clause therefore specifies a reference index, which may be a consumer price index (CPI), a construction cost index, or any other relevant index linked to the covered risk. The clause also defines how retention and limit adjustments are made. For example, let us take an XL treaty per risk 100 € M xs 100 € M. In the case where the indexation clause is contractually based on a given country CPI issued by a specified institution, and that index increases by, say, 2% over a given period, the retention and limit could be increased by 2%. In this project, by denoting $X_{k,t}$ the payment of claim k at time t, R the retention and L the limit of the treaty, $v_{indexation,t}$ the deflating factor for payment made at time t, the indexed limit $L_{k,T}^{indexed}$ and the indexed retention $R_{k,T}^{indexed}$ for the claim k at time T are calculated as follows:

$$L_{k,T}^{indexed} = L * \frac{\sum_{t=1}^{T} X_{k,t}}{\sum_{t=1}^{T} X_{k,t} * v_{indexation,t}}$$

$$R_{k,T}^{indexed} = R * \frac{\sum_{t=1}^{T} X_{k,t}}{\sum_{t=1}^{T} X_{k,t} * v_{indexation,t}}$$

A concrete application of the indexation clause an XL treaty will be provided in section 5.1.3.1.

1.2.3.2.2 Stop Loss or Aggregate

Stop-loss, or aggregate, is a form of non-proportional reinsurance. This type of treaty covers all claims whose aggregated amount exceeds a retention threshold, up to a limit. Unlike XL treaties, where retentions and limits are defined in financial terms, a stop-loss treaty uses the loss ratio - claims as a percentage of earned premiums - to define its retention and limit. If, at the end of the treaty period, the cedent's loss ratio exceeds the retention, the reinsurer intervenes to cover the excess losses, reducing the cedent's loss ratio back to the retention level, but only up to the maximum limit established by the treaty's terms and conditions.

Mathematically, by noting P the covered portfolio premium, $R_{\%}$ the retention and $L_{\%}$ the limit (as a percentage of P), S the annual loss experience, the amount paid by the reinsurer underwriting an aggregate treaty $L_{\%}\ xs\ R_{\%}$ will be:

$$C_{reins} = P * \min\left(L_{\%}, \max\left(\frac{S}{P} - R_{\%}, 0\right)\right)$$

For example, let us assume that an insurer has taken out a stop-loss 50% xs 110%. The insurer's premiums for the year is 100 € M, and the insurer's loss ratio is 140%, meaning the claims incurred total 140 € M. Under the terms of the treaty, the reinsurer is responsible for covering the portion of the loss ratio that exceeds the 110% retention point. Therefore, the reinsurer will cover the difference between the 140% loss ratio and the 110% retention, which is 30%. As a result, the reinsurer will pay 30 € M, effectively reducing the cedent's loss ratio from 140% to 110%.

Stop-Loss enables smoother results and stabilize annual results as other forms of reinsurance, but it is mainly used to hedge against frequency risk due to its aggregate nature. For instance, in the case of storms, although each individual storm may not result in catastrophic losses, the cumulative loss experience of storms over a year could lead to significant losses. In this case, stop-loss comes into play to stabilize annual results by covering excess losses over the set threshold.

The characteristics of reinsurance and the various forms it can take having been described, let us now present the objective of the study and the context in which it took place.

2. Context of the study

2.1 AXA Group Ceded Reinsurance

AXA is a French insurance company founded in 1816 under the name *Mutuelle de l'Assurance contre l'Incendie*. AXA is one of the world's largest insurance companies with over 94 million customers worldwide, and total revenue reaching 102.7 € billion as of December 31, 2023.

AXA's reinsurance operations are centralized through AXA Group Ceded Reinsurance (AGCRe), which resulted from the recent merger between the former AXA Global Re entity and AXA's holding company AXA SA. The primary responsibilities of AXA Group Ceded Reinsurance include:

- Defining the AXA Group's reinsurance strategy.
- Analyzing risks within the Group's insurance portfolios.
- Protecting the balance sheets of AXA entities.
- Ensuring the security of the Group's insurance operations.
- Implementing solutions to optimize internal capacities and transfer risks to external markets.

To do so, AXA Group Ceded Reinsurance functions both as an internal reinsurer for all AXA Group entities and as a buyer of reinsurance on the global market.

At the operational level, when a reinsurance treaty needs to be underwritten or renewed for one of its entities, AGCRe employs various mechanisms:

- **Back-to-back reinsurance:** The risks underwritten by AGCRe are transferred to the external reinsurance market without retention.
- **Own funds reinsurance:** AGCRe assumes full responsibility for the risks and obligations associated with the reinsurance treaty. It acts as the sole reinsurer for the treaty, taking on all the associated risks without transferring them to external reinsurers.
- **Reinsurance Pooling:** With reinsurance pooling, a portion of the treaty is reinsured by AGCRe, while the remaining portion is placed into a reinsurance pool. This pooling operation increases AGCRe's capacity to cover risks it might not be able to assume individually. AXA Group Ceded Reinsurance's pools are covered by external reinsurers through "Group covers" reinsurance treaties. The results of the pools are also shared with ceding entities through QS treaties.

The figure below presents a schematic representation of the reinsurance process within AGCRe.

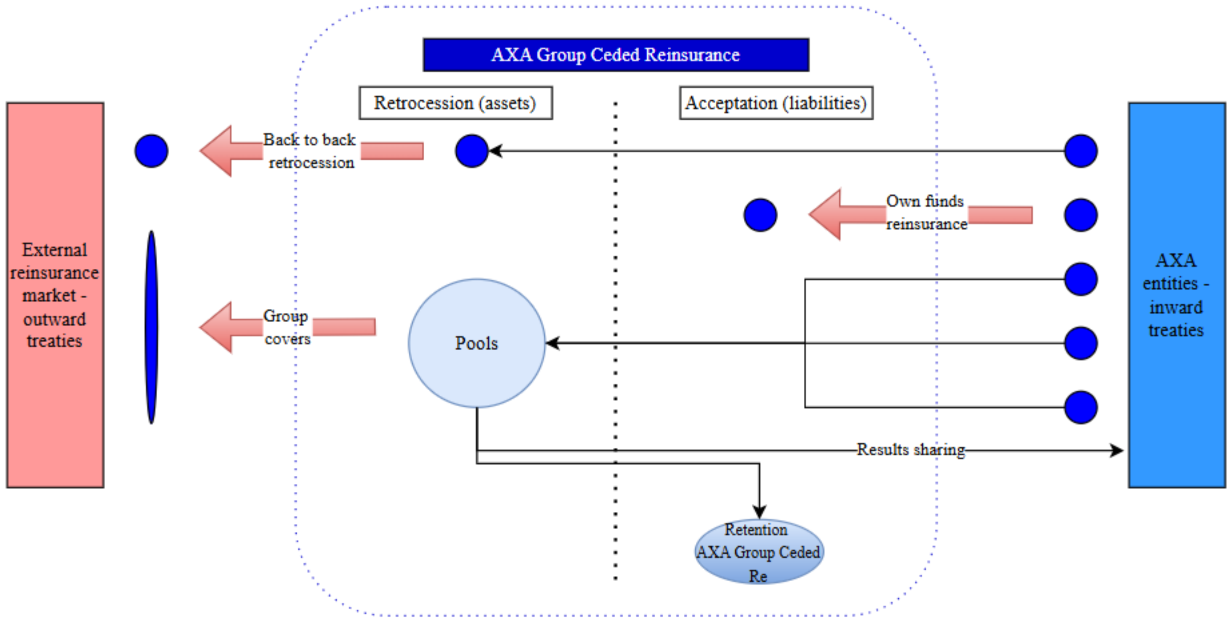

Figure 15: Diagram of AGCRe commitments

The reinsurance treaties detained by AGCRe concern six major lines of business, which are the six "Lines of Business" (LoB) of the reinsured insurance policies:

- **Property**: Property LoB covers all insurance policies related to property-related risks, such as fire, theft, and certain types of weather damage.
- **Motor Third Party Liability (MTPL)**: MTPL LoB groups together reinsurance treaties whose underlying insurance policies protect both the owner and driver of a vehicle against third-party liability claims in the event of damage caused by the vehicle. It can also offer protection against vehicle theft and damage caused by events other than traffic accidents.
- **General Third-Party Liability (GPTL)**: GTPL LoB concerns general liability insurance policies covering claims or damage imposed to life, health or property of third parties because of events related to the insured's activities.
- **Marine**: Marine LoB covers insurance for physical loss or damage related to maritime and transport activities.
- **Life**: Life LoB encompasses reinsurance treaties covering life insurance policies which are designed to provide financial security to policyholders' beneficiaries in the event of death, critical illness, disability, etc.
- **Cyber**: Cyber LoB focuses on reinsurance treaties for cyber insurance policies. These policies protect against risks like data breaches, cyberattacks, and other digital threats that could lead to economic loss.

2.2 Project objective and structure

To support its operations, AGCRe needs to accurately assess the reinsurance treaties it manages. This task falls to the Analytics & Pricing team, which is responsible for modeling reinsurance treaties and assessing their profitability through the generation of several indicators. However, the current models used by the team to assess reinsurance treaties are calibrated at group level. As a result, these models often lack flexibility and transparency concerning the modeling process and do not allow AGCRe to have an internal vision, based on their own data and independent of group guidelines. Furthermore, these models are not capable of modeling all types of treaties and focus mainly on pooled XL treaties and do not encompass all LoBs or cedents, leaving SP, QS and AGG treaties unmodeled by the internal model. In all, around 20% of reinsurance treaties are not modeled, requiring case-by-case studies. The models are also difficult to customize, making it difficult to effectively consider new treaties or to adapt treaty conditions to the specific needs of AXA entities.

Given AXA's strategic goal of minimizing ceded premium and ceded margin while reducing the net result volatility and complying with AXA Risk Appetite limit, developing an internal reinsurance pricing tool based on internal data which could increase the scope of analyzed treaties and provide the entity with an internal opinion on their treaties is of great interest. It would thus respond to the problem at hand in this paper which is: How can AGCRe, through a centralized, reliable, flexible, and fully controlled modeling approach, evaluate the risks associated with reinsurance treaties using internal data available across all cedents and risk classifications?

In this work, we will try to give a first answer to this problem by assessing four types of reinsurance treaties: XL per Risk, QS, AGG and SP[1] treaties. The study will be limited to non-life reinsurance treaties, as life reinsurance treaties constitute less than 3% of AXA's total reinsurance portfolio and were not considered as a priority. Additionally, treaties within the CAT LoB will not be examined, as their modeling is significantly more complex and falls under the purview of other specialized teams. However, the tool is aimed at ultimately encompassing life and CAT modeling through the integration of Event Loss Tables provided by the risk management. Furthermore, only one-year horizon reinsurance treaties will be considered.

To develop the pricing methodologies forming the basis of the pricing tool, several key steps were undertaken:

1. **Data Selection and Transformation**
   The first step involved identifying all the data required for quantifying the risk transferred under reinsurance treaties. Since each treaty type has distinct data requirements, an inventory of the available databases and the specific data needs for each treaty type was conducted. Subsequently, the necessary operations to adapt these datasets for modeling were defined for each type of treaty, with for instance the assessment of the ultimate loss. This step is particularly important and represents one of the key value-added features of the tool, as it centralizes and formats all data necessary for treaty evaluation while allowing users to add new data over time. Moreover, incorporating the estimation of the ultimate loss within the tool is a new development for the Analytics and Pricing team, as they are not usually responsible for this task. This addition gives them direct control over the estimation process, which could be improved by working with the reserving team.
2. **Claims Modeling**
   Evaluating reinsurance treaties necessitates modeling the underlying claims to estimate future losses associated with the treaty. Loss modeling differs based on the treaty type and its specific characteristics. For instance, in the case of QS or AGG treaties, which are not modeled within the internal model, the focus will be on modeling annual loss ratios. Conversely, for XL (respectively SP) treaties, one will focus on the annual recoveries (respectively annual ceded losses) modeling. This part serves as the cornerstone of the pricing tool, where several pricing methodologies were explored. These include a frequency-severity model utilizing a broader range of statistical distributions, and the development of a ranking process to identify the most suitable distribution, as well as an exposure curve-based methodology inspired by (Mata, Fannin, & Verheyen, 2002) for SP and XL treaties within the property LoB.
3. **Pricing and Indicator Calculation**
   This part is responsible for the final pricing of the reinsurance treaty as it computes risk and profitability indicators for overall reinsurance evaluation. It centralizes all indicators calculated for underwriting committees in addition to cost of capital indicators.

These steps formed the foundation of the methodology conceptualization and guided the development of the tool's structure. The architecture of the pricing tool was designed based on these stages, easing its development. Throughout this work, the pricing methodology will be demonstrated through the analysis of reinsurance treaties. An initial overview of the tool's inputs, outputs, processes and flows to clarify its structure and the developed methodology is presented below.

[1]Only those covering the property LoB

Figure 16: Overview of the reinsurance pricing tool architecture

3. Databases description

To address our problem statement, it is first necessary to identify the data required for pricing each treaty and the transformations that need to be applied to it for modeling purposes. For the sake of reproducibility and future additions, which is part of our goal, each of the databases defined below was elaborated with the objective of creating a standardized data structure. This structure could later be provided to entities, enabling them to supply data if AGCRe's historical data is insufficient to reliably quantify the risk associated with the treaty the entity wishes to underwrite. Furthermore, the databases were designed so that all entities could complete the required information within the datasets, ensuring no entity is unable to fill in the necessary details. Thus, this part of the actuarial thesis will focus on describing the various databases used for evaluating the diverse reinsurance treaties, as well as the operations applied to obtain the necessary datasets for modeling.

3.1 Reinsurance Terms and Conditions (T&C)

To assess risks linked to a reinsurance treaty, one needs to identify which information about the evaluated treaties should be retained or added. To achieve this, the "Reinsurance T&C" database was formed based on AGCRe's internal database, retaining only the columns essential for treaty pricing and incorporating additional columns required for this purpose. The database serves as a reference for treaty details and contains, among other things, all reinsurance treaties underwritten by AGCRe in 2024. Additionally, it is designed to be updated with new treaties, whether they are treaties underwritten for 2025 or new ones that the Analytics & Pricing team intends to study. The variables in this database will be used throughout the treaty pricing process. The table below lists the variables included in the reinsurance T&C database:

Table 3: List of variables - reinsurance T&C database

Variable ID	Description
PROGRAM	Reinsurance program name
ID	Unique ID of the reinsurance treaty
TREATY	Name of the treaty
COUNTRY	Name of the country
COUNTRY_CODE	Country alpha 3 code
CEDENT	Cedent(s) covered by the treaty
RISK_CLASSIFICATION	Risk(s) covered by the treaty
LOB	Line of Business
ATTACHING_TYPE	Attaching type: Loss Occurring or Risk Attaching
REINSURANCE_TYPE	Reinsurance type (proportional or non-proportional)
TREATY_TYPE	Treaty type (XL, QS, AGG, SP)
PER_EVENT	Binary variable (only for XL treaty): 1 if the treaty is an XL per event, 0 otherwise
NB_LAYERS	Reinsurance program number of layers
TREATY_LAYER_NR	Treaty layer within the reinsurance program
POOLED	Binary variable: 0 if the treaty is not pooled, 1 if it is pooled
POOL	Name of the pool
POOL_SHARE	Pool share rate
ACCEPTED_SHARE	Retention share rate
REINSTATEMENTS	Reinstatement conditions
CURRENCY	Currency code in which the treaty is issued
RETENTION	Retention (amount or percentage)
LIMIT	Retention (amount or percentage)

AAD	Annual Aggregate Deductible amount
AAL	Annual Aggregate Limit amount
INDEXATION_CLAUSE	Binary variable: 1 if the treaty has an indexation clause, 0 otherwise
INDEXATION_CLAUSE_THRESHOLD	Indexation clause threshold
INDEXATION_CLAUSE_INFLATION_RATE	Indexation clause rate
INDEXATION_CLAUSE_INDEX	Index used for the indexation clause
CESSION_RATE	Cession rate (only for QS treaties)
COMMISSION_RATE	Commission rate (if fixed commission)
SLIDING_SCALE_LR_UPPER_BOUND	Upper bound for loss ratio (if sliding scale commission)
SLIDING_SCALE_LR_LOWER_BOUND	Lower bound for loss ratio (if sliding scale commission)
SLIDING_SCALE_COMMISSION_LOWER_BOUND	Commission rate lower bound (if sliding scale commission)
SLIDING_SCALE_COMMISSION_UPPER_BOUND	Commission scale rate lower bound (if sliding scale commission)
LOSS_CORRIDOR	Binary variable: 1 if the treaty has a loss corridor, 0 otherwise
LOSS_CORRIDOR_LR_LOWER_BOUND	Loss corridor lower bound (if loss corridor)
LOSS_CORRIDOR_LR_UPPER_BOUND	Loss corridor upper bound (if loss corridor)
TARGET_MARGIN	Treaty's target margin rate
PROFIT_SHARING	Treaty's profit sharing
PRICING_METHOD	Pricing methodology: Exposure Pricing or Experience Pricing
INURING_CESSION_RATE	Inuring cession rate to apply on covered claims
EXPERIENCE_PRICING_FREQ_RISK_DRIVER	Selected exposure measure
DISCOUNT_RATE	Discount rate to use
EXPOSURE_CURVE	Selected exposure curve

3.2 Historical individual claims

To accurately quantify the risk associated to the reinsurance treaty, it is essential to analyze and model the underlying loss experience of the reinsurance treaty. Before diving into the details of the historical claims database, let us define some insurance characteristics that are crucial for understanding the elements of the database.

With the inverted production cycle, an insurer collects premiums from clients before fully knowing the ultimate cost of the insurance policy or contract: the insurer provides a guarantee without knowing the exact cost at the time of sale. This uncertainty creates a regulatory obligation for the insurer to ensure it can meet its future obligations, which leads the insurer to set aside technical reserves at the end of each budget year. These reserves represent the estimated amount of future liabilities that the insurer must cover. In non-life insurance, which is the scope of this project, there are two main technical reserves:

- **Unearned premiums reserves**: The reserves formed must correspond to the premium of the policy pro rata temporis the duration of the remaining guarantee.
- **Outstanding claims reserves:** This is a type of technical reserve designed to quantify and provide for commitments relating to insurance losses that have occurred but not yet been settled.

Considering AGCRe's activities, outstanding claims reserves are more important than unearned premium reserves since most reinsurance treaties are taken out on January 1 and cover an annual period. Thus, only outstanding claim reserve will be considered in this reinsurance pricing tool development.

Concerning the outstanding claims reserves, they are composed of two main quantities:

- **Case reserves**: This is a case-by-case estimate of what remains to be paid by the insurer for each claim.
- **IBNR (Incurred But Not Reported) reserves:** This technical reserve corresponds to the sum of the IBNYR (Incurred But Not Yet Reported) provision, intended to cover claims already incurred but not yet reported at the balance sheet date, and the IBNER (Incurred But Not Enough Reported) provision, intended to cover claims already

incurred and reported, but for which the amount communicated may be insufficient. The IBNER and IBNYR are evaluated by statistical methods for a set of contracts.

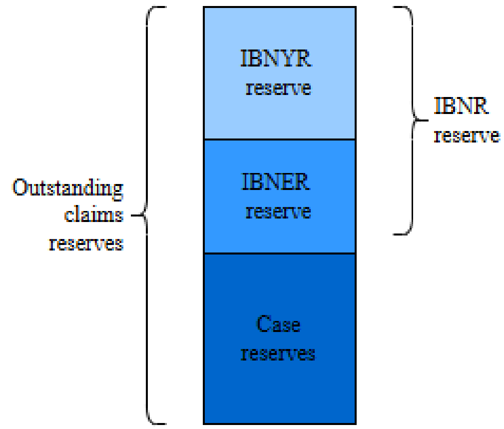

Figure 17: Overview of outstanding claims reserves

When a claim occurs over a certain threshold depending on the coverage scope, ceding companies provide AGCRe with the following information: the cumulative amount paid, which represents the total amount settled by the ceding company to its policyholders, and the corresponding case reserves. The combination of paid claims and case reserves constitutes the claims incurred. Additionally, a range of variables offering a detailed description of the incurred claims is provided. These variables cover various aspects, including financial, geographical, contractual, and descriptive information. All this data is then stored in an internal database, which contains claims records from 1964 to 2024.

This database has been processed by removing outliers, duplicates, claims with missing values or incoherent ones, and keeping, for a given development year and claim, only the most recent evaluations to ensure that there is only one data point per claim and development year. Additionally, only claims with an attachment year (i.e., the year of the accident or the underwriting year) from 1995 onwards were saved to ensure the inclusion of claims with sufficient representativeness. The variables used are detailed below:

Table 4: List of variables - individual historical claims database

Variable ID	Description
CEDENT	Cedent
RISK_CLASSIFICATION	Risk classification of the claim
LOB	Line of Business
CLAIM_ID	Unique ID of the claim
CURRENCY	Currency
OCCURENCE_DATE	Claim occurrence date
OCCURENCE_YEAR	Claim accident year
EVALUATION_DATE	Claim evaluation date
EVALUATION_YEAR	Claim evaluation year
ATTACHMENT_YEAR	Claim attachment year
ATTACHMENT_YEAR_TYPE	Type of attachment: Underwriting year or Accident year
DEVELOPMENT_YEAR	Development year
INURING_CESSION_RATE_TO_APPLY	Inuring cession rate to apply
CLAIM_PAID_FGU_CUMULATIVE_AMOUNT	Claim paid cumulative amount (FGU)
CLAIM_CASE_RESERVES_FGU_AMOUNT	Case reserves amount (FGU)
CLAIM_INCURRED_FGU_CUMULATIVE_AMOUNT	Claim incurred cumulative amount (FGU)

This database of 211 306 rows contains approximately 58 640 claims, covering a total of 57 cedents and 80 risk classifications. The distribution of the total number of claims per year is given in the Figure 18.

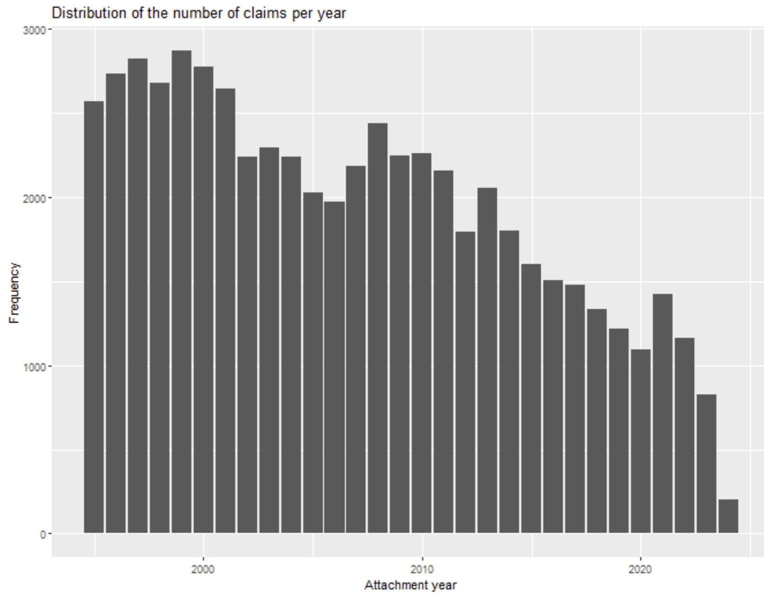

Figure 18: Distribution of the number of historical claims per year – historical individual claims

In the historical individual claims database, FGU stands for "From Ground Up". It refers to the gross amount (paid or provisioned) of a claim, i.e., the total losses associated with that claim, before adjustments and restatements related to underlying reinsurance treaties. It therefore represents the gross loss of a given claim. The inuring rate refers to the percentage applied to the total amount of claims (including paid and provisioned amounts) to account for amount reduction due to prior proportional reinsurance treaties. It could thus represent the cession rate of an underlying QS or SP treaty.

The attachment year variable corresponds to the year in which the claim occurred, or the year in which the underlying insurance contract was underwritten. This variable is essential for filtering claims according to the way the treaty accounts for claims covered by the reinsurance treaty (Risk Attaching or Loss occurring).

3.3 Historical aggregated claims

When pricing AGG or QS contracts, the reinsurer focuses on the underlying aggregated claims experience rather than the distribution of individual claims. Therefore, one should analyze the underlying aggregated claims experience. To this end, a database containing all the historical claims data provided by ceding companies to AGCRe was constructed. It is derived from the historical individual claims database by aggregating amounts per year of attachment, type of attachment, and development year. This dataset is made up of 53 390 rows, encompassing 57 cedents and 80 risk classifications. The data specification is as follows:

Table 5: List of variables - historical aggregated clams database

Variable ID	Description
CEDENT	Cedent
RISK_CLASSIFICATION	Risk classification of the claim
LOB	Line of Business
CURRENCY	Currency
ATTACHMENT_YEAR	Claim attachment year
ATTACHMENT_YEAR_TYPE	Type of attachment: Underwriting year or Accident year
DEVELOPMENT_YEAR	Development year

INURING_CESSION_RATE_TO_APPLY	Inuring cession to apply
AGGREGATE_CLAIM_PAID _FGU_CUMULATIVE_AMOUNT	Paid cumulative amount during the development year (FGU)
AGGREGATE_CLAIM_CASE_RESERVES_FGU_AMOUNT	Case reserves amount for the attachment year (FGU)
AGGREGATE_CLAIM_INCURRED_FGU_CUMULATIVE_AMOUNT	Aggregate incurred cumulative amount of the attachment year (FGU)

3.4 Historical exposure

When pricing a reinsurance treaty, we will also consider the underlying exposure of the cedent covered by the treaty. Indeed, changes in exposure affect the number of claims experienced by the cedent, as well as their severity, and therefore the reinsurer's potential total losses. It was therefore required to create such a data structure and select the variables it should contain, so that the pricing method could be as accurate as possible. The variables composing the pricing database are listed below:

Table 6: List of variables - historical exposure database

Variable ID	Description
CEDENT	Cedent
RISK_CLASSIFICATION	Risk classification
CURRENCY	Currency
YEAR	Year
GROSS_WRITTEN_PREMIUM	Gross Written Premium amount
GROSS_EARNED_PREMIUM	Gross Earned Premium amount
SUM_INSURED	Sum Insured amount
NB_RISKS	Number of risks

The term "Gross Written Premium" (GWP) is used to designate the total sum of insurance premiums that an insurance company has underwritten (or written) over a given period, usually one year, before any deductions. It thus corresponds to the total amount of premiums that customers agree to pay for insurance policies issued by the insurer. Conversely, Gross Earned Premium (GEP) represents the portion of the insurance premium that an insurer has earned over a given period.

Regarding its construction, this dataset was created using the exposure of the entities covered by AGCRe from 1995 to 2024. Only cedents and classifications for which at least one of the exposure columns (GEP, GWP, Number of risks, or SI) is non-empty across the years covered were included. In cases where data was not available for all years, the closest known value in time was used to fill the missing lines. Moreover, since GWP values could not be obtained, they were assumed to be equal to the GEP values. It ultimately comprises 14 291 lines with 55 risk classifications and 30 cedents.

3.5 Risk profile

The risk profile of ceding companies is also considered within the developped pricing process. A risk profile provides a snapshot of an insurance portfolio, categorizing risks into different ranges based on their exposure value. Each range is defined by its minimal exposure and maximal exposure. The number of risks per range indicates how many individual risks or insurance policies are included in each exposure range. In this actuarial project, risk profiles will be used for pricing XL per risk and SP treaties, specifically for the property Line of Business (LoB). Ideally, the database would include risk profiles for 2025. However, due to the unavailability of 2025 data, only the 2024 risk profiles will be used. After filtering, the risk profile database includes 3 risk classifications within the property LoB (commercial, industrial, and residential) and 15 cedents. The key variables selected for constructing the risk profile are as follows:

Table 7: List of variables - risk profile database

Variable ID	Description
YEAR	Year
CEDENT	Cedent
RISK_CLASSIFICATION	Risk classification
CURRENCY	Claim accident year
RANGE_MIN	Minimal SI value
RANGE_MAX	Maximal SI value
NB_RISKS	Number of risks within the range
SUM_INSURED	SI
GROSS_WRITTEN_PREMIUM	Total GWP amount within the range
GROSS_EARNED_PREMIUM	Total GEP amount within the range
BUDGETED_LOSS_RATIO	Budgeted LR of the range
SUM_INSURED_MEAN	SI mean within the range
SUM_INSURED_VARIANCE	SI variance within the range

3.6 Pricing data

During the calculation of the indicators, we will need different variables coming from various sources. These variables will be explained in detail threw the part 6. The table below provides a list of these variables and a brief description of each one:

Table 8: List of data – pricing data

Variable	Description
Tax rates	Various tax rates representing tax on benefits, C3S, CVAE, ACPR fees, etc.
SCR values	AXA SA's SCR of the premium, reserve and CAT modules. The SCR is a Solvency II concept that will be introduced in detail later.
Coverage ratio	Coverage ratio defined as the ratio AXA SA SCR to AXA SA actual capital.
Cost of capital rate	Return on investment expected by the company's shareholders. This concept will be developed later.
Marginal impact rate	Mmarginal impact on the SCR reserve and operational.
Underwriting expenses	Typical underwriting fee.
Durations	Duration of the different LoBs.
Transmission factors	Transmission factors account for diversification effects between AGCRe pools and the SCR of AXA SA.
Solvency II premium and reserve risks covariances	Premium and Reserves risks covariances as specified by Solvency 2 regulatory framework in the context of Premium and Reserves Risks estimation through defined standard formulas.
Modeled results and recoveries for pooled treaties	Monte Carlo simulated distributions (Remetrica software) of all AXA SA Reinsurance Pools (Property per risk & per event, Liability, Motor, Marine, and Cyber) underwriting net profits, and all pooled individual treaties.

3.7 Other databases

During this project, other data structures were handled to support the modeling needs, including an index database and an exchange rate database. The index database contains the evolution of the Consumer Price Index (CPI) and the Construction Producer Prices Index (CPPI) from 1954 to 2054 for all countries covered by the reinsurance treaties within the AGCRe portfolio. Based on the World Economic Outlook and Eurostat data, this index database frequently employed within the Analytics & Pricing team will be used without further modifications. It will be exploited for adjusting historical claims and premiums to reflect inflationary impacts over time. The variables contained in this database are detailed below:

Table 9: List of variables – index database

Variable ID	Description
YEAR	Year
COUNTRY	Country name
COUNTRY_CODE	Country code
INDEX_TYPE	CPI or CPPI
INDEX	Index value

The exchange rate database contains exchange rate values at the first quarter of 2024 for all currencies covered by the AGCRe treaties. The variables included in this dataset are as follows:

Table 10: List of variables – FX rates database

Variable ID	Description
CURRENCY	Currency
EXCHANGE_RATE	Exchange rate

Once all the data specifications for pricing have been set, the following part will present the developed methodology for selecting and, if necessary, transforming the data required for pricing the diverse types of treaties.

4. Data processing

This section will describe how the different databases are selected and transformed within the pricing tool based on treaty characteristics in order to develop a global pricing process that will increase the team pricing capacities. The following figure details the implemented steps to do so. Each process will be explained within this part.

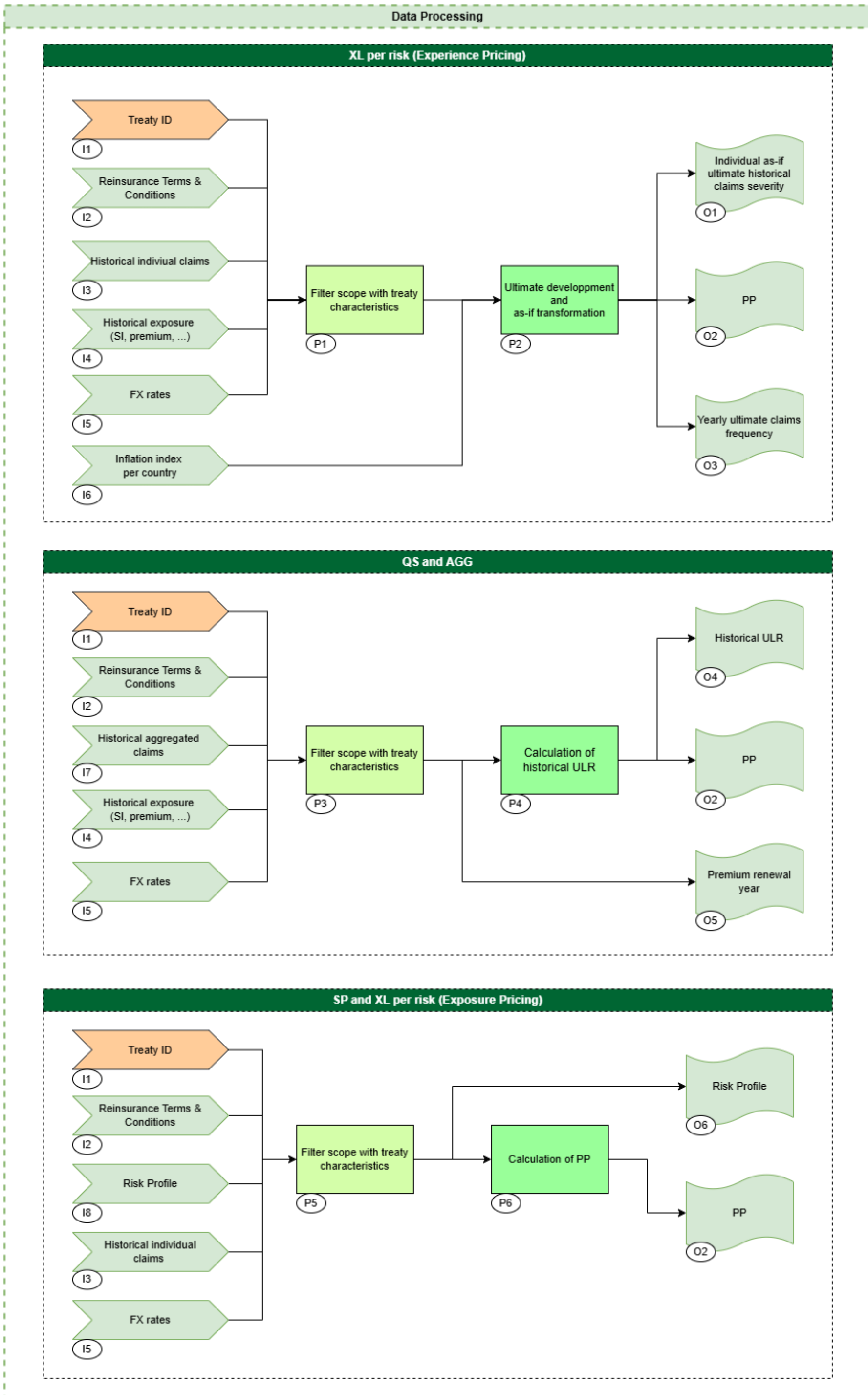

Figure 19: Data processing process

4.1 Data selection

In this section, let us assume one wishes to price a treaty covering one or more ceding companies on some risk classification(s). The first step of the selection process involves retrieving all relevant information about the treaty from the Reinsurance T&C database. The table below summarizes, for each type of treaty, which data from the Reinsurance T&C database will be required to generate the indicators:

Table 11: Variables needed from reinsurance T&C per treaty type

Treaty type	Variable Name
All treaties	ID
	COUNTRY_CODE
	CEDENT
	LOB
	RISK_CLASSIFICATION
	ATTACHING_TYPE
	POOLED
	POOL
	POOL_SHARE
	CURRENCY
	DISCOUNT_RATE
	INURING_CESSION_RATE
XL per risk	NB_LAYERS
	REINSTATEMENTS
	RETENTION
	LIMIT
	AAD
	AAL
	INDEXATION_CLAUSE
	INDEXATION_CLAUSE_THRESHOLD
	INDEXATION_CLAUSE_INFLATION_RATE
	INDEXATION_CLAUSE_INDEX
	PRICING_METHOD
	EXPERIENCE_PRICING_FREQ_RISK_DRIVER
	EXPOSURE_CURVE
AGG	RETENTION
	LIMIT
QS	CESSION_RATE
	CAPACITY
	COMMISSION_RATE
	SLIDING_SCALE_LR_LOWER_BOUND
	SLIDING_SCALE_LR_UPPER_BOUND
	SLIDING_SCALE_COMMISSION_LOWER_BOUND
	SLIDING_SCALE_COMMISSION_UPPER_BOUND
	LOSS_CORRIDOR
	LOSS_CORRIDOR_LR_LOWER_BOUND
	LOSS_CORRIDOR_LR_UPPER_BOUND
	TARGET_MARGIN
	PROFIT_SHARING
SP	CAPACITY
	COMMISSION_RATE
	SLIDING_SCALE_LR_LOWER_BOUND
	SLIDING_SCALE_LR_UPPER_BOUND
	SLIDING_SCALE_COMMISSION_LOWER_BOUND
	SLIDING_SCALE_COMMISSION_UPPER_BOUND
	TARGET_MARGIN
	PROFIT_SHARING

Once this information is retrieved, one needs to select the relevant data from the previously described data structures. Since data requirements vary depending on the treaty nature, not all databases will be used for pricing every treaty type. The table below details which databases are required for pricing each type of treaty:

Table 12: List of data tables required for pricing each type of treaty

Treaty type	Databases required
All treaties	Reinsurance T&C FX rates Pricing parameters
XL per risk	Inflation indices Historical exposure Historical individual claims Risk profile
Aggregate	Historical exposure Historical aggregated claims
Quota Share	Historical exposure Historical aggregated claims
Surplus	Historical individual claims Risk profile

Thus, knowing the type of treaty, each of the relevant databases will be selected based on the previous table. Subsequently, each dataset will be filtered to retain only the data related to the specific treaty. The following filtrations will be applied for each database:

- **Historical Exposure, Risk Profile, Historical Individual/Aggregated Claims Databases**: All variables where the ceding companies and risk classifications match those covered by the treaty will be selected.
- **FX Rates and Index Database**: All variables where the currency or country matches those present in the databases will be selected.

Once these transformations have been applied, further operations are needed for the historical claims databases. Due to the specific nature of insurance and the uncertainty surrounding the final cost of a claim, the ultimate loss of the aggregated or individual claims must be estimated to model losses properly. The ultimate loss is calculated as the sum of paid claims and outstanding reserves (which include both IBNR and case reserves). The next section will focus on describing the process of estimating aggregate losses, whether individual or collective.

4.2 Ultimate loss calculation

Let us consider the historical individual/aggregated claims database. First, depending on how the claims covered by the treaty are determined (Loss Occurring or Risk Attaching), the claims with the appropriate attachment type need to be selected. For a Loss Occurring (respectively Risk Attaching) treaty, claims whose attaching type corresponds to “Accident Year” (respectively “Underwriting Year”) are gathered. Once this operation has been made, the financial amounts are converted in the treaty currency thanks to the FX rates database.

To estimate the ultimate amount and avoid biases when modeling ultimate amounts, one needs to calculate the IBNR of individual/aggregated claims. To do so, we first determine which claims will be used to calibrate the IBNRs. Indeed, to calculate IBNR, homogeneous types of policies are necessitated. If a treaty covers multiple risks and ceding companies, calibrating the IBNR using all the historical claims could introduce biases and violate the assumptions of the IBNR calculation model. The selection of claims is thus based on the following criteria:

1. **Number of individual claims greater than 20 for each cedent and risk classification:**

   If the number of claims for every cedent and risk classification combination exceeds 20 in the filtered historical claims database, and if the oldest claim is more than 5 years old for short-tail LoBs or more than 15 years old for long-tail LoBs, then the IBNR is calibrated on the claims within the filtered database. Regarding the aggregated claims database, since the number of individual claims is not available, only the historical depth restriction is considered.

2. **Number of claims less than 20 for one cedent and risk classification, but total greater than 20:**

   If the number of historical claims for a given cedent and risk classification is less than 20 in the historical individual claims database, but the total number of claims within the filtered database exceeds 20, and if the oldest claim is more than 5 or 15 years old (depending on whether it is short-tail or long-tail LoBs), then the IBNR is calibrated on all historical individual claims covered by the treaty. Regarding the filtered aggregated claims database, only the historical depth restriction is considered.

3. **None of the above conditions are met:**

   If none of the above conditions is met, the IBNRs are calibrated using all the claims within the historical individual claims database whose risk classification is covered by the treaty. The same applies to aggregated claims, where the IBNR is calibrated on the historical aggregated claims database.

Let us now delve into the IBNR estimation procedure. First, the incurred amounts are aggregated by development year and attachment year to form the matrix below, known as the cumulative incurred amount triangle:

$$\begin{bmatrix} C_{1,1} & \cdots & C_{1,n} \\ \vdots & \ddots & \vdots \\ C_{n,1} & \cdots & 0 \end{bmatrix}$$

Where:

- n designates the maximum development year
- $C_{i,j}$ designates the cumulative FGU incurred amount for the period i and the development year j. Thus, $C_{i,j} = 0$ for $i + j - 1 > n$.

The main objective is then to estimate the unknown part of the triangle and deduce from it the ultimate loss. To do this, we will use the Chain Ladder - Mack model, widely employed in insurance.

Let us consider a probabilized space (Ω, F, P). We define the real random variables $c_{i,j} = C_{i,j} - C_{i,j-1}$ for $1 \leq i \leq n,\ 2 \leq j \leq n$, $C_{i,1} = c_{i,1}$ for $1 \leq i \leq n$ where $n \in \mathbb{N}$ is a number of years. $c_{i,j}$ is the random variable corresponding to the aggregated amount incurred FGU for claims occurring in the attachment year i for the development year j. By construction, only random variables $c_{i,j}$ such as $i + j - 1 > n$ are observed. We also introduce the filtration $\mathcal{F}_k = \sigma(c_{i,j} \mid i + j \leq k + 1)$. By construction, variables $C_{i,j}$ are $\mathcal{F}_k$-measurable if $i + j \leq k + 1$.

The Chain Ladder-Mack model is based on the following assumptions:

- Hypothesis 1: The random variables $\left(C_{i_1,j}\right)_{1 \leq j \leq n}$ and $\left(C_{i_2,j}\right)_{1 \leq j \leq n}$ are independent for $i_1 \neq i_2$.
- Hypothesis 2: For $1 \leq j \leq n - 1$, there exists $f_j \geq 0$ such that for $1 \leq i \leq n$:

$$\mathbb{E}\left(C_{i,j+1} \middle| \mathcal{F}_{i+j-1}\right) = f_j C_{i,j}$$

- Hypothesis 3: For $1 \leq j \leq n - 1$, there exists $\sigma_j \geq 0$ such that for $1 \leq i \leq n$:

$$Var(C_{i,j+1} \mid \mathcal{F}_{i+j-1}) = \sigma_j^2 C_{i,j}$$

For $1 \le k \le n$, the filtration $\mathcal{B}_k = \sigma(C_{i,j} \mid i + j \le n + 1,\ j \le k) \subset \mathcal{F}_k$ is introduced

Under these three assumptions, it can be shown that $\mathbb{E}(C_{i,j+1} \mid \mathcal{B}_j) = f_j C_{i,j}$, $Var(C_{i,j+1} \mid \mathcal{B}_j) = \sigma_j^2 C_{i,j}$. The best linear estimator of f_j in $\left(\hat{f}_{i,j} = \frac{C_{i,j+1}}{C_{i,j}}\right)_{1 \le i \le n-j}$ for the quadratic conditioning error at $\mathcal{B}_j$ is for $1 \le j \le n - 1$:

$$\hat{f}_j = \frac{\sum_{i=1}^{n-j} w_{i,j} \hat{f}_{i,j}}{\sum_{i=1}^{n-j} w_{i,j}} = \frac{\sum_{i=1}^{n-j} C_{i,j} \hat{f}_{i,j}}{\sum_{i=1}^{n-j} C_{i,j}} = \frac{\sum_{i=1}^{n-j} C_{i,j+1}}{\sum_{i=1}^{n-j} C_{i,j}}$$

Link-ratios $\hat{f}_{i,j}$ can be interpreted as a weighted mean of the observed individual link-ratios $\hat{f}_{i,j}$, weighted by the $C_{i,j}$. Other weights can be chosen, allowing some observed link-ratio to be excluded if considered as “outlier.” To consider these outliers, the following weights were defined relying on expert judgement in the context of a previous study at AGCRe:

- If $\hat{f}_{i,j} \le 2$, $w_{i,j} = 1$
- If $2 < \hat{f}_{i,j} \le 5$, $w_{i,j} = 0.66$
- If $5 < \hat{f}_{i,j} \le 7$, $w_{i,j} = 0.33$
- If $7 < \hat{f}_{i,j}$, $w_{i,j} = 0$

Furthermore, if an individual development factor is less than 1 (a situation where case reserves were overestimated), the value of the individual coefficient is fixed at 1. The estimators $\hat{f}_j$ for each development year are then derived by calculating the weighted average of each individual development factor per development year.

To obtain the IBNR amount per claim, one needs to consider the most recent incurred amount $X_{i,n-i+1}$ and multiply it by $\prod_{j=n-i+1}^{n-1} \hat{f}_j - 1$:

$$IBNR_{individual\ claim} = (\prod_{j=n-i+1}^{n-1} \hat{f}_j - 1) * X_{i,n-i+1}$$

For historical aggregated claims database, one needs to apply the same operation on the most recent cumulated amount per year $C_{i,n-i+1}$:

$$IBNR_{aggregated\ claim} = (\prod_{j=n-i+1}^{n-1} \hat{f}_j - 1) * C_{i,n-i+1}$$

From the previous IBNRs, the ultimate loss per claim/year can be computed:

$$Ultimate\ loss = Claim\ inccurred + IBNR = Claim\ Paid + Case\ Reserve + IBNR$$

In the case of AGG or QS treaties, instead of considering the ultimate loss per year, the historical Ultimate Loss Ratio (ULR) will be used. These ULRs will be estimated based on historical exposure data. Regarding a loss-occurring treaty, the GEP will be used to calculate the ULR by dividing the ultimate loss per year with the corresponding GEP. For risk-attaching treaties, the same logic applies, with the GWP instead.

To illustrate the methodology, let us consider an example using a development triangle for an XL treaty. The analysis focuses on a treaty that covers a single cedent, involves one specific risk category, and pertains to a short-tail LoB. Consequently, all historical individual claims falling within the treaty's scope have been incorporated into the construction of this development triangle.

Table 13: Example of a development triangle for an XL treaty

	1	2	3	4	5	6	7	8	9	10	11
2014	1569875	2457482	2881275	3168144	3400194	3502200	3572244	3575816	3575816	3575816	3575816
2015	2365478	3178114	3701041	3991410	4320582	4455557	4550995	4578301	4587457	4587457	
2016	1269875	1890361	2259895	2460987	2706667	2800751	2857774	2883494	2883494		
2017	3256844	4046153	4927791	5447180	5718831	5945468	6096899	6145675			
2018	2763543	4568966	5397022	5738303	6089694	6319884	6454497				
2019	1325698	1953620	2291616	2476546	2619929	2706334					
2020	1789563	2801632	3323296	3646701	3863388						
2021	1124593	1658257	1913629	2075463							
2022	1365897	1892628	2249305								
2023	1532463	2385340									
2024	1145863										

Using the previous development triangle, it is possible to calculate the individual development factors $\hat{f}_{i,j}$:

Table 14: Example of individual development factors

	1	2	3	4	5	6	7	8	9	10
2014	1,57	1,17	1,10	1,07	1,03	1,02	1,00	1,00	1,00	1,00
2015	1,34	1,16	1,08	1,08	1,03	1,02	1,01	1,00	1,00	
2016	1,49	1,20	1,09	1,10	1,03	1,02	1,01	1,00		
2017	1,24	1,22	1,11	1,05	1,04	1,03	1,01			
2018	1,65	1,18	1,06	1,06	1,04	1,02				
2019	1,47	1,17	1,08	1,06	1,03					
2020	1,57	1,19	1,10	1,06						
2021	1,47	1,15	1,08							
2022	1,39	1,19								
2023	1,56									

Since no individual development factors exceed two, all the weights are set to one. This allows us to calculate the development factors for each development year:

Table 15: Example of development factors

Development year	1	2	3	4	5	6	7	8	9	10
$\hat{f}_j$	1,47	1,18	1,09	1,07	1,03	1,02	1,01	1,00	1,00	1,00

The individual IBNRs can thus be calculated in our case, constituting the ultimate loss amount of each individual claim.

To ensure the proper application of the Mack chain-ladder model, one needs to verify its assumptions. The hypothesis regarding the independence between $\left(C_{i_1,j}\right)_{1\leq j\leq n}$ and $\left(C_{i_2,j}\right)_{1\leq j\leq n}$ for $i_1 \neq i_2$, can be checked graphically by plotting the development factors row by row against their median. If the development factors are not too dispersed around the median value, it indicates that the independence assumption is satisfied. *Figure 20* compares the dispersion around the median for development years 1 to 4: the development factor values are not excessively dispersed around the median, validating the hypothesis.

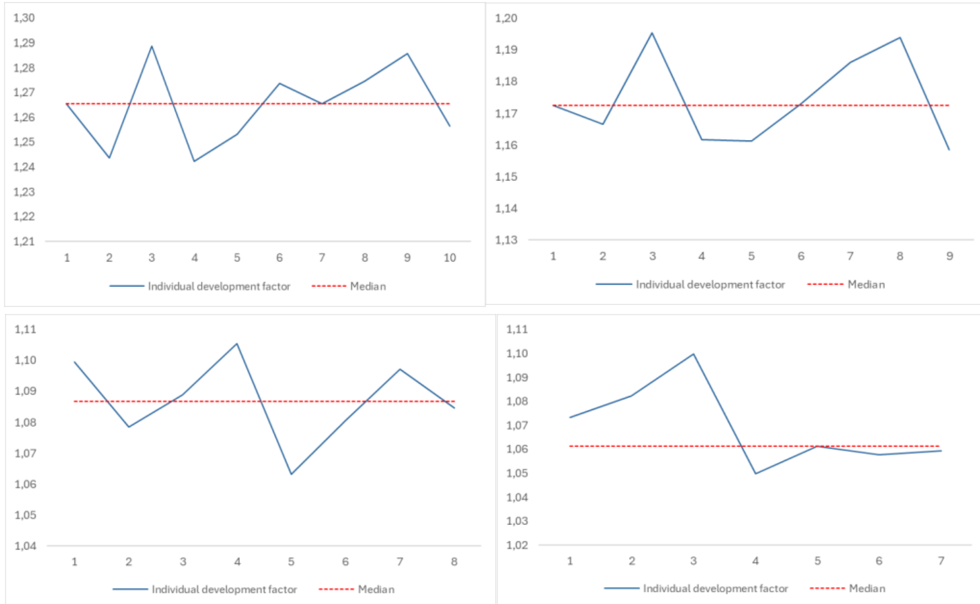

Figure 20: Median comparison for individual development factors

To verify Hypothesis 2, one can plot the linear regression line of cumulative incurred amounts $C_{i,j+1}$ against $C_{i,j}$. The figure below presents the regression plots for development years 1 to 4. The hypothesis appears to be validated, particularly since the R^2 values of the regressions exceed 0.9, and the regression coefficients are statistically significant at the 5% level.

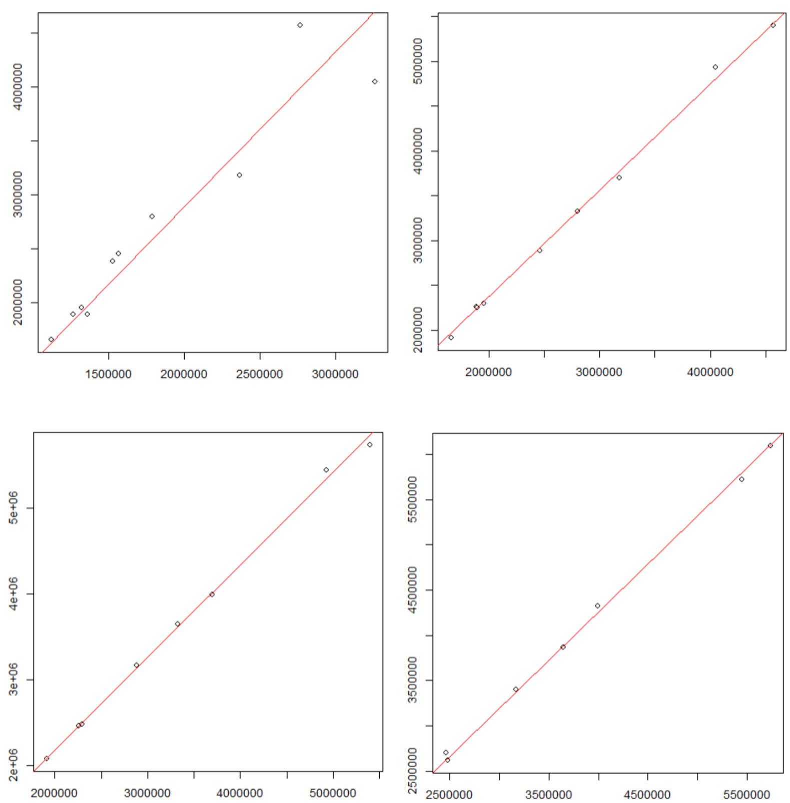

Figure 21: Linear regression of $C_{i,j+1}$ on $C_{i,j}$

Regarding Hypothesis 3, it can be checked using the graph of standardized residuals $\frac{C_{i,j+1}-\hat{f}_j C_{i,j}}{\sqrt{C_{i,j}}}$. The figure below displays the standardized residuals for development years 1 to 4. The residuals do not display any specific trend and appear random, which provides graphical validation of the assumption regarding the variance hypothesis of the chain-ladder model.

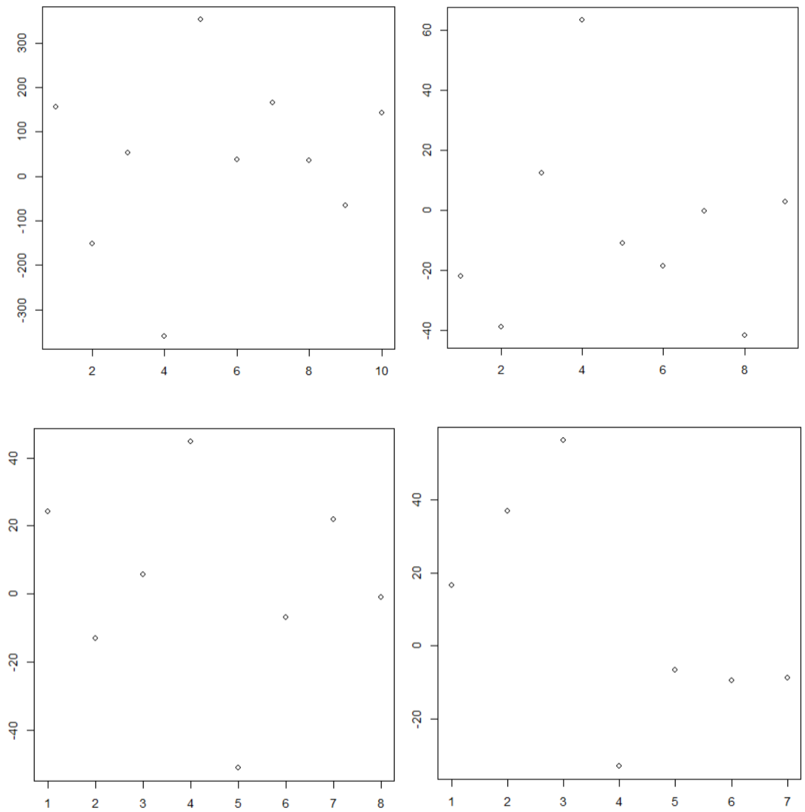

Figure 22: Standardized residuals

The Chain-Ladder method has certain limitations that can affect its reliability. One key issue is its reliance on historical claims. An atypical data point can distort an entire row and, consequently, the ultimate loss estimate. However, incorporating different weights in the provisioning process can help mitigate this issue during IBNR estimation. Another concern is the dependence on the last development periods, where limited data is available to calculate development factors. Nonetheless, this impact is generally less critical, as claims tend to stabilize in later development periods. Additionally, the method assumes strong independence of development factors across accident years, an assumption that may not always hold due to changes in claims handling or reporting practices. It is therefore essential to check the model's assumptions for each type of treaty and interpret results cautiously if these assumptions are not met.

For this study, the Chain-Ladder method was selected due to its simplicity, ease of implementation, and the ability to quickly and graphically check its assumptions. Other methods for IBNR calculation, such as the Loss Ratio method, the Bornhuetter-Fergusson method, or the Benktander method, also exist. Integrating these alternative approaches in the future could provide a comparative perspective on ultimate loss estimates. Collaborating with the AGCRe Reserving team could also be a valuable step in successfully implementing these enhancements.

4.3 Payment pattern computation

The previous section provided estimates for individual or aggregated ultimate claim amounts. In this part, in order to consider discounting issues and as-if calculations, we focused on modeling how claims are paid over time. To do so, a payment pace, known as a payment pattern, is computed at a 30-year horizon. The payment patterns calibrated in this study are based on an annual scale, with the simplifying assumption that all claims are settled in the middle of the year. These simplifications enable the calibration of payment patterns on an annual basis.

To estimate the payment pattern, the Chain-Ladder methodology has been opted for. Like the estimation of IBNR, cumulative development triangles are constructed. However, unlike the IBNR estimation, cumulative FGU paid amounts will be considered instead of incurred amounts since one is interested in the payment frequency. Regarding the selection of historical claims, the same selection process will be used to determine which claims will be included for payment pattern computation. Following the same approach as for IBNR estimation, development factors $\hat{f}_j$ by development year j are calculated.

A cumulative payment pattern is thus defined based on these development factors:

$$PP_{cum,j} = \frac{1}{\prod_{k=j}^{n-1} \hat{f}_k} \text{ for } 1 \leq j \leq n-1 \text{ and } PP_{cum,k} = 1 \text{ for } k \in \{n, \dots, 30\}$$ [1]

This variable represents the cumulative paid proportion of the claim for the development year j. The incremental payment pattern over a development year j noted PP_j is defined by:

$$PP_j = PP_{cum,j} - PP_{cum,j-1} \text{ for } 2 \leq j \leq 30 \text{ and } PP_1 = PP_{cum,1}$$

Depending on the number of triangles calibrated, one or more payment patterns are computed for each treaty. The payment pattern vector will be noted $PP = (PP_j)_{1 \leq j \leq 30}$.

Let us take as an example two reinsurance treaties: both are loss-occurring treaty, one is covering a single cedent on the property LoB, and another covering two different cedents on the liability LoB. Each of these payment patterns is calibrated using the complete set of historical claims data.

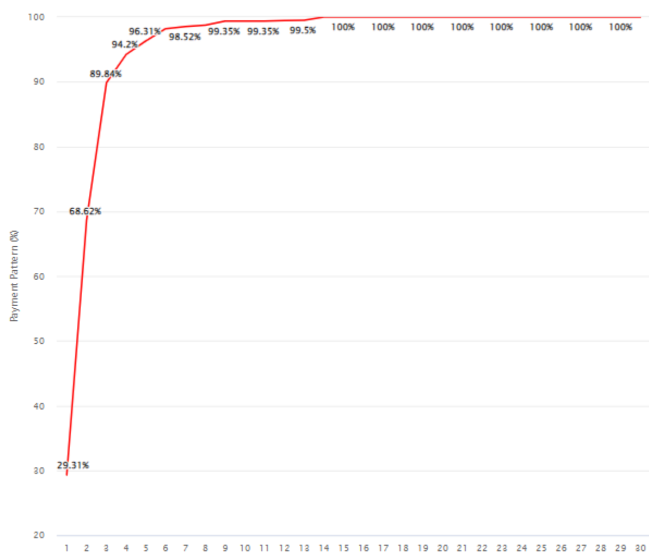

Figure 23: Cumulated PP of a property reinsurance treaty's covered underlying claims

[1] *The extension of the payment pattern to 30 years is carried out to obtain an estimate of the payment pace over a 30-year horizon.*

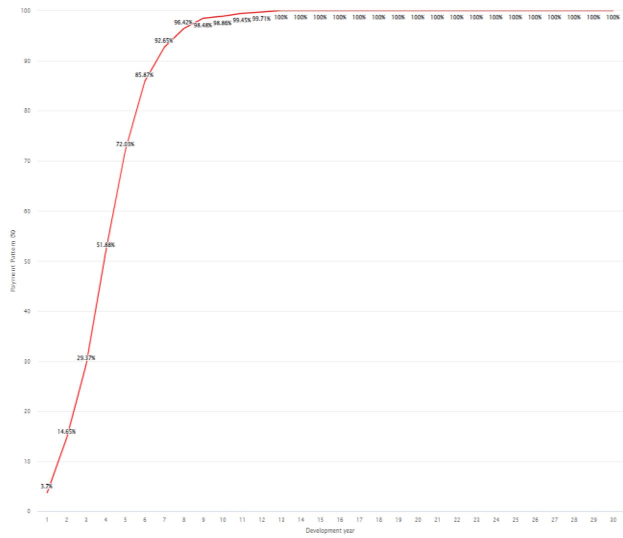

Figure 24: Cumulated PP of a liability reinsurance treaty's covered underlying claims

Regarding the coverage perimeter of the considered property treaty, 96% of the underlying/covered ultimate claims amounts are paid after 4 years. In contrast, for the liability treaty, only 51% of the claim has been paid within this period. Additionally, a notable difference is observed in the early years of development, where claims for the property treaty are settled more quickly compared to the liability treaty. Since a material or property damage claim settles faster than a serious bodily injury claim, these results were expected and help to highlight the characteristics of short-tail versus long-tail business.

4.4 As-if adjustments (XL treaty)

When considering the ultimate loss of a claim, one does not necessarily account for the economic changes that may have occurred, especially when the oldest claims date back many years. Directly using the historical ultimate loss amounts in modeling could lead to underestimating or overestimating the underlying risks. To address this issue, the ultimate claim amounts should be estimated "as-if" it had occurred during the period covered by the treaty to be analyzed, considering current economic conditions. For example, the gross amount of a claim incurred in 2003 would not be equivalent to the amount of the same claim if it occurred in 2024, due to factors such as inflation. This adjustment would allow historical amounts to reflect their true value in 2024, avoiding any bias in future severity modeling.

The following section will explain how individual amounts are converted into "as-if" values for XL per-risk treaties within the reinsurance pricing tool developed in this paper. In the case of AGG or QS treaties, it is assumed that the ultimate loss ratios are less dependent on the "as-if" calculation than individual claim amounts. We assumed that applying the "as-if" adjustment would multiply both the premium and the total amount by approximately the same factor. Consequently, by considering ULRs, this "as-if" adjustment does not impact the ULRs values. Therefore, we decided to simplify the process and omit the "as-if" adjustment for QS and AGG treaties. Concerning SP treaties, they will be modeled using risk profiles and do not necessitate as-if transformations.

Once the payment pattern vector has been estimated for the considered XL treaty, the last known provisioned amount of each claim will be projected using the calibrated payment pattern. Regarding the payment pattern vector, it is important to note that one or more payment patterns may have been calibrated. If only one payment pattern has been calibrated, it will be used to project reserves for all claims. However, if multiple payment patterns have been computed, the payment pattern corresponding to the cedent/risk classification that matches the claim will be used for the projection.

For a given claim, we define $RA_{i,e-i+1}^{e}$ as the last known amount provisioned amount where:

- i is the attachment year

- e is the most recent evaluation year
- $e - i + 1$ corresponds to the development year

The reserve amount corresponds to the sum of the case reserves and the individual IBNR as estimated in the previous section. Let PP be the incremental payment pattern vector used for an individual claim. If $RA_{i,t-i+1}^{e} \neq 0$, the claim cash-flow $CF_{e+k}(.)$ over the years $e + k$ with $k \in \{1, \dots, 30 - (e - i)\}$, function of the last amount provisioned $RA_{i,e-i+1}^{e}$, is defined by :

$$CF_{e-i+k}\left(RA_{i,e-i+1}^{e}\right) = \frac{PP_k}{1 - \sum_{j=1}^{t-i} PP_j} * RA_{i,e-i+1}^{e}$$

If $\sum_{j=1}^{t-i} PP_j = 1$ which corresponds to the case where the entire claim should have been settled, but a provisioned amount $RA_{i,t-i+1}^{e}$ is not zero, all reserves are assumed to be released during the year, i.e. $CF_{e+k}\left(RA_{i,e-i+1}^{e}\right) = RA_{i,e-i+1}^{e}$.

For a given claim, the following vector of cash flows is thus computed:

$$CF = \left(X_{i,1}, X_{i,2}, \dots, X_{i,e-i}, CF_{e-i+1}\left(RA_{i,e-i+1}^{e}\right), \dots, CF_{30}\left(RA_{i,e-i+1}^{e}\right)\right) = \left(\left(\widetilde{CF_{i,j}}\right)_{1 \leq j \leq 30}\right)$$

It is then necessary to as-if these various payments. The CPI or the CPPI of the country where the concerned cedent is located is retrieved. Let us note $\left(index_{t,cedent\ country}\right)_{1954 \leq t \leq 2050}$ the corresponding vector. Any other kind of relevant inflation measure indices could be used and may be more relevant according to the covered line of business. In our specific case, we resorted mainly to CPI (for all LoBs except property per risk) and CPPI (for property per risk treaties). The ultimate as-if amount $X_{ultimate,\ as-if}$ of the claim is therefore calculated by summing together all the as-if cash flows:

$$X_{ultimate,\ as-if} = \sum_{j=1}^{30} \frac{index_{renewal\ year+j-1,cedent\ country}}{index_{i+j-1,cedent\ country}} * \widetilde{CF_{i,j}}$$

These various operations are carried out for each claim in the selected historical individual database. The figure below schematically summarizes all the operations applied in the as-if adjustment process.

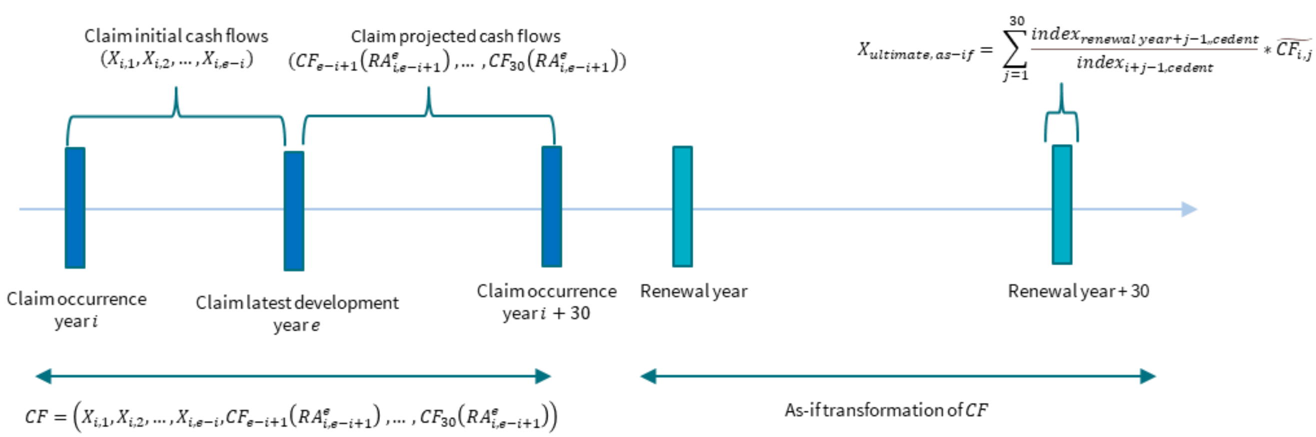

Figure 25: Overview of the as-if calculation process

Let us consider a property XL treaty 1M€ *xs* 2M€ and a liability XL treaty 10M€ *xs* 10M€. Historical claims for both treaties are shown in *Figure 26* and *Figure 27*. It illustrates how historical losses are distributed. The "as-if" adjustments applied to older claims reveal a proportionally greater impact, indicating that the size of those claims would have been significantly higher in value under current macroeconomic and market conditions, reflecting an upward trend in the index over time. Additionally, for both treaties, reserves (case reserves and IBNR) are notably larger for recent claims while being nearly null for claims older by a few years which have been fully settled and are no longer reserved. However, we observe that claims covered by the liability treaty hold higher reserve amounts for older claims compared to the property treaty. This highlights the difference between the short-tail and long-tail nature of the risks covered by the considered treaties. To sum up, we can see clearly on the following chart that the as-if effect prevails for older claims, while the ultimate projection effect is predominant for more recent ones.

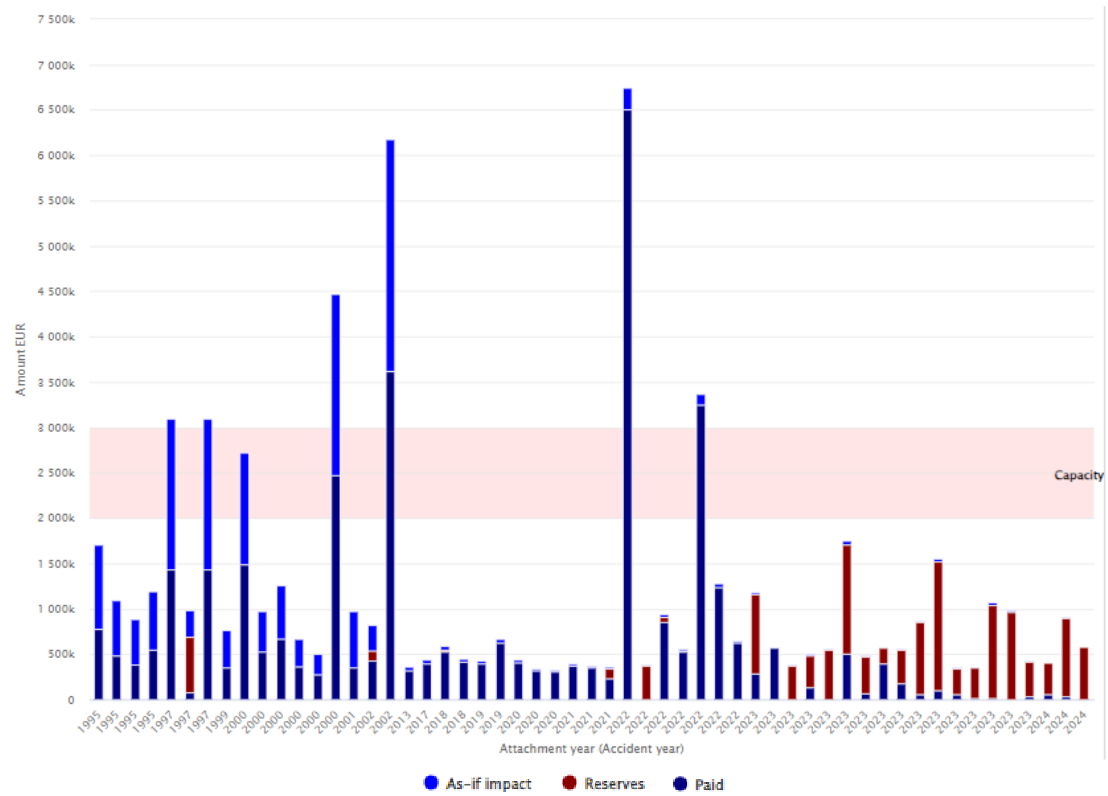

Figure 26: Historical claims amount for a property 1M€ xs 2M€ treaty

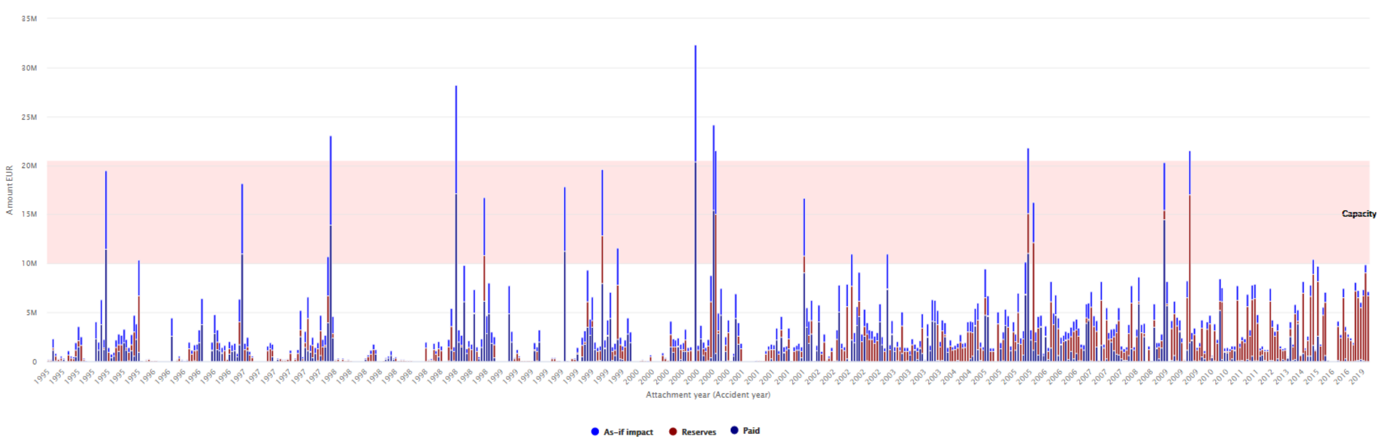

Figure 27: Historical claims amount for a liability 10M€ xs 10M€ treaty

4.5 Ultimate historical frequency calculation (XL treaty)

When modeling XL treaties, due to the model selected to model individual claims, it was crucial to approximate the distribution of the number of claims per year, as this serves as an input for modeling underlying claims. Models and statistical frameworks for claim counts can be constructed in either discrete or continuous time.

To model claims counts in continuous time, the use of counting processes would allow us to represent the number of claims covered by the treaty over a given period. A stochastic process $N = \{N(t);\ t \geq 0\}$ is defined as a counting process if it satisfies the following four conditions:

- $N(t) \geq 0$
- $N(t)$ is integer valued
- If $s < t$, then $N(s) \leq N(t)$
- For $s < t$, $N(t) - N(s)$ equals the number of claims that have occurred in the time interval $(s, t]$.

The most popular claims-counting process is the Poisson process and its extensions, including homogeneous, inhomogeneous, mixed, doubly stochastic, and even multivariate Poisson processes. The multivariate extension can be particularly useful for modeling the number of claims when multiple cedents or various risk classifications are covered by a reinsurance treaty. While other counting processes could also be considered, they will not be detailed in this study.

Table 16: Overview of Poisson process properties

Homogeneous Poisson process • $N_{\mu(t)}$ • $\mu(t) = \lambda t$ • $\mathbb{E}(N_{\mu(t)}) = Var(N_{\mu(t)}) = \lambda t$	**Inhomogeneous Poisson process** • $N_{\mu(t)}$ • $\mu(t) = \int_0^t \lambda(u)du$ • $\mathbb{E}(N_{\mu(t)}) = Var(N_{\mu(t)}) = \mu(t)$
Mixed Poisson process • $N_{\mu(t)}$ • $\mu(t) = \Lambda t$ • $\mathbb{E}(N_{\mu(t)}) = t\mathbb{E}(\Lambda)$ • $Var(N_{\mu(t)}) = t\mathbb{E}(\Lambda) + t^2 Var(\Lambda)$	**Doubly stochastic Poisson process** • $N_{\mu(t)}$ • $\mu(t) = \int_0^t \Lambda(u)du$ • $\mathbb{E}(N_{\mu(t)}) = \int_0^t \mathbb{E}(\Lambda(u))du$ • $Var(N_{\mu(t)}) = \int_0^t \mathbb{E}(\Lambda(u))du + Var(\int_0^t \Lambda(u)du)$

When modeling claims for reinsurance treaties with a one-year coverage period, our goal is to predict the number of claims on an annual scale. A homogeneous Poisson process is not well-suited for this purpose due to its fixed mean. Such processes are therefore better suited for modeling portfolios stable over time. To calibrate homogeneous Poisson process, one could assume that only the most recent year is considered, as underlying portfolios are less likely to change significantly. However, this approach would limit the data available for calibration, likely leading to underestimation since some claims might still be unreported and thus omitted in the calibration process. The same limitation applies to mixed Poisson processes. As an alternative, inhomogeneous or doubly stochastic Poisson processes seem more appropriate for modeling claim numbers. These approaches allow for varying rates over time, reflecting changes in the portfolio or other factors. However, calibrating such models requires defining rate parameters and mean value functions, which makes parameter estimation more complex. Furthermore, in the context of insurance, claims can be reported to the insurer several years after the occurrence of the event or the inception of the policy. Accurately modeling claim numbers evolution and linking them to their accident/underwriting year would thus be challenging to implement. More generally, the time process would also require detailed knowledge of claim attachment dates, potentially down to the month or day level. To ensure data consistency across entities, only the attachment year has been considered.

Given these challenges, we chose to focus on modeling the number of claims by attachment year using a discrete approach, which appears more suitable. To effectively model the number of claims, the ultimate number of claims per attachment year is also needed. However, the use of the Chain-Ladder model for projecting claim counts would not be appropriate. Due to the purely multiplicative nature of the Chain-Ladder method, if the number of claims in the first development year is zero, projecting claims using this method would lead to a null ultimate number of claims. Furthermore, the annual number of claims seems more sensitive to exposure evolution than individual claim amounts. For instance, a portfolio covering more risks is likely to face a higher number of claims than a smaller portfolio, whereas individual claim amounts might exhibit less variation. Considering the portfolio exposure - measured either in total insured value, number of risks or premium volumes - might thus be relevant when evaluating the ultimate number of claims.

To address these issues, a methodology based on the Schnieper model has been preferred to calculate the ultimate number of claims. In accordance with the Schnieper model, let us note:

- $N_{i,j}$ the number of annual claims for an attachment year i and a development year j.
- $C_{i,j}$ the cumulative number of claims for an attachment year i and a development year j. The cumulative number of claims is then written as : $C_{i,1} = N_{i,1}$ for $1 \leq i \leq n$and $C_{i,j+1} = C_{i,j} + N_{i,j+1}$ for $1 \leq i \leq n, 1 \leq j \leq n-1$.
- E_i is the underlying portfolio exposure measure for the attachment year i.

Let us introduce filtration $\mathcal{F}_k = \sigma(N_{i,j} \mid i + j \leq k + 1)$. Schnieper's model is based on the following assumptions.

- Assumption 1: The random variables $\left(N_{i_1,j}\right)_{1 \leq j \leq n}$ and $\left(N_{i_2,j}\right)_{1 \leq j \leq n}$ are independent for $i_1 \neq i_2$.
- Assumption 2: For $1 \leq j \leq n$ there exists $\lambda_j \geq 0$ such that for $1 \leq i \leq n$:

$$\mathbb{E}\left(N_{i,j} \middle| \mathcal{F}_{i+j-2}\right) = \lambda_j E_i$$

- Assumption 3: For $1 \leq j \leq n-1$ there exists $\sigma_j \geq 0$ such that for $1 \leq i \leq n$:

$$Var\left(N_{i,j} \middle| \mathcal{F}_{i+j-2}\right) = \sigma_j^2 E_i$$

By introducing filtration $\mathcal{B}_k = \sigma\left(N_{i,j} \middle| i + j \leq n + 1,\ j \leq k\right) \subset \mathcal{F}_k$ for $1 \leq k \leq n$ and under the previous assumptions, we see that $\mathbb{E}(N_{i,j}) = \lambda_j E_i$ and $Var\left(N_{i,j}\right) = \sigma_j^2 E_i$. As in the Chain-Ladder model, it can be shown that, under the above assumptions, the best linear estimator of λ_j in $\left(\hat{\lambda}_{i,j} = \frac{N_{i,j}}{E_i}\right)_{1 \leq i \leq n-j+1}$ conditional on $\mathcal{B}_j$ for the mean-squared error is :

$$\widehat{\lambda_j} = \frac{\sum_{i=1}^{n+j-1} E_i \hat{\lambda}_{i,j}}{\sum_{i=1}^{n-j+1} E_i} = \frac{\sum_{i=1}^{n+j-1} N_{i,j}}{\sum_{i=1}^{n-j+1} E_i}$$

For $1 \leq i \leq n$ and $n - i + 1 \leq j \leq n$, the estimator $\hat{C}_{i,j} = C_{i,n+1-i} + E_i \sum_{k=n+2-i}^{j} \hat{\lambda}_k$ is an unbiased estimator of $C_{i,j}$. The ultimate number of claims per year can finally be deduced by developing the number of claims triangle and conserving the ultimate column $\left(\hat{C}_{i,n}\right)_{1 \leq i \leq n}$.

Concerning the number of triangles, if a treaty covers multiple risks or cedents, and the historical number of claims per cedent/risk classification combination exceeds 20, there will be as many triangles as there are cedent and risk classification combinations. Otherwise, a single triangle will be calibrated.

For exposure data, one could use GWP, GEP, Sum Insured, or the Number of Risks per year for a specific cedent/risk classification combination. If a single triangle is used, the corresponding exposure values will be aggregated to provide a single exposure value. Regarding the choice of exposure type, the number of risks per year is more appropriate, as it is not inherently sensitive to economic fluctuations, unlike monetary values, which could introduce biases in the calculation of development factors. However, the final choice of exposure depends on data availability. If historical exposure data are insufficient for accurately estimating the ultimate loss, additional data may need to be requested from ceding entities.

Let us illustrate the methodology over a fictive example. We consider the following triangle of the number of annual claims $N_{i,j}$:

Table 17: Example of a development triangle for the number of claims

	1	2	3	4	5	6	7	8	9	10
2015	15	2	2	1	0	0	0	0	0	0
2016	14	5	3	0	0	0	0	0	0	
2017	19	1	1	2	0	0	0	0		
2018	17	4	1	2	0	0	0			
2019	21	7	2	0	1	0				
2020	23	5	3	0	0					
2021	26	4	2	0						
2022	25	5	1							
2023	23	4								
2024	36									

The historical GEP are used to model the exposure. The values applied in this example are provided in the table below.

Table 18: Example of historical exposure value

Attachment year	E_i
2015	59 333 657 €
2016	60 569 438 €
2017	61 535 436 €
2018	61 398 589 €
2019	64 589 136 €
2020	65 789 351 €
2021	66 589 632 €
2022	64 893 598 €
2023	67 968 326 €
2024	69 875 325 €

Once these foundations are established, it is possible to calculate the values of $\widehat{\lambda_j}$ for each development year. These values are listed below.

Table 19: Development factors for ultimate number of claims

	1	2	3	4	5	6	7	8	9	10
$\widehat{\lambda_j}$	3,25E-07	6,46E-08	2,97E-08	1,14E-08	2,68E-09	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00

It is then feasible to develop the previous triangle and estimate the ultimate number of claims. The ultimate number of claims is provided in the table below.

Table 20: Ultimate claim frequency

Attachment year	$\widehat{C}_{i,n}$
2015	20
2016	22
2017	23
2018	24
2019	31
2020	31
2021	32
2022	32
2023	30
2024	34

The model proposed earlier has several shortcomings. It relies on strong assumptions, notably the independence of the number of claims based on the development years, an assumption that may not always hold true. To verify hypotheses 2 and 3, it would also be possible to plot the values of $N_{i,j}$ against the values of E_i to check the model's assumptions and examine the standardized residuals. Additionally, this model is highly dependent on data and requires reliable exposure data, as well as sufficient historical depth to make accurate estimations. However, we have decided to apply this method due to its relative simplicity in implementation, and because it accounts for the underlying evolution of exposure in the calculation of the ultimate number of claims, both for risk attaching and loss occurring treaties.

To conclude this section, the following figures summarize the process implemented during the processing phase to obtain the outputs necessary for modeling the underlying claims experience of the treaties. Including this aspect within the pricing tool is one of the strengths of this project, as it addresses part of the research question by enabling the team to take control of the data transformation processes and clarifying the requirements necessary for modeling.

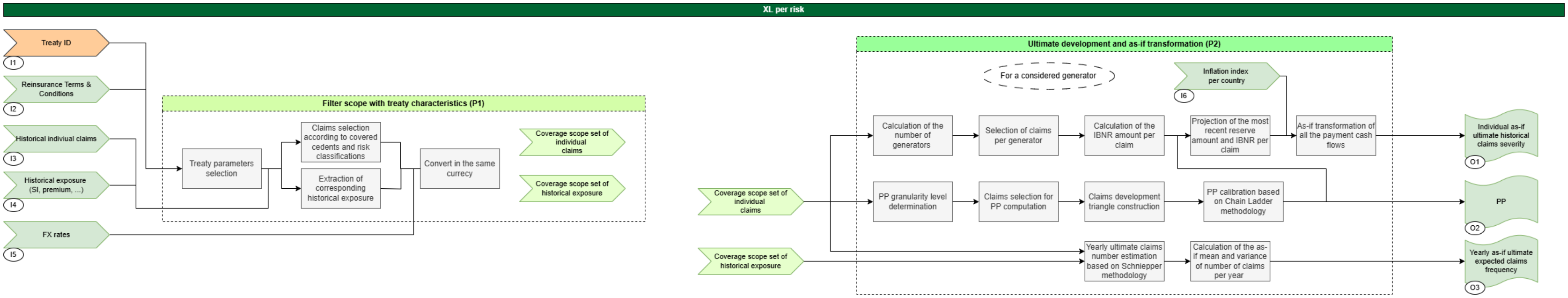

Figure 28: Data processing module overview – XL per risk treaties

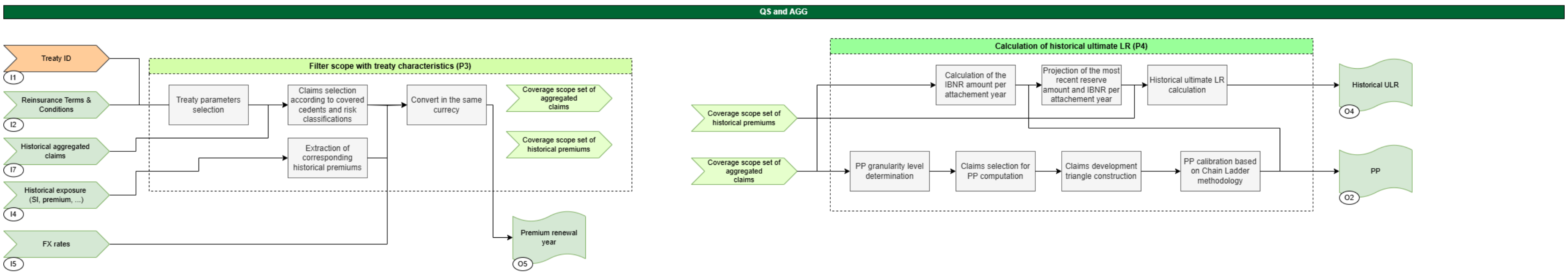

Figure 29: Data processing module overview – AGG and QS treaties

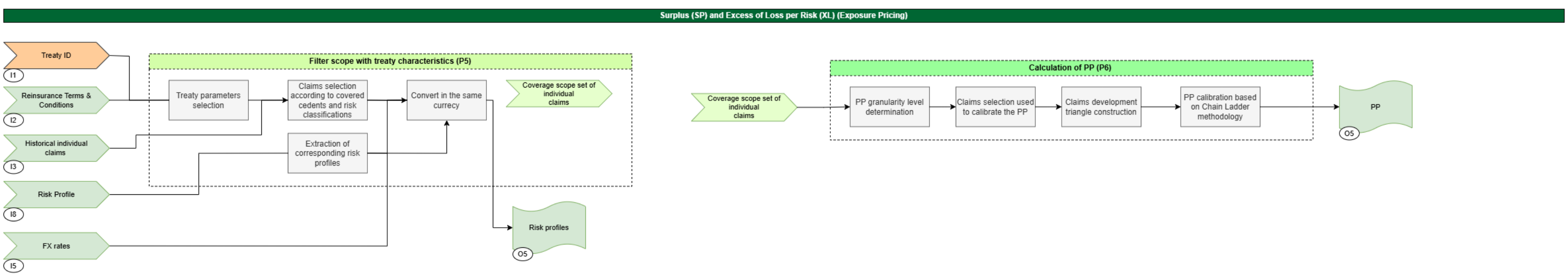

Figure 30: Data processing module overview – SP treaties

5. Modeling losses

To build AGCRe's own perspective on reinsurance treaties and increase pricing capacities, modeling the losses ceded to each treaty is crucial and widely employed since the treaty's risk assessment is based on the distributions of these underlying losses. Several methods can be implemented for this purpose. For instance, numerous resources enable the calibration of statistical distributions using historical claims data to derive ceded loss distributions through mathematical modeling. In order to get an estimate of the total underlying loss experience S per year of coverage of a reinsurance treaty, one could use the normal approximation based on the first few moments of the distribution S. The Panjer recursion or the Fast Fourier transform can also be implemented to approximate the distribution of the yearly loss experience. The distribution of recoveries for QS and AGG can then be deduced mathematically in a relatively simple way using the cumulative distribution function of the distribution S. However, such approaches have limitations due to the complexity and unique features of reinsurance treaties which make it challenging to model reinsurer losses accurately.

Given the growing computational power available for handling complex calculations, the Monte Carlo method has been preferred. It is a computational technique based on repeated random sampling in order to estimate numerical values. This approach offers several advantages for reinsurance modeling since treaty risk and profitability indicators are difficult to compute directly. Its flexibility makes it suitable for accommodating complex treaty structures and non-linear features, like indexation clauses, which analytical models struggle to handle. The Monte Carlo method is thus used to model the underlying claims covered by any reinsurance treaties. By applying treaty conditions to simulated claims amounts, it becomes possible to derive a distribution of losses ceded to the reinsurer. This loss distribution is then used to calculate various indicators, which will be detailed in the next section.

To model reinsurer losses, two distinct methodologies has been selected, depending on the type of treaty:

- **Excess of Loss per risk:** The modeling focuses on individual underlying claims. The collective risk model, a widely used framework in insurance, has been selected for this purpose. In cases where historical claims data is insufficient or unreliable, the exposure pricing method will also be employed as an alternative.
- **Surplus:** Surplus treaties are currently not modeled within the internal model. In this work, they will be evaluated using exposure curves, and only for the property LoB. SP treaties, which are influenced by several factors such as sums insured, has been modeled in this way due to the practicality of exposure curves. These curves allow for the derivation of various quantities from the data contained in risk profiles.
- **Aggregate and Quota Share:** The methodology used to model these previously unmodeled treaties relies on modeling the ultimate loss ratio. This approach focuses on the aggregate behavior of claims rather than individual claim modeling to estimate reinsurer losses.

The following section will focus on presenting the implemented methodology through the analysis of various treaties. The figure below helps get a good grasp of the modeling process, including its inputs and outputs. The elements marked with a "P" in this diagram will be detailed throughout this section.

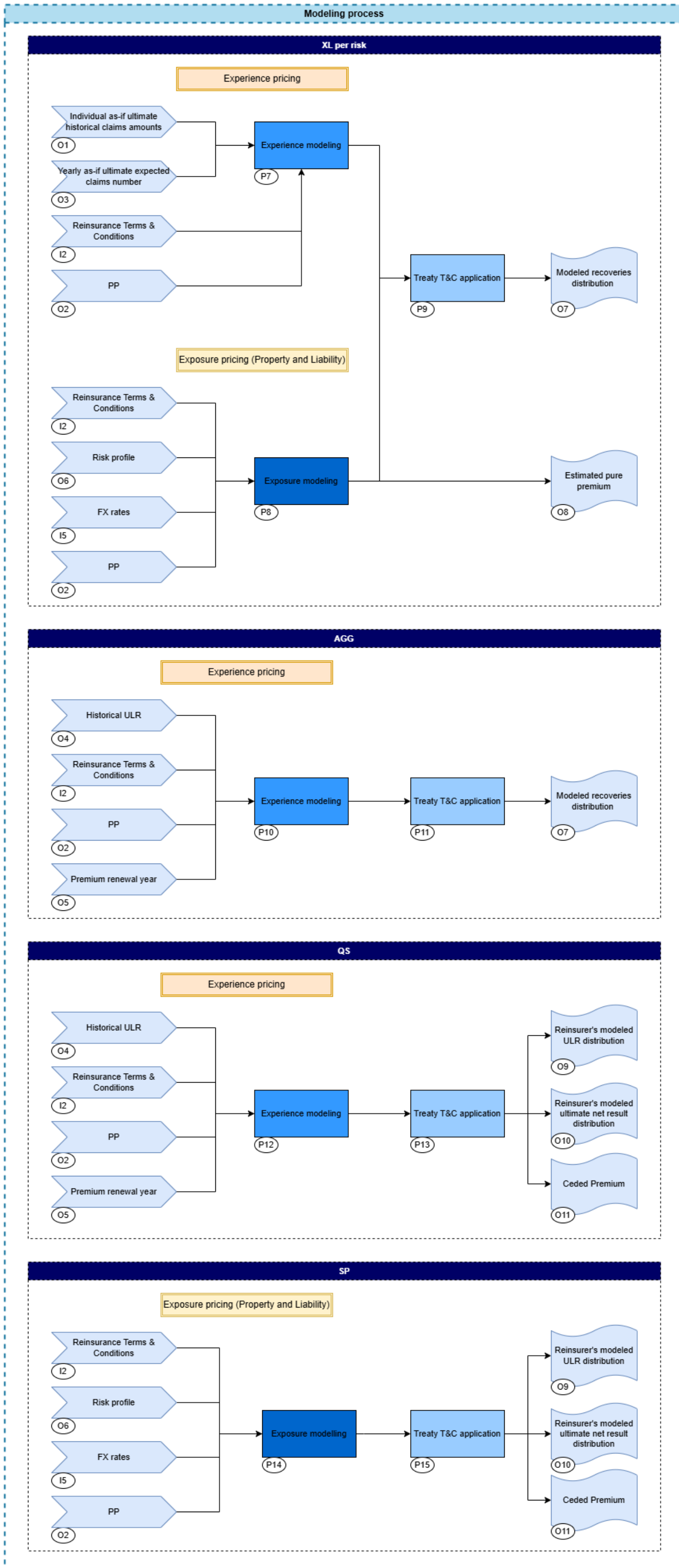

Figure 31: Modeling process

5.1 Modeling losses by frequency severity (XL per Risk)

For an XL per risk treaty, the focus lies on modeling the individual underlying claims that fall within the scope of the treaty. To achieve this, the collective model has been selected. This choice is justified by the model's flexibility and adaptability to the available data. Additionally, for XL treaties, the treaty terms are applied at the level of individual claims, making the collective model a coherent choice for this type of treaty. XL treaties are studied in this actuarial work because they represent an important part of underwritten treaties: providing the Analytics and Pricing team with an internal perspective on theses treaties is one of the objectives of this paper, in addition to addressing treaties that are not modeled within the internal model (specifically non-pooled treaties and for certain cedents/risk classifications).

In the collective model, the total underlying loss experience per year of coverage of a reinsurance treaty is denoted by the random variable S, defined as a random sum:

$$S = \sum_{k=1}^{N} X_k$$

Where N denotes a discrete random variable corresponding to the number of claims attached to the reinsurance treaty and $(X_k)_{1 \leq k \leq N}$ is a sequence of independent and identically distributed random variables corresponding to the amount attached to the claim k. Furthermore, it is assumed that for all k, N and X_k are independent. In this study, N will be referred as the frequency distribution while X corresponds to the severity distribution.

To calibrate the frequency and severity distributions, the outputs of the data processing module for XL treaty will be used:

- The historical individual as-if ultimate claims amount distribution
- The historical ultimate frequency per attachment year distribution

In our model, the calibration of frequency and severity distributions depends on the number of available severity values for each cedent-risk crossover covered by the treaty:

- When the number of historical claims exceeds 20 for all cedent-risk crossovers within the treaty, specific claims generators are calibrated for each crossover.
- When the number of severity values is 20 or less, a single claims generator is calibrated for all claims.

In practice, treaties covering multiple cedents and various risks encounter significant variability in claim frequency and severity, depending on the cedent and the risk characteristics. This variability challenges the assumptions of independence and identical distribution of claims, as well as the independence between frequency and severity. By calibrating specific frequency and severity distributions for each cedent-risk crossover, the model groups claim related to more homogeneous policies, better aligning with the frequency-severity model's assumptions. However, this approach introduces a trade-off: fine segmentation reduces the number of claims available for each group. The choice of a threshold of 20 claims was made to balance these factors. On one hand, the threshold ensures the model's assumptions are respected by avoiding overly heterogeneous risk groupings. On the other hand, it maintains a sufficient sample size to allow statistically reliable calibration. However, by separating the modeling processes based on cedent and risk classification crossovers, the correlations between the evolution of claim frequency and severity are not accounted for. Each crossover is modeled independently, which constitutes a limitation in calibrating multiple generators. Despite this drawback, the method will still be applied since reinsurance treaties generally cover a single cedent and a single risk classification.

This methodology has also other drawbacks: for XL treaties with high retentions, there may not be enough loss experience to produce credible results. The method also assumes that the treaty will face similar loss experiences, which may not hold true if the covered insurance portfolio composition changes over time. Even though the underlying portfolio exposure evolution is backed in the Schnieper claims frequency statistical model, it is a priori not the case in the claim's sizes statistical models. Indeed, the claim amounts are simply corrected through an as-if index which does not consider the underlying insurance portfolio evolution: for instance, if the cedent writes bigger risks, bigger losses can be expected.

It is also important to note that if the total number of historical claims above a certain threshold covered by the treaty is insufficient, additional data must be sought from the cedents, if possible. In such situations, the exposure rating method, which will be detailed later, can officiate as an alternative approach.

To clarify the approach used, a specific XL treaty with the following parameters will be studied throughout this part:

- Limit: 1 500 000 €.
- Retention: 1 000 000 €.
- Reinstatement: 3 reinstatements available, each at 100% of the original premium.
- Indexation Clause: None.
- Loss Basis: Loss Occurring (losses are covered based on when they occur, regardless of the policy's inception date).
- AAD/AAL: No Annual Aggregate Deductible (AAD) and an Annual Aggregate Limit (AAL) of 6 000 000 €, covering only one cedent and one risk classification.

This XL treaty will be referred to as "XL treaty number 1".

5.1.1 Frequency modeling

Let us $(N_i)_{1 \le i \le n}$ represent the vector of observations corresponding to the ultimate number of claims recorded per attachment year for the considered generator.

In the context of frequency distribution modeling, the candidate distributions used to model claims frequency are some of the (a,b,1) class from (Klugman, Panjer, & Willmot, 2012):

- Poisson distribution
- Negative Binomial distribution
- Binomial distribution

The main properties of each distribution are described in Appendix A.

5.1.1.1 Parametric estimation

The figure below highlights the historical ultimate number of claims reported to AXA Group Ceded Re for the XL treaty number 1. Over time, the number of claims has decreased, primarily due to a reduction in exposure.

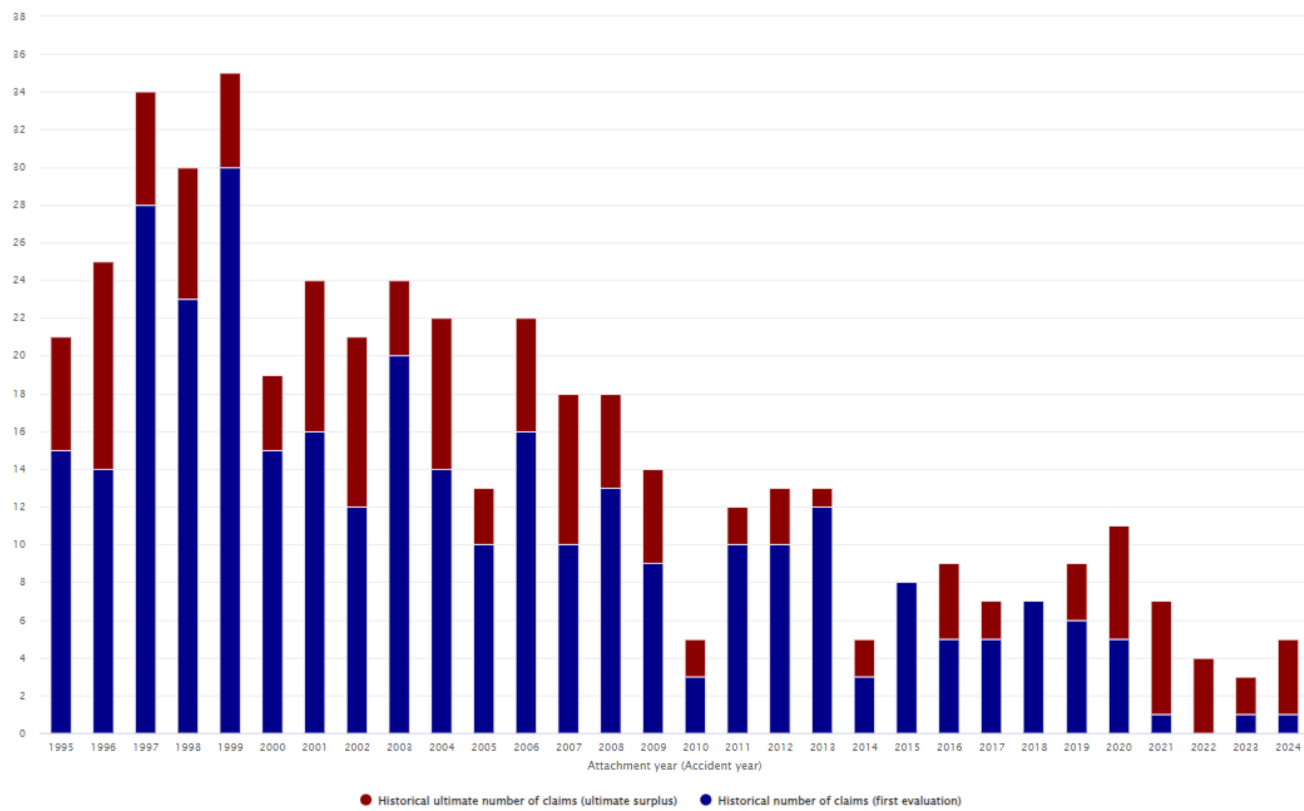

Figure 32: Historical ultimate number of claims of the XL treaty number 1

Once the historical claims data has been collected, the next step is to estimate the parameters of each candidate distribution to calibrate the frequency model. Let us note θ the vector corresponding to the parameters of the distribution and $f(x;\theta)$ the probability density function distribution.

To estimate θ, it would be possible to use the distribution $(N_i)_{1\leq i\leq n}$ of the ultimate number of annual claims. However, this methodology would not consider changes in exposure for the incoming year. Let us note E_{n+1} the exposure value of the incoming year (for the purposes of this report, this year would correspond to 2025 but here 2024 has been retained due to a lack of data). Using the historical ultimate number of claims distribution, one can calculate the estimator $\hat{\lambda} = \frac{\sum_{i=1}^{n} N_i}{\sum_{i=1}^{n} E_i}$ and deduce an estimator of the average number of ultimate claims per year for the coming year $\widehat{\mu_N} = \hat{\lambda} E_{n+1}$. Concerning the variance of the ultimate number of claims per year, one can make the additional assumption, aligned with the Schnieper model, that $Var(N_i) = \sigma^2 E_i$. From this assumption, we derive the estimator $\widehat{\sigma^2} = \frac{1}{n-1}\sum_{i=1}^{n} \frac{\left(N_i - \hat{\lambda} E_i\right)^2}{E_i}$ and the variance of the number of ultimate claims per year $\widehat{\sigma_N^2} = \widehat{\sigma^2}\, E_{n+1}$. The parameters of the frequency distribution can thus be computed from the mean $\widehat{\mu_N}$ and the variance $\widehat{\sigma_N^2}$ using the method of moments.

The method of moments is a statistical method which enables us to estimate the parameters of a probability distribution. It is based on the equalization of the theoretical and empirical moments of a considered distribution. The moments of a distribution are measures that summarize certain characteristics of the distribution, such as mean (moment of order 1), variance (moment of order 2), skewness (moment of order 3), and kurtosis (moment of order 4). The empirical moments are then equated with the theoretical, leading to a system of equations. To estimate k parameters, the first k moments are considered. Solving the system of equations yields the parameter estimators.

Since the candidate distributions have at most two parameters, only the first two moments (theoretical and empirical) are exploited for parameter estimation. Regarding the empirical moments, the moments previously calculated can be used ($m_1 = \widehat{\mu_N}$ and $m_2 = \widehat{\sigma_N^2}$). Solving the moments systems yields the following parameter estimators for each of the candidate distributions:

- Poisson distribution : $\hat{\lambda} = \widehat{\mu_N}$
- Negative Binomial distribution: $\hat{n} = \frac{\widehat{\mu_N}^2}{\widehat{\sigma_N^2} - \widehat{\mu_N}}$ and $\hat{p} = \frac{\widehat{\mu_N}}{\widehat{\sigma_N^2}}$
- Binomial distribution: $\hat{n} = \frac{\widehat{\mu_N}^2}{\widehat{\mu_N} - \widehat{\sigma_N^2}}$ and $\hat{p} = 1 - \frac{\widehat{\sigma_N^2}}{\widehat{\mu_N}}$

5.1.1.2 Frequency distribution selection

Once the parameters have been estimated for each distribution (if possible), the frequency model that best matches the observations is to be selected. The method proposed to select the best frequency distribution is relatively simple and is based on the properties of each distribution using the dispersion ratio $\frac{\widehat{\sigma_N^2}}{\widehat{\mu_N}}$:

- The Negative Binomial distribution is chosen if the dispersion ratio exceeds 1.1, since the Negative Binomial distribution inherently allows the variance to be greater than the mean.
- The Poisson distribution is picked if the dispersion ratio is between 0.9 and 1.1, as the Poisson distribution has the same mean and variance.
- Binomial distribution is considered if the dispersion ratio is less than 0.9, because the Binomial distribution has a smaller variance than the mean.

For the XL treaty under analysis:

- The estimated mean $\widehat{\mu_N}$ is 13.87.
- The estimated variance $\widehat{\sigma_N^2}$ is 58.35.
- The dispersion ratio $\frac{\widehat{\sigma_N^2}}{\widehat{\mu_N}}$ exceeds 1.1.

Thus, the Negative Binomial distribution is selected for modeling annual claims frequency. The parameters of the Negative Binomial distribution are:

- $\hat{n} = 4.33$
- $\hat{p} = 0.24$

The density functions of the Poisson and Negative Binomial distributions, calibrated on the coverage scope of the considered XL treaty, are compared in the figure below. The Binomial distribution could not be calibrated, as the variance exceeded the mean, making it unsuitable.

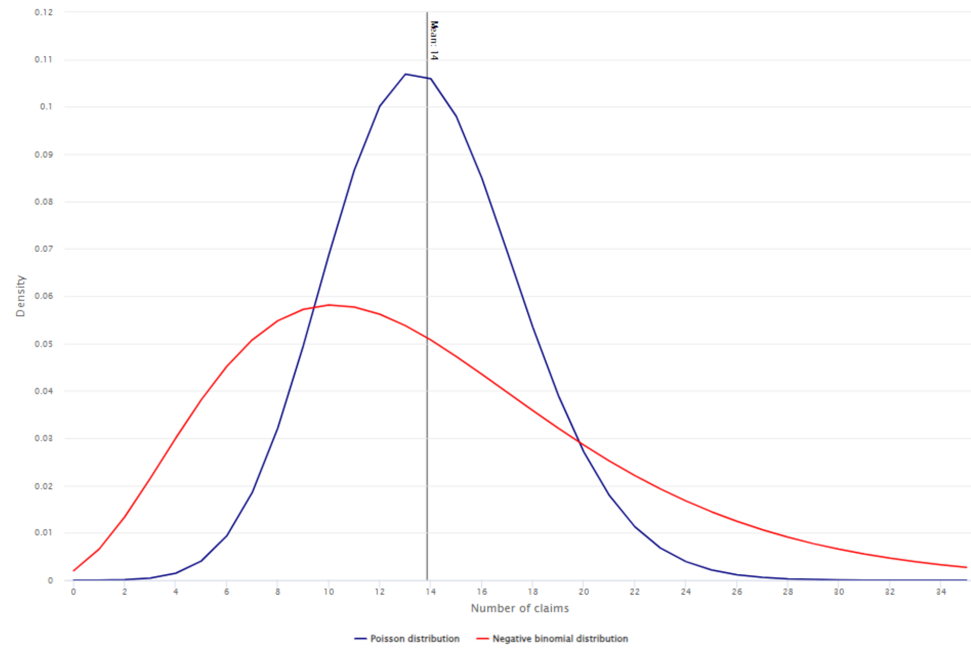

Figure 33: Comparison of Poisson and Negative Binomial probability distribution - XL treaty number 1

5.1.2 Severity modeling

Once the frequency distribution has been chosen and can be modeled, the severity distribution must be calibrated. Let $(X_i)_{1 \le i \le n}$ represent the as-if ultimate amounts related to a considered loss generator, which is assumed to be a set of continuous iid random variables. The figure below presents the historical distribution of claims for the XL treaty number 1, showing both the reserve amounts, the paid amounts, and the as-if ultimate amounts, with 448 claims, spanning from 1995 to 2024. Out of this historical data, only 13 claims have consumed the capacity.

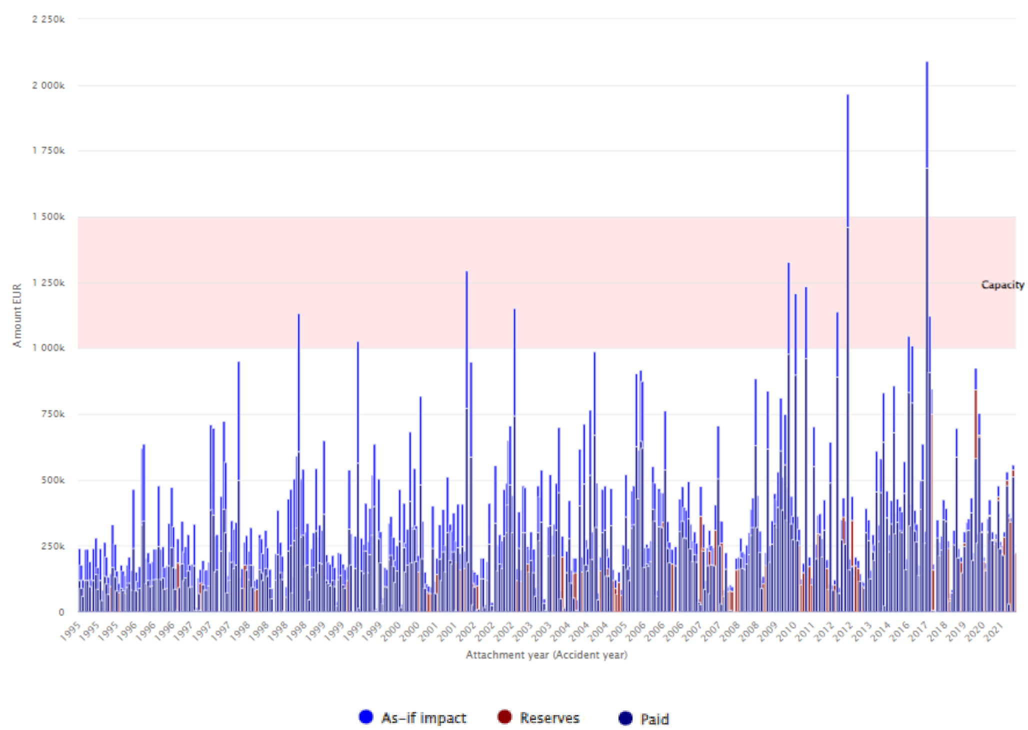

Figure 34: Distribution of historical as-il ultimate claims - XL treaty number 1

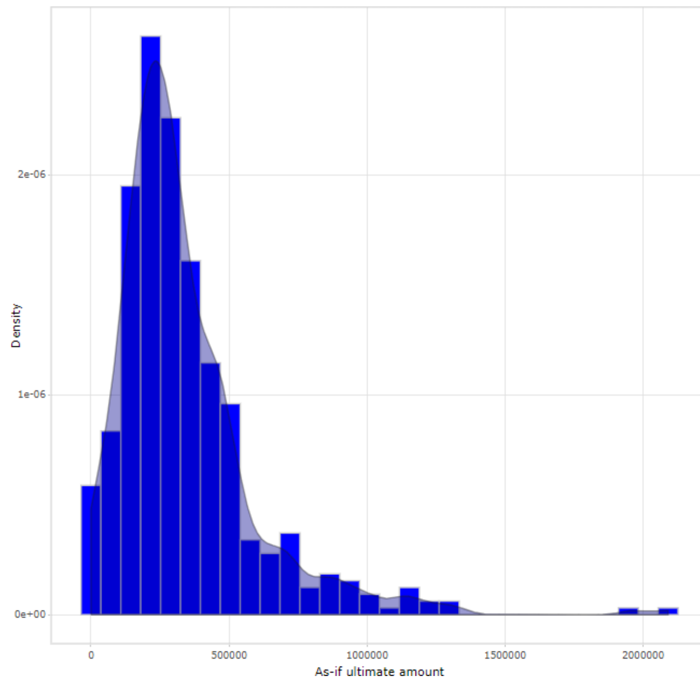

Figure 35: Density of historical as-if ultimate claims - XL treaty number 1

A large array of both parametric and non-parametric continuous distributions were considered as severity models candidates.

5.1.2.1 Continuous severity distributions

First, let us assume that the severity distribution is the same for the entire range of possible claim sizes, without any specific consideration for the tail of the distribution. Thus, a continuous distribution is fitted on all observations $(X_i)_{1 \leq i \leq n}$. The list below exhibits the so-called continuous severity distributions that are calibrated on all observations:

- Exponential distribution
- Log-Normal distribution.
- Gamma distribution
- Generalized Pareto distribution
- Weibull distribution
- Fréchet distribution
- Estimation by kernel method

These distributions have been selected within the context of modeling large claims in reinsurance: distributions with heavier tails are thus expected (except for the gaussian kernel and exponential distribution[1]). Additionally, all these distributions are defined on a positive and unbounded support, which is the appropriate domain here. The parameters and properties of each distribution are listed in Appendix B.

5.1.2.2 Spliced severity distributions

Large claims liable to exceed the retention of XL reinsurance treaty can significantly vary in size. Fitting a single distribution across the entire range of those atypical claims could lead to a poor fit regarding the tail of the distribution. This is problematic because the underlying risk distribution tails are crucial for accurately modeling XL treaties, where retentions are typically high.

[1] *The exponential distribution is often considered a reference distribution to compare distribution tails which justifies its use.*

A well-fitting severity model that captures the entire range of loss sizes - ranging from many smaller claims to a few large ones - is thus needed for effectively modeling severity.

One way to deal with this problem is by fitting a spliced distribution. The basic idea here is to stick pieces of two (or more) different distributions together. An m-component spliced distribution then has a density $f(.)$ expressed as:

$$f(x) = \begin{cases} \pi_1 \dfrac{f_1(x)}{F_1(c_1) - F_1(c_0)},\ c_0 < x \leq c_1 \\ \quad\quad\quad\quad \cdots \\ \quad\quad\quad\quad \cdots \\ \quad\quad\quad\quad \cdots \\ \pi_m \dfrac{f_m(x)}{F_m(c_m) - F_m(c_{m-1})},\ c_{m-1} < x \leq c_m \end{cases}$$

Where $\pi_i > 0$ and $\sum_{j=1}^{m} \pi_j = 1$ and f_j, respectively F_j for $j = 1, .., m$ are densities and CDFs of random variables. Restrictions on the parameters can be imposed, requiring continuity of the density f_j at the junction points c_j.

For the sake of simplicity and to have enough data to fit severity distributions, we will restrict ourselves to the case of $m = 2$. Furthermore, a spliced distribution will only be fitted when the number of historical claims is sufficient, i.e., more than 50 in this work. The table below indicates the spliced distributions implemented in this actuarial work:

Table 21: Spliced severity distribution retained for severity modeling

Bulk Distribution	Tail Distribution
Normal	GPD
Log-Normal	Pareto *(only for a Mixed Erlang bulk distribution)*
Gamma	
Weibull	
Gaussian Kernel	
Mixed Erlang	

We will now discuss the properties of the Mixed Erlang distribution and explain our choice of the Pareto and Generalized Pareto distributions to model the tail distribution.

5.1.2.2.1 Mixed Erlang

The Erlang distribution is a two-parameter family of continuous probability distributions with support $x \epsilon [0; +\infty)$. The two parameters are a positive integer r defined as the shape of the distribution and a positive real number λ defined as the rate. Let $r \in \mathbb{N}$ and $\lambda > 0$. We list below the various properties of the random variable X which we assume to follow an Erlang distribution with parameters r and λ:

- Probability density: For all $t \in \mathbb{R}^{+*}$, $f(t) = \frac{\lambda^k t^{k-1} \exp(-\lambda t)}{(k-1)!}$
- Mean: $\mathbb{E}(X) = \frac{r}{\lambda}$
- Variance: $Var(X) = \frac{r}{\lambda^2}$

In this work, a mixture of M Erlang distributions defined with a common scale parameter λ was considered. The density function of such distribution for $t > 0$ is:

$$f_{ME}(t, r, \alpha, \lambda) = \sum_{j=1}^{M} \frac{\lambda^{r_j} t^{r_j - 1} \exp(-\lambda t)}{(r_j - 1)!}$$

where the positive integers $r = (r_1, \dots, r_M)$ with $r_1 < \dots < r_M$ are the shape parameters of the Erlang distributions, and $\alpha = (\alpha_1, \dots, \alpha_M) > 0$ with $\sum_{j=1}^{M} \alpha_j$ are the weights of the mixture. Mixed Erlang are also dense in the space of positive continuous distributions on $\mathbb{R}^+$, which is an interesting property as it allows us to use a unique class of distributions to model different forms of distributions.

5.1.2.2.2 Tail modeling with Extreme Value Theory

The decision to model the tail of the distribution using a GPD distribution is based on Extreme Value Theory (EVT). To begin, let us introduce the Fisher-Tippet theorem, which characterizes the limiting behavior of normalized sample maxima.

Fisher-Tippet Theorem:

We define $M_n = max\,\{X_1, \dots, X_n\}$, where $X_1, \dots, X_n$ is a sequence of independent random variables having a common unknown cumulative distribution function F. Suppose there exist sequences of real valued numbers $\{a_n\} > 0$ and $\{b_n\}$ such that $P(\frac{M_n - b_n}{a_n} \leq t) \to G(t)$ as $n \to \infty$ where G is a non-degenerate distribution function. Then G is of the same type[1] as the following distribution called the Generalized Extreme Value distribution $GEV_{\mu,\sigma,\gamma}$:

$$GEV_{\mu,\sigma,\gamma}(t) = \begin{cases} \exp\left(-\left(1 + \gamma\left(\frac{t-\mu}{\sigma}\right)\right)_+^{-\frac{1}{\gamma}}\right), & \gamma \neq 0 \\ \exp\left(-\exp\left(-\frac{t-\mu}{\sigma}\right)\right), & \gamma = 0 \end{cases}$$

Where $x_+ = \max(x, 0)$ and $\sigma > 0$. Thus, the GEV is defined on $(\mu - \frac{\sigma}{\gamma}; +\infty)$ for $\gamma > 0$, on $(-\infty; \mu - \frac{\sigma}{\gamma})$ for $\gamma < 0$ and on $\mathbb{R}$ if $\gamma = 0$.

One can thus define the max-domain of attraction, which contains the distributions for which the previous theorem applies. Within the max-domain of attraction, distributions can be described in terms of extremal classes using the tail quantile function. Given that F is a CDF, its quantile function is defined by the inverse function $Q(p) = \inf\{x | F(x) \geq p\}, p \in (0, 1)$. The tail quantile function is thus $U(t) = Q\left(1 - \frac{1}{t}\right), t > 1$. One can say that F belongs to the extremal class $C_\gamma(a)$ if there exists $\gamma \in \mathbb{R}$ and an ultimately positive function $a(.)$ such that: $\frac{|U(tz) - U(t)|}{a(t)} \to \int_1^z v^{\gamma - 1} dv$ for all $z \geq 1$ as $t \to \infty$. Depending on the value of γ, one can define the three extremal classes:

- The Fréchet/Pareto class when $\gamma > 0$
- The Gumbel class when $\gamma = 0$
- The extremal Weibull class when $\gamma < 0$

The following table displays some of the previous distributions that belong to a max-domain of attraction, with their tail characteristics:

[1] *If F_1 and F_2 are two distribution functions and there exist constants $a > 0$ and b such that $F_2(ax + b) = F_1(x)$ for all x then F_1 and F_2 are of the same type.*

Table 22: Overview of extremal classes and their component distributions

Distribution	**Extremal class $C_\gamma(a)$**	**Sign of γ**	**Tail**
Beta	Extremal Weibull	$\gamma < 0$	Light tail
GPD ($\gamma < 0$)	Extremal Weibull	$\gamma < 0$	Light tail
Exponential and GPD ($\gamma = 0$)	Gumbel	$\gamma = 0$	Light tail
Weibull ($\alpha > 1$)	Gumbel	$\gamma = 0$	Light tail
Weibull ($\alpha < 1$)	Gumbel	$\gamma = 0$	Heavy tail
Log-Normal	Gumbel	$\gamma = 0$	Heavy tail
Gamma	Gumbel	$\gamma = 0$	Heavy tail
GPD ($\gamma > 0$)	Fréchet/Pareto	$\gamma > 0$	Heavy tail
Pareto	Fréchet/Pareto	$\gamma > 0$	Heavy tail
Fréchet	Fréchet/Pareto	$\gamma > 0$	Heavy tail

Rather than modeling block maxima of losses' amounts, we are more interested in the modeling of extremes losses over a high threshold u to model the tail of the distribution. The distribution of excesses over a high threshold u is defined as $F_u(x) = \mathbb{P}(X - u \leq x \mid X > u)$ for $0 \leq x \leq x^F - u$ where $x^F = \sup\{x : F(x) > 1\}$.

The following theorem states that the convergence of the maximum of appropriately normalized iid random variables is equivalent to the convergence of the exceedance distribution to a Generalized Pareto Distribution (GPD):

Let $\mu \in \mathbb{R}$, $\sigma > 0$ and $\gamma \in \mathbb{R}$ and $t^F = \sup\{x : F(x) > 1\}$. The following propositions are equivalent:

Prop 1: There exist two sequences a_n and b_n such that:

$$P\left(\frac{M_n - b_n}{a_n} \leq t\right) = [F(a_n t + b_n)]^n \underset{n\to\infty}{\longrightarrow} GEV_{\mu,\sigma,\gamma}(t).$$

Prop 2: There exists a positive function $\sigma(.)$ such that:

$$\lim_{u \to t^F} \sup_{t \in [0, t^F - u)} | F_u(t) - GPD_{\sigma(u),\gamma}(t) | = 0$$

Where $GPD_{\sigma(u),\gamma}$ is defined in Appendix B.

Thus, the GPD is the natural model for the distribution of excess values above sufficiently high thresholds. One can also notice the following relationship between the GEV and the GPD:

$$GPD_{\sigma,\gamma}(t) = 1 + \log(GEV_{0,\sigma,\gamma}(t))$$

In this actuarial project, the GPD has been adopted for modeling the tail of the distribution, as it represents the theoretical limit for distributions above a threshold and can model both heavy and light tails, depending on the shape parameter γ. This approach is thus known as the Peaks-Over-Threshold (POT) method. Additionally, the Pareto distribution has also been added for comparison to the spliced Mixed Erlang distribution given that most claims exhibit heavy-tailed characteristics. However, since the GPD is a limit distribution, incorporating alternative distributions, such as the Fréchet distribution for heavier tail, or the Log-Normal/Weibull distribution for lighter tail, could enhance the range of tail modeling options. Furthermore, practical problems can arise when using GPD or Pareto distributions. Indeed, these models may allocate some probability mass to loss amounts that are unrealistically large, without any consideration to the maximum possible loss amount. Thus, using upper-truncation of a Pareto-type distribution such as the GPD can be of interest to address this issue and could be a future improvement of the methodology.

5.1.2.3 Parameter estimation

We will now proceed with the estimation of the parameters for our different candidate distributions.

5.1.2.3.1 In the case of simple severity distributions

In the case of distributions defined in 5.1.2.1, two methods will be considered for parametric estimation:

- Maximum likelihood estimation (see Appendix C)
- Estimation using the method of moments (see 5.1.1.1)

For each of the continuous severity distributions, these two parameter estimation methods will be applied. Additionally, normalizing the ultimate loss distributions will be performed to facilitate the parameter estimation process if necessary.

Other methods can be developed for estimating the parameters of a distribution such as percentile matching or least square estimation. Percentile matching is similar to the method of moments and consists in equalizing arbitrarily chosen theoretical and empirical percentiles. The least square estimation, also known as the method of probability plotting, works by linearizing the theoretical CDF and then using least squares estimation to determine regression parameters, which are subsequently used to derive the parameters of the probability distribution. These methodologies have not been implemented in the tool but could be considered for future development.

5.1.2.3.2 In the case of spliced severity distributions

For spliced severity distributions, the number of parameters to be determined is slightly greater. Indeed, one needs to estimate the parameters of the distributions considered for the bulk and tail of the severity distribution, as well as the threshold at which the distribution changes, and the weights π_1 and π_2 of the spliced distribution.

However, before opting to fit a spliced distribution, it is necessary to examine the tail of the distribution to determine whether it can be properly modeled by a Generalized Pareto Distribution or a Pareto.

A preliminary criterion is to examine the shape of the distribution tail. Let us introduce the notion of QQ plot. By denoting $\hat{q}_i$ as the quantiles of the empirical distribution and q_i as those of the theoretical distribution, the QQ plot represents the points $M(\hat{q}_i, q_i)$. If the two distributions being compared are similar, the points in the QQ plot will align along a straight line. Conversely, deviations from the straight line indicate differences between the theoretical and observed distributions. This graph is particularly useful for assessing the quality of fit and will be analyzed once various distributions have been fitted. To analyze tail shapes, specific types of QQ plots will be displayed.

The first QQ plot to be constructed is the exponential QQ plot, which helps determine whether the claim distribution has a heavier tail than the exponential distribution. Let $X_{1,n} \leq \cdots \leq X_{n-k+1,n} \leq \cdots \leq X_{n,n}$ be the ordinate vector of observations. The exponential QQ plot is defined as $\left(-\log\left(1-\frac{i}{n+1}\right); X_{i,n}\right)$. If the data originates from a distribution with a tail heavier than the exponential, which is expected when modeling large claims, the exponential QQ plot will ultimately exhibit a convex shape. Conversely, for tails lighter than the exponential, the QQ plot will eventually appear concave. In the following example (*Figure 36*), the exponential QQ plot is ultimately convex, indicating a tail heavier than the exponential distribution. However, this characteristic is less pronounced in the case of the XL treaty studied in this section (*Figure 37*). Indeed, while the exponential QQ plot is convex, it is noticeably less marked compared to *Figure 36*. This suggests that the tail of the distribution is only slightly heavier than the exponential distribution, implying that the extreme value index γ is positive but close to 0.

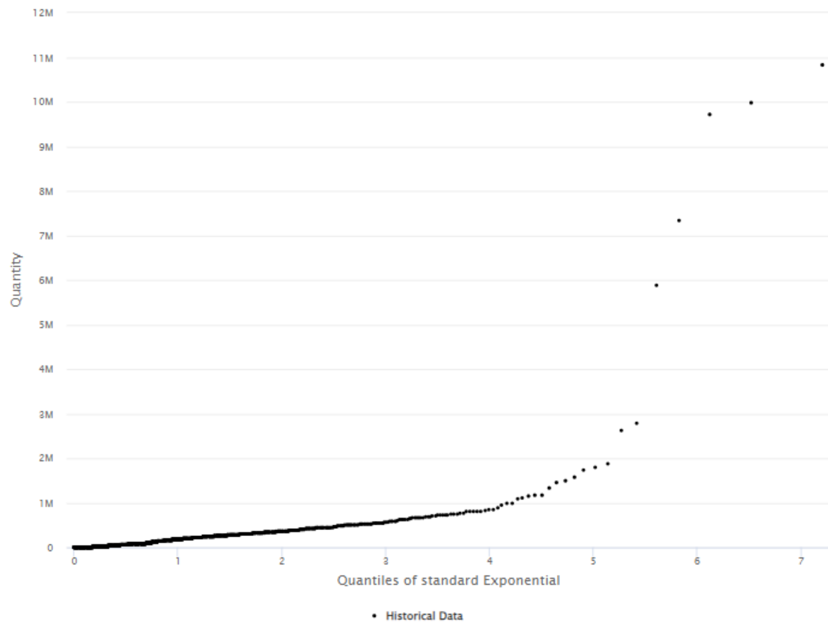

Figure 36: Example of exponential QQ-plot

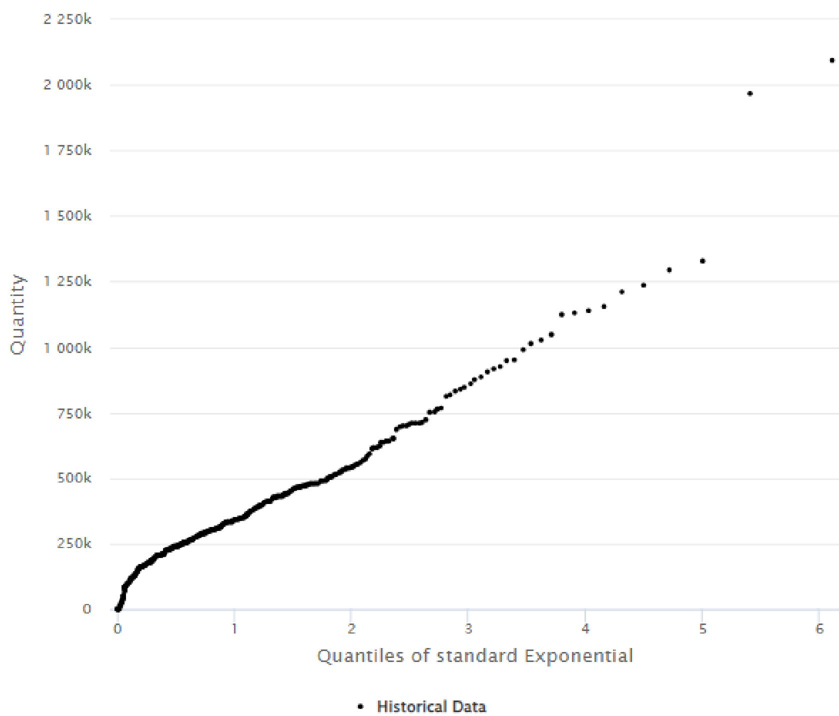

Figure 37: Exponential QQ-plot – XL treaty number 1

If the exponential QQ plot is ultimately convex, the next step is to examine Log-Normal or Pareto QQ-plots to gain insights into the value of the extreme value index γ and the tail behavior:

- The Pareto QQ-plot is defined as $(-\log\left(1-\frac{i}{n+1}\right); \log(X_{i,n}))$
- The Log-Normal QQ-plot is defined as $(\phi^{-1}\left(\frac{i}{n+1}\right); \log(X_{i,n}))$, where ϕ^{-1} is the inverse of the standard Normal cumulative distribution function.

These plots allow us to visually assess whether the tail behavior aligns with a Log-Normal or Pareto distribution, based on the linearity observed in the upper tail of the plot.

In the following example, the data appears to exhibit a Pareto tail behavior, as indicated by the linearity observed in the upper end of the Pareto QQ plot. This suggests that the extreme value index γ is strictly positive.

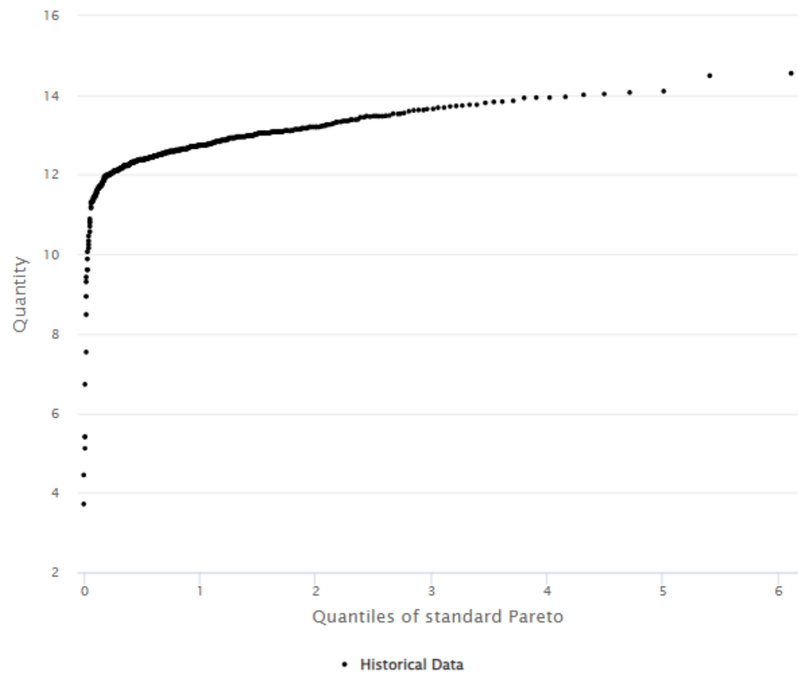

Figure 38: Pareto QQ plot – XL treaty number 1

However, since the Log-Normal QQ plot also exhibits a linear trend, we can expect the extreme value index γ to be close to 0, as the Log-Normal distribution belongs to the Gumbel domain of attraction, characterized by an extreme value index equal to 0.

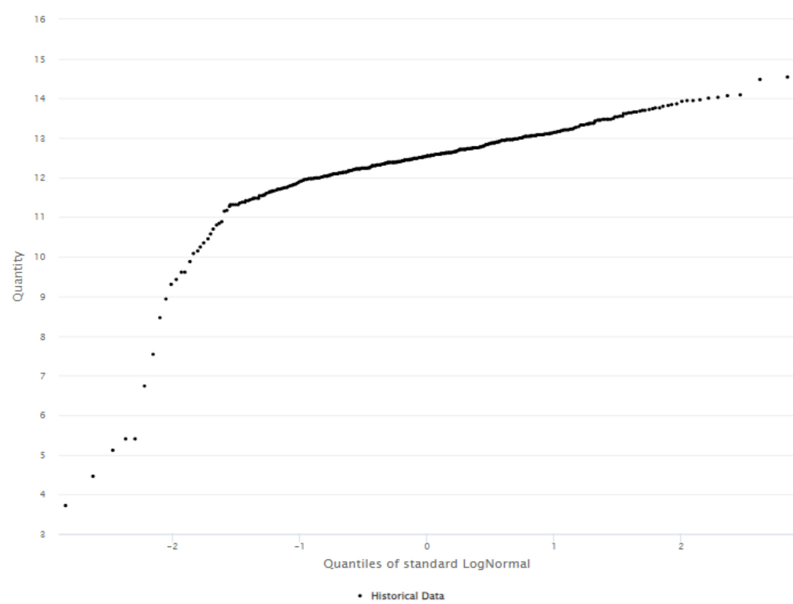

Figure 39: Log-Normal QQ plot of the XL treaty number 1

Choice of threshold and weights π_1 and π_2

Once the relevance of modeling the distribution trial using a GPD has been established, as part of the Peaks-Over-Threshold approach, the appropriate threshold u and the weights of our spliced distribution must be determined. In practice, the most critical step is the threshold selection. Using a low threshold increases the amount of data, but the approximation by the GPD becomes less accurate. On the other hand, using a too high threshold reduces the amount of data available, making the estimates of the tail distribution parameters less reliable. Several methods are available for choosing an appropriate threshold, but keeping in mind the operational objective of simplifying the tool's use as much as possible, only two methods were employed:

- By graphical method
- By minimizing the approximate mean squared error

Let us look at threshold determination using the graphical method.

As a first step, we can graphically estimate the threshold u using the mean excess function. Indeed, suppose that exceedances $(X_i - u)_{1 \le i \le n}$ above the threshold u follow a distribution $GPD_{\sigma(u),\gamma}$ such that $\mathbb{P}(X - u \ < x \,|X > \ u) = \ 1 - \left(1 + \gamma \left(\frac{x}{\sigma(u)}\right)\right)_+^{-\frac{1}{\gamma}}$.

If $X - u \mid X > u \sim GPD_{\sigma(u),\gamma}$then for $v > u$ then $\mathbb{E}(X - v \,|X > v) = \frac{\sigma(u)+\gamma(v-u)}{1-\gamma}$: $\mathbb{E}(X - v \,|X > v)$ is therefore linear in v beyond the threshold u.

Based on the sample $(X_i)_{1 \le i \le n}$ the mean excess function $e(t) \ = \ \mathbb{E}(X \ - \ t|X \ > \ t)$ can then be estimated:

$$\widehat{e_n}(t) = \frac{\sum_{i=1}^{n} X_i \, 1_{t,\infty}(X_i)}{\sum_{i=1}^{n} 1_{t,\infty}(X_i)} - t$$

Let $X_{1,n} \le \cdots \le X_{n-k+1,n} \le \cdots \le X_{n,n}$ be the ordinate vector of observations. The value t is here equal to the $(k \ + 1)$-largest observation $X_{n-k,n}$ of the data points for some $k = 0, 1, \dots n - 1$. Thus, one has:

$$e_{k,n} = \widehat{e_n}\left(X_{n-k,n}\right) = \frac{1}{k}\sum_{j=1}^{k} X_{n-j+1,n} - X_{n-k,n}$$

The mean excess plot can be displayed by plotting values $e_{k,n}$ as a function of the threshold $X_{n-k,n}$ or as a function of the inverse rank k. If the random variable follows a GPD above the threshold, u then the mean excess plot $(t, \widehat{e_n}(t))$ should be approximately linear. Furthermore, when the tail of the distribution exhibits a heavy tail, the mean excess function should increase for small values of k. In contrast, for lighter tails, the mean excess plot is expected to decrease.

In the following example, the mean excess plot of claims covered by the same XL treaty as in *Figure 36* has been plotted. Since for small values of k, the mean excess plot increases, one suspects a tail heavier than the exponential distribution, which is consistent with the previous exponential QQ-plot. One can also observe a linear behavior up to approximately $k = 40$, which suggests modeling the tail distribution using a GPD. This corresponds to a threshold of approximately 4 million euros. The threshold value indicated on the graph corresponds to the threshold selected during the minimization of the AMSE (Average Mean Squared Error), a method that will be described later.

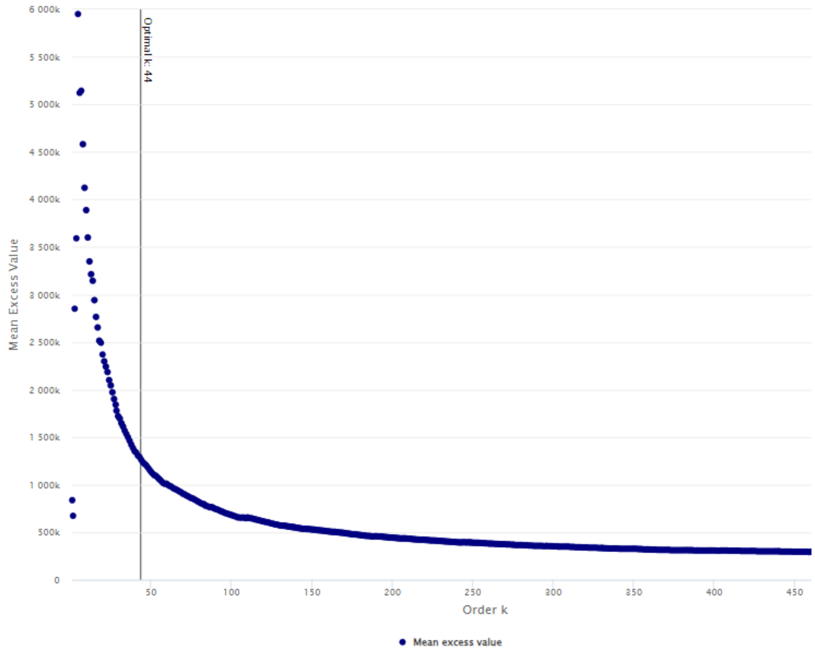

Figure 40: Example of a mean excess plot

For the XL treaty considered, the linear trend of the mean excess plot is less apparent: the mean excess plot does not directly suggest modeling the tail distribution with a GPD, even if a linear trend does seem to emerge for high threshold.

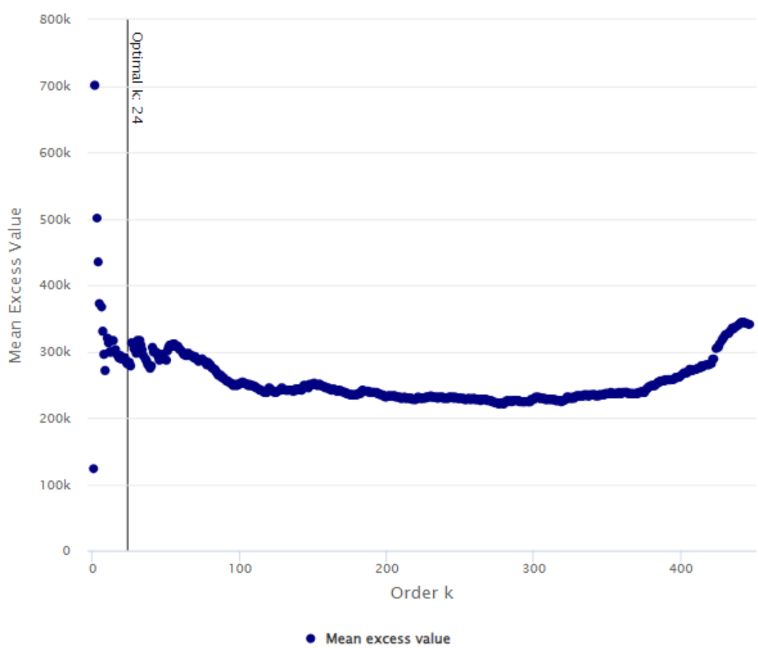

Figure 41: Mean excess plot – XL treaty number 1

Faced with the uncertainty in determining the threshold graphically, other graphs that can help identify the threshold can be used. By estimating the parameter γ of the GPD, it is also possible to determine the threshold graphically. If we suppose that $\gamma > 0$, i.e. assuming the distribution belongs to the Pareto domain of attraction, the most popular estimator of γ is the Hill estimator:

$$\gamma_{k,n}^{Hill} = \frac{1}{k}\sum_{i=1}^{k}(\log(X_{n-i+1,n}) - \log(X_{n-k,n}))$$

The Hill estimator can be interpreted as an estimate of the mean excess function of the log-transformed data, which is $e_{\log(X)}(\log(t)) = \mathbb{E}(\log(X) - \log(t) \mid X > t)$ with the threshold value t substituted by $X_{n-k,n}$. Using this estimator, one can define the Hill plot as $\left(\left(k, \gamma_{k,n}^{Hill}\right)\right)_{1\leq k\leq n}$. The Hill plot is a common graph for examining the estimates of the extreme value index $\gamma > 0$ and determining the optimal threshold rank k^*. Since the Hill estimator is weakly convergent, the plot becomes horizontal for $k > k^*$, indicating stability in the estimation of γ. Based on this, the threshold u can be set as $X_{n-k^*,n}$, the value corresponding to the rank k^* in the ordered data.

To allow tail modeling with Log-Normal or Weibull tails, one must incorporate the case where the γ can be 0 or negative. For $\gamma \in \mathbb{R}$, (Albrecher, Beirlant, & Teugels) introduce the Generalized Hill estimator as:

$$\gamma_{k,n}^{Gen\ Hill} = \frac{1}{k}\sum_{j=1}^{k}\log\left(UH_{j,n}\right) - \log(UH_{k+1,n})$$

Where $UH_{j,n} = X_{n-j,n}\gamma_{j,n}^{Hill}$.

As for the Hill estimator, one can plot $\left(\left(k, \gamma_{k,n}^{Gen\ Hill}\right)\right)_{1\leq k\leq n}$ and select the appropriate threshold rank k^* and the appropriate threshold $u = X_{n-k^*,n}$ when the estimator converges.

The second method for choosing an appropriate threshold is based on minimizing the mean squared error of the Hill estimator. $\gamma_{k,n}^{Hill}$ estimator used to estimate γ when $\gamma > 0$. (Albrecher, Beirlant, & Teugels) approximate the mean squared error of the estimator $\gamma_{k,n}^{Hill}$ by:

$$AMSE\left(\gamma_{k,n}^{Hill}\right) = \frac{\gamma^2}{k} + \left(\frac{b_{k,n}}{1+\beta}\right)^2$$

Where the parameters $\gamma, b_{k,n}$ and β are estimated by non-linear regression of $Z_j = \left(\gamma + b_{k,n}\left(\frac{j}{k+1}\right)^{\beta}\right)E_j$ for $j = 1, \dots, k$. E_j denotes a sequence of independent standard exponentially distributed random variables and $Z_j = j(\log\left(X_{n-j+1,n}\right) - \log(X_{n-j,n}))$. The appropriate threshold rank k^* and the appropriate threshold $u = X_{n-k^*,n}$ are then chosen by taking the value k^* for which $AMSE\left(\gamma_{k^*,n}^{Hill}\right)$ is minimal. For the treaty considered in this part, since we consider that the extreme value index is positive, one can compute the AMSE. In our case, the AMSE is minimal for $k^* = 24$ which corresponds to a threshold $u = 838\ 379$ € (see *Figure 42)*.

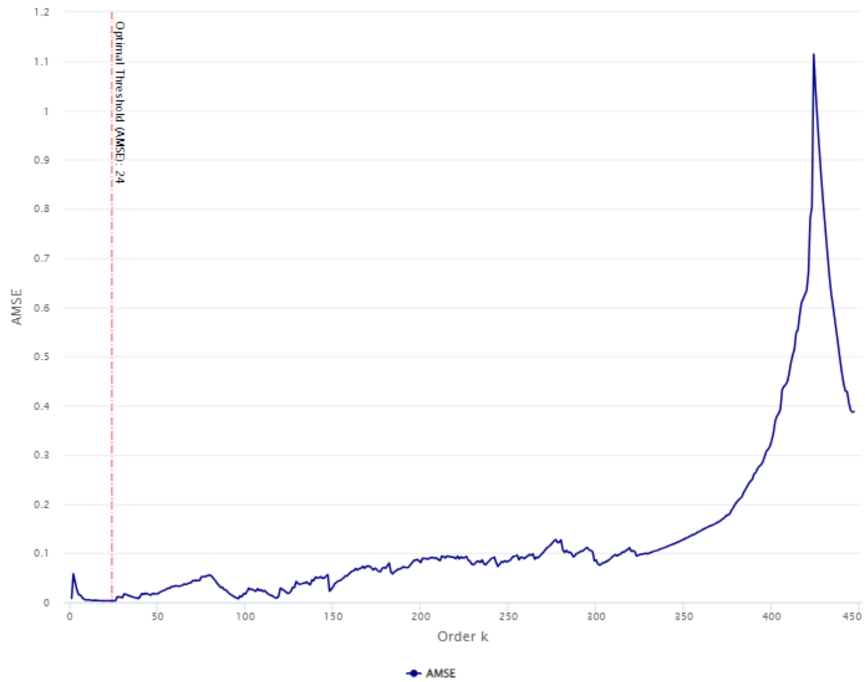

Figure 42: AMSE plot of the XL treaty number 1

The choice of the threshold appears reasonable, as the value of the extreme value index seems to converge for both the generalized estimator and the simple Hill estimator, as illustrated in the figure below. Therefore, the threshold $k^* = 24$ is retained.

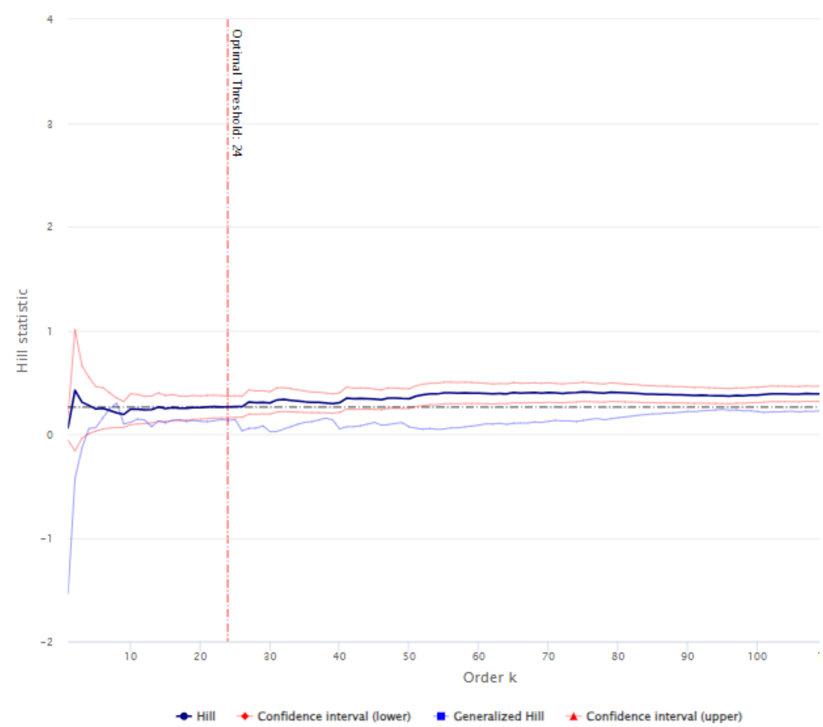

Figure 43: Hill plot of the XL per risk number 1

The method of estimating the threshold by minimizing the error is applied only in the case where it is suspected that $\gamma > 0.$, which is the case for most treaties. Else, the threshold u will be estimated graphically using the Hill and mean excess plot. Concerning the weight estimates $\widehat{\pi_1}$ and $\widehat{\pi_2}$ of the spliced distribution, these will be set by $\widehat{\pi_1} = 1 - \frac{u}{X_{n,n},}$ and $\widehat{\pi_2} = 1 - \widehat{\pi_1}$ where u is the chosen threshold. In our case, $\widehat{\pi_1} \approx 0.4$ and $\widehat{\pi_1} \approx 0.6$.

To wrap up, the discussed EVT plots and their corresponding properties are listed below

Table 23: Description of EVT plots

Plot	Formula	Information
Exponential QQ plot	$(-\log\left(1 - \frac{i}{n+1}\right); X_{i,n})$	A linear exponential QQ plot indicates an exponential distribution. The slope of the plot can be used as an estimate of the distribution's parameter. An ultimately convex/concave exponential QQ plot indicates a tail heavier/lighter than the exponential.
Pareto QQ plot	$(-\log\left(1 - \frac{i}{n+1}\right); \log(X_{i,n}))$	An ultimately linear pareto QQ plot indicates a tail distribution that belongs to the Pareto class of distributions, i.e. with an extreme value index $\gamma > 0$
Log-Normal QQ plot	$(\phi^{-1}\left(\frac{i}{n+1}\right); \log(X_{i,n}))$	An ultimately linear Log-Normal QQ plot suggests a tail Log-Normal distribution. The extreme value index should be equal to 0.
Mean Excess plot	$(k, \frac{1}{k}\sum_{j=1}^{k} X_{n-j+1,n} - X_{n-k,n})$	A horizontal mean excess plot indicates an exponential distribution. Convex or concave mean excess plot suggests a heavy or light tail, respectively. Additionally, if the mean excess plot is linear, the GPD can be used to model the tail of the distribution.
Hill plot	$(k, \frac{1}{k}\sum_{i=1}^{k}(\log(X_{n-i+1,n}) - \log(X_{n-k,n})))$	The Hill plot is used to estimate the extreme value index. The optimal value of the extreme value index and the threshold should be selected at the point where the Hill plot begins to stabilize.

<u>Estimation of the parameters of the GPD</u>

Once the threshold u has been picked, the parameters of the GPD used to model the tail of our severity distribution can be fitted. Since the GPD is the limit distribution of the excess values $X - u$ given $X > u$ when $u \to x^F$, assuming the Fisher-Tippet theorem holds, we aim to model the values $Y = X - u$ given that $X > u$ using the $GPD_{\sigma(u),\gamma}$. The MLE method will be employed to estimate the parameters $\sigma(u), \gamma$.

If the number of exceedances over u is denoted by N_u, the log-likelihood is given by:

$$l(\sigma(u), \gamma \mid Y_1, \dots, Y_n) = -\mathrm{N_u}\log\left(\sigma(u)\right) - (\frac{1}{\gamma} + 1)\sum_{i=1}^{N_u} \log(1 + Y_i \frac{\gamma}{\sigma(u)})$$

Using a reparameterization with $\tau(u) = \frac{\gamma}{\sigma(u)}$, one has the log likelihood equations:

$$\begin{cases} \frac{1}{\hat{\gamma} + 1} = \frac{1}{N_u}\sum_{i=1}^{N_u} \frac{1}{1 + \widehat{\tau(u)}Y_i} \\ \hat{\gamma} = \frac{1}{N_u}\sum_{i=1}^{N_u} \log(1 + \widehat{\tau(u)}Y_i) \end{cases}$$

In our case, if we replace u by its intermediate order statistic $X_{n-k,n}$ (which corresponds to the chosen threshold), one has:

$$\begin{cases} \dfrac{1}{\widehat{\gamma_{k,n}}+1} = \dfrac{1}{k}\displaystyle\sum_{i=1}^{k} \dfrac{1}{1+\widehat{\tau_{k,n}}(X_{n-j+1,n}-X_{n-k,n})} \\ \widehat{\gamma_{k,n}} = \dfrac{1}{k}\displaystyle\sum_{i=1}^{k} \log(1+\widehat{\tau_{k,n}}(X_{n-j+1,n}-X_{n-k,n})) \end{cases}$$

Solving the above equations leads to the MLE estimators.

Furthermore, $\widehat{\gamma_{k,n}} - \gamma$ is asymptotically normal under some regularity conditions on the underlying distributions when $k, n \to \infty$ and $\frac{k}{n} \to 0$, with mean 0 and asymptotic covariance matrix given by:

$$\Sigma_{\widehat{\gamma_{k,n}},\widehat{\sigma_{k,n}}} \sim \frac{1+\gamma}{k}\begin{pmatrix} 1+\gamma & -\sigma(u) \\ -\sigma(u) & 2\sigma(u)^2 \end{pmatrix}$$

This property allows us to build confidence intervals for the parameters γ and $\sigma(u)$.

The estimation of the parameters of the GPD in the splicing model will be done using the previous MLE method.

Choice of parameters for bulk distribution

To estimate the parameters for the bulk distribution, the MLE method described earlier will be used. The observations used for parameter estimation are $\left(X_i 1_{X_i \le u}\right)_{1\le i\le n}$ where u denotes the selected threshold. Density estimation using the kernel method will be performed directly on the previous observations.

For the Mixed Erlang distribution, the parameters (α, λ) are estimated using the EM algorithm as developed by (Verbelen, Gong, Katrien , Badescu, & Sheldon). This algorithm consists of two steps, repeated iteratively until convergence:

- E-step: Compute the conditional expectation of the log-likelihood given the observed data and previous parameter estimates.
- M-step: Determine a subsequent set of parameter estimates in the parameter range through maximization of the conditional expectation computed in the E-step.

For the number M of ME components and the shapes $r = (r_1, \dots, r_M)$, we follow the method proposed by (Albrecher, Beirlant, & Teugels). The number M of ME components is estimated using a backward stepwise search, starting from a certain upper value, whereby the smallest shape is deleted if this decreases an information criterion such as the BIC or the AIC (see 5.1.2.4). Moreover, for each value of M, the shapes $r = (r_1, \dots, r_M)$ are adjusted based on maximizing the likelihood starting from $r = (s, \dots, M * s)$ where s is a chosen spread factor.

5.1.2.4 Selecting the severity distribution

5.1.2.4.1 Ranking the distributions

Once the different severity distributions have been fitted to the as-if ultimate historical claims amounts, they must be compared to each other and ranked in order to select the "best" one(s). To help us classify the different fits, statistical indicators will be examined.

Statistical indicators

- First, one can compare the relative differences between empirical and theoretical values of various statistics, such as the mean, standard deviation, and quantiles (for example extreme ones such as the 90th, 95th and 99th ones)
- Furthermore, to compare the distributions fitted using MLE, indicators such as the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) can be relevant.

- ✓ The AIC helps in comparing different statistical models to determine which one best fits a given set of data while balancing model complexity. A lower AIC value suggests a better fit. The formula for AIC is provided below:

$$AIC = 2k - \ln(L)$$

  where k is the number of parameters and L is the likelihood of the model. The AIC thus penalizes models for having more parameters to prevent overfitting.

- ✓ The second criterion is the BIC. Like the AIC, the BIC is based on the likelihood function but includes a stricter penalty for models with more parameters. A lower BIC still indicates a better fit. The BIC is calculated as:

$$BIC = k\ln(n) - 2\ln(L)$$

  where k is the number of parameters, n is the number of observations in the data, L is the likelihood of the model.

- ➢ Finally, statistical tests and distance metrics can also be used to evaluate the goodness of fit. In this project, the Kolmogorov-Smirnov (KS) test and the Cramer Von Misses (CVM) test have been retained
  - ✓ For the KS test, let us take an iid sample $(X_i)_{1\leq i\leq n}$ which is assumed to follow a probability distribution $\mathbb{P}$. The null hypothesis is: H_0 the distribution $\mathbb{P}$ has the distribution function F_0 where F_0 is the distribution function of a given continuous distribution. The main idea of the test is that if the null hypothesis is true, the empirical cumulative distribution function $\widehat{F_n}(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{X_{(i)}\leq x}$ should be close to F_0.
    The distance between the empirical distribution function and the function F_0 is measured by the Kolmogorov-Smirnov distance. To calculate it, one evaluates the difference between $\widehat{F_n}(x)$ and $F_0(x)$ at the points $(X_{(i)})$:
    $$D_{KS}(F_0, \widehat{F_n}) = \max_{i=1,\dots,n}\{\left|F_0(X_{(i)}) - \frac{i}{n}\right|, \left|F_0(X_{(i)}) - \frac{i-1}{n}\right|\}$$
    Under the null hypothesis H_0, one can prove that for all $t > 0$:
    $$\lim_{n\to+\infty}\mathbb{P}(\sqrt{n}\,D_{KS}(F_0, \widehat{F_n}) > t) = 2\sum_{k=1}^{+\infty}(-1)^{k+1}\exp(-2k^2t^2)$$
    In practice, for $t > 0.56$, the sum of the first three terms already provides an approximation with an error less than 10^{-4}. If the null hypothesis H_0 is false, $\sqrt{n}\,D_{KS}(F_0, \widehat{F_n})$ tends towards $+\infty$ as n increases. The test is therefore necessarily one-sided to the right (rejecting overly large values). If we define the test statistic by $t = \sqrt{n}\,D_{KS}(F_0, \widehat{F_n})$, the corresponding p-value is approximately $2\sum_{k=1}^{+\infty}(-1)^{k+1}\exp(-2k^2t^2)$. We reject the null hypothesis if the p-value is inferior to the considered level $\alpha = 0.05$.
  - ➢ The CVM test can also be used to test the goodness-of-fit of a distribution, but unlike the KS test, it is based on a quadratic measure of the difference between the empirical and theoretical CDFs. The CVM test computes the average squared difference between the empirical CDF $\widehat{F_n}$ and the theoretical CDF F_0. It considers the entire range of differences, rather than focusing only on the largest difference, as the KS test does. The CVM distance is being defined as follows:

$$D_{CVM}(F_0, \widehat{F_n})^2 = n\int_{-\infty}^{+\infty}|\widehat{F_n}(x) - F_0(x)|^2 dF_0(x)$$

Ranking metric

On the basis of the statistical indicators introduced previously and computed for all the candidate fitted severity distributions, the following methodology to rank those fits and highlight the most suitable ones have been adopted.

First, for each of the fitted distributions, the following indicators are considered:

Table 24: Considered ranking metrics

Metrics
Mean (absolute relative difference[1])
Quantile 90th (absolute relative difference)
Quantile 99th (absolute relative difference)
Negative log likelihood
KS distance
CVM distance

The AIC and BIC were not considered because both are based on the log-likelihood, which could result in it having a disproportionate influence in the ranking of fitted distributions. However, we could consider an indicator that represents the average of the BIC, AIC, and the negative log-likelihood, thereby accounting for the degrees of freedom of each distribution in the global ranking.

For a fitted distribution, the objective is to minimize each of these indicators to guarantee a good fit. Thus, for each indicator, distributions are ranked in ascending order. For example, considering the KS distance, the distribution with the lowest KS distance will be assigned the highest rank for this indicator, i.e. the first rank. From there, if we note $(Rank_i)_{1 \le i \le 6}$ the rank per indicator of a fitted distribution, a global ranking index is calculated for each fitted distribution:

$$Global\ rank\ index = \frac{1}{6}\sum_{i=1}^{6} Rank_i^2$$

The fitted distributions are then definitively ranked according to this index, with the distribution having the lowest $Global\ rank\ index$ being the distribution considered the most appropriate.

Regarding the treaty we aimed to price, given that the number of observations exceeded 50, all the spliced and continuous distributions were fitted. Among these, only the top four distributions identified through the previously established ranking methodology were retained. The table below presents the values of the metrics calculated to determine this ranking, while the p-values for KS and CVM test are also provided for reference. The term GAMMA_MME denotes the Gamma distribution obtained with the method of moments, while the ME_GPD (respectively ME_PARETO) corresponds to the Mixed Erlang with a GPD (respectively Pareto) tail.

Table 25: Fitting comparison - XL treaty number 1

Distributions	Mean	Standard deviation	Quantile (90)	Quantile (99)	Log likelihood	KS distance	KS p-value	CVM distance	CVM p-value
OBSERVED	339 890	257 157	637 695	1 223 812	/	/	/	/	/
GAUSSIAN_KERNEL	340 096	261 266	643 334	1 245 726	/	0,03	0,88	0,05	0,89
ME_GPD	339 947	252 940	599 222	1 294 909	-60,86	0,04	0,53	0,11	0,55
ME_PARETO	340 551	263 407	598 924	1 287 995	-60,93	0,04	0,55	0,11	0,56
GAMMA_MME	340 263	256 865	683 870	1 197 180	/	0,09	0,00	0,82	0,01

The most appropriate distribution appears to be the Gaussian Kernel. This makes sense since it directly reflects the empirical distribution. However, the issue with modeling using the Gaussian Kernel lies in the tail of the distribution, which tends to be thin, making it less effective for representing significant claim amounts. Indeed, it is due to limited historical depth of observations, which may not include large claims with return periods of 50 or 100 years. It leads the Gaussian Kernel to underestimate the probability of extreme events. Gaussian kernels are also characterized by normal distribution tails, which may not realistically

[1] *The absolute relative difference is calculated between the fitted and the observed distribution*

represent the heavy-tailed nature of reinsurance risks. Spliced distributions, on the other hand, appear to be strong candidates, as they exhibit low KS and CVM distances. Moreover, the p-values are greater than 5%, so we do not reject the null hypothesis, unlike the Gamma distribution calibrated through the method of moments. The following figure displays a spider web chart representing the rank of the fitted distributions for each calculated metric. The chart on the left illustrates the rankings of the metrics for the top four distributions, while the one on the right represents the rankings across all fitted distributions.

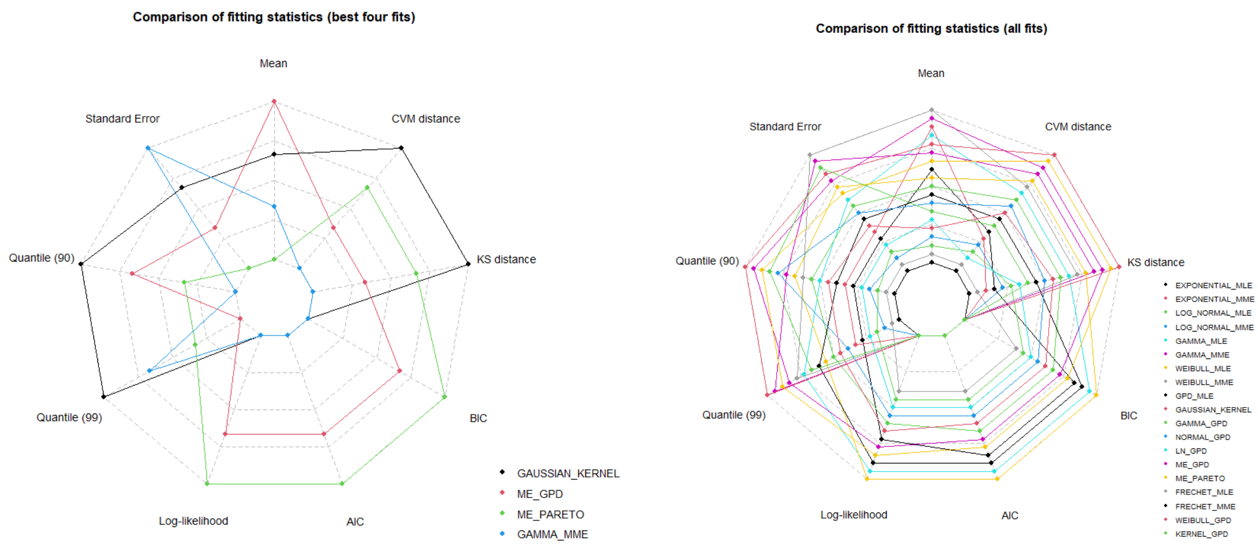

Figure 44: Spider web chart – XL treaty number 1

The table below displays the calculation of the index, and the distribution ranks. While the Gaussian Kernel is the most appropriate distribution due to its nature, the ME-GPD distribution is of greater interest and appears to be a more suitable choice.

Table 26: Rank of the best 4 distributions - XL treaty number 1

Distributions	Mean	Quantile (90)	Quantile (99)	Log likelihood	KS distance	CVM distance	*Global rank index*	Rank
GAUSSIAN_KERNEL	5	1	1	/	1	1	**5,80**	**1**
ME_GPD	2	2	4	5	3	3	**11,16**	**2**
ME_PARETO	7	3	3	4	2	2	**15,16**	**3**
GAMMA_MME	6	6	2	/	4	4	**21,60**	**4**

The parameters of these four distributions and the ranks of all distributions are provided in the appendix.

5.1.2.4.2 Qualitative approach

To assess the fit of ranked distributions for severity modeling, it is possible to plot CDF, density and QQ plots. These plots can be used to visually evaluate the goodness of fit and determine whether the ranked distributions appear consistent to model claims amounts.

The CDF represents the probability that a random variable X takes a value less than x, expressed as $F(x) = \mathbb{P}(X \leq x)$. The empirical CDF of the observed data is thus computed and plotted alongside the theoretical CDFs of each distribution. Additionally, the retention and the limit of the treaty are indicated on the graph. If the CDF of a distribution closely matches the empirical one, the distribution is suitable for modeling severity.

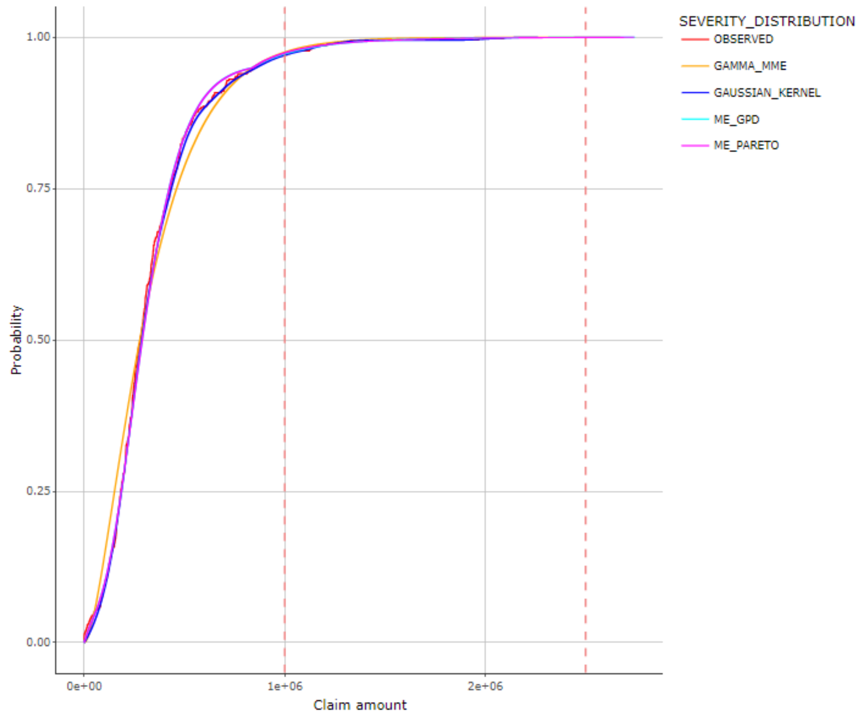

Figure 45: CDF of fitted distributions – XL treaty number 1

The CDF of XL treaty number 1 suggests that the ranking of distributions appears appropriate, as the CDFs of the Mixed Erlang and Pareto/GPD distributions seem quite close to the observed distribution, while the Gamma distribution appears to deviate more significantly. Additionally, a slight inflection point can be noted around 840 000 € for the Mixed Erlang – Pareto/GPD distributions. This inflection corresponds to the threshold above which severity is modeled using a GPD or a Pareto distribution.

Let us focus on the part of the curve between the retention and the treaty limit (*Figure 46*).

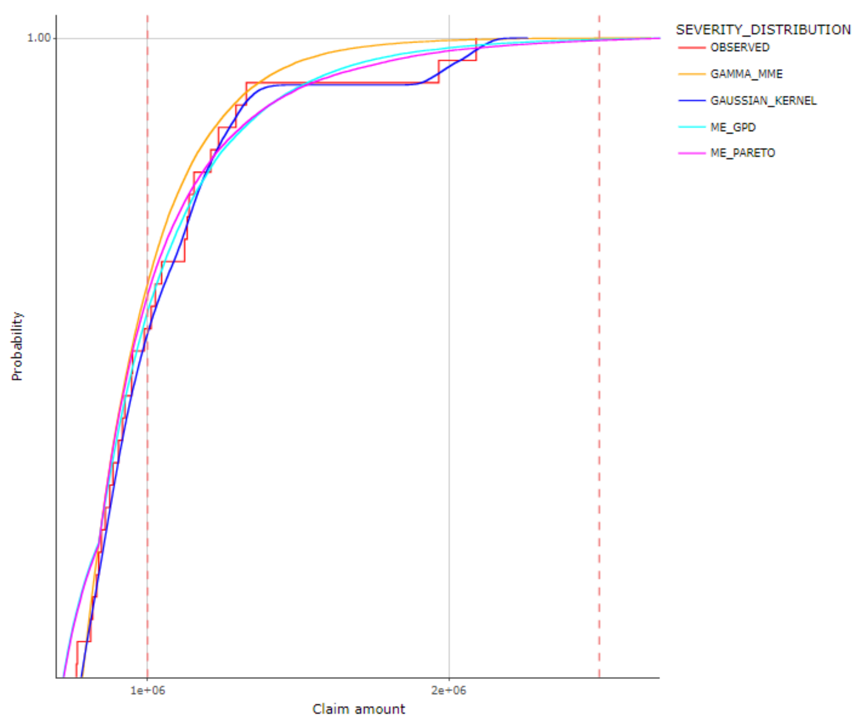

Figure 46: CDF of fitted distributions (zoom) – XL treaty number 1

On this graph, the previously mentioned inflection point is more pronounced. Additionally, the Gamma distribution appears to have a thinner tail compared to the other distributions. The Gamma distribution would thus result in a lower expected recovery amount. In contrast, the ME Pareto and ME GPD distributions are quite similar and thus expected to lead to comparable recovery amounts. Furthermore, it is noteworthy that the likelihood of a claim exceeding the retention threshold remains relatively low, estimated at around 5% to 10%.

Concerning the QQ plot, the figure below presents the QQ plot of the four previously selected distributions. It seems to confirm that the distributions were appropriately selected and calibrated, as the quantiles align well with the bisector. Consistent with the observations from the CDF, the Gamma distribution exhibits lower high quantiles compared to the observed distribution and the other fitted distributions. This was expected, given that the Gamma distribution has a thinner tail. Consequently, this characteristic would likely result in lower recovery amounts for the reinsurer.

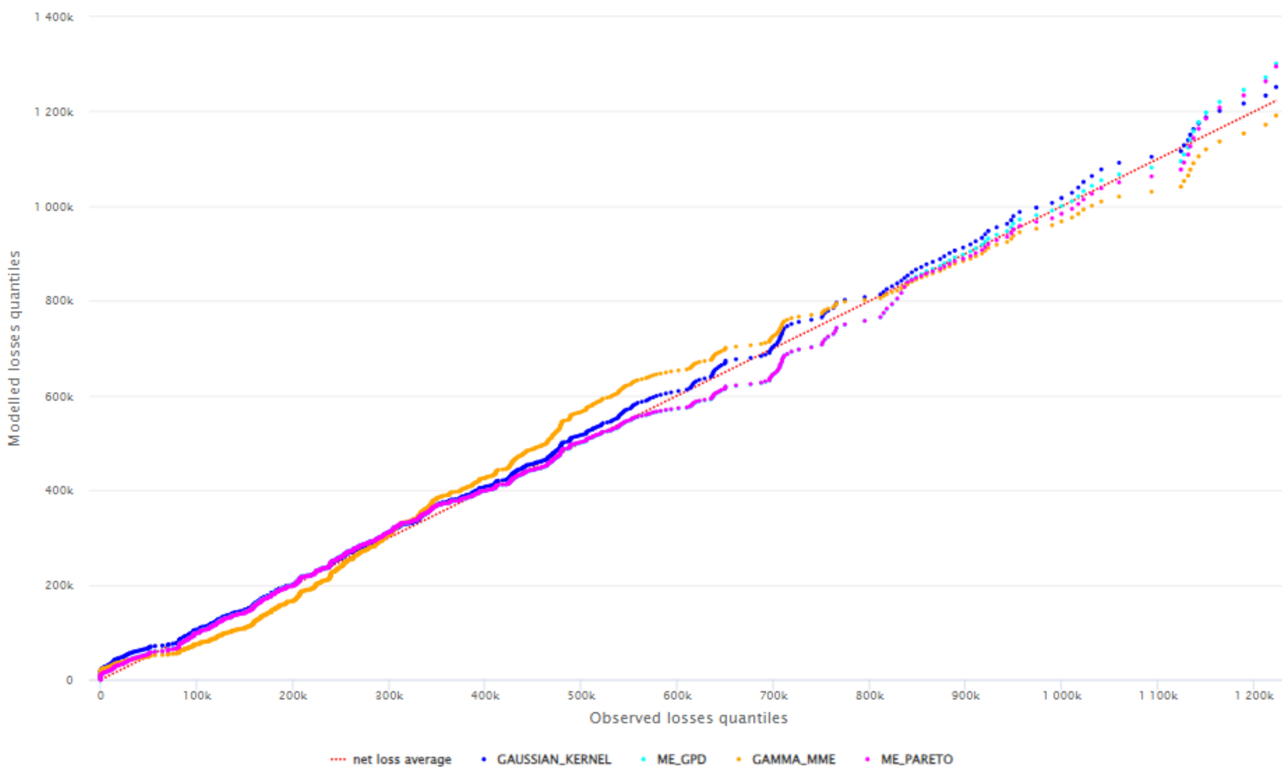

Figure 47: QQ plot of fitted distributions – XL treaty number 1

To sum up, the following figure details the underlying claims modeling process, defined as “P7” in *Figure 31*.

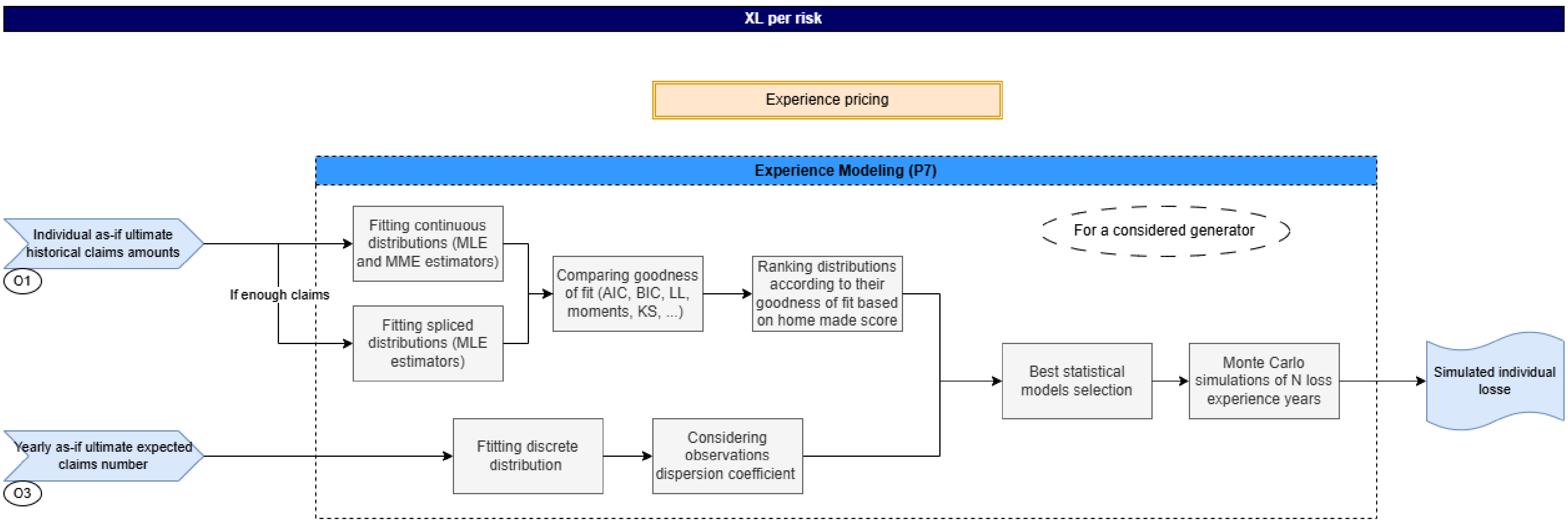

Figure 48: Experience modeling process – XL per risk treaties

5.1.3 Recoveries modeling

Once the parameters of the frequency and severity distributions have been established and the appropriate distributions selected for each loss-generator, the next step in the Monte Carlo estimation process is to simulate a distribution of amounts ceded to reinsurance, also known as recoveries.

To achieve this, 50,000 years of loss experience for each generator will be simulated. In this project, the number of simulations has been set to $N_{simul} = 50\,000$ following AGCRe usual process within the internal model. However, the number of simulations can be adjusted within the reinsurance pricing tool. Let us consider a loss experience year for a fixed generator. Let i be a simulated year, N_i be the number of claims simulated and $\left(X_k^i\right)_{1\leq k\leq N_i}$ the individual amount of each claim k for the loss experience year i considered.

When pricing an XL treaty, one is also reminded that the treaty parameters include:

- A limit L and retention R
- The Annual Aggregate Deductible noted AAD, and the Annual Aggregate Limit noted AAL
- The total number of reinstatements noted nb_{rec}. If the treaty reinstatements are 2@50%, 3@100%, then $nb_{rec} = 5$. In the case where the number of reinstatements is fixed, the AAL is equal to: $(1 + nb_{rec}) * L$.
- The presence of a indexation clause, the indexation clause threshold noted $t_{indexation}$, the indexation rate to be used $r_{indexation}$ and the discount rate to be used $r_{discount}$.

5.1.3.1 Application of the indexation clause

Before applying the retention limit to individual claims, the indexation clause, which can significantly impact the amounts related to the retention and the treaty limit, must be considered since it is a frequent clause among AXA underwritten treaties. In this project, the Annual Aggregate Deductible and Annual Aggregate Limit are not considered in the application of the indexation clause, since few AGCRe treaties include such an indexation clause. For future considerations about AAD and AAL indexation clauses, one could look at (Yeung, 2011).

Suppose that the XL treaty includes an indexation clause. Let us consider an index vector $v_{indexation} \in \mathbb{R}^{30}$ equal to $\left(\left(1 + r_{indexation} * 1_{(1+r_{indexation})^j > 1+t_{indexation}}\right)^{-j}\right)_{1\leq j\leq 30}$. This vector $v_{indexation}$ is then used to index individual payments over time through their payment pattern PP, to ultimately deduce the indexed vector $PP_{indexed} \in \mathbb{R}^{30}$:

$$PP_{indexed} = \left(PP_j * (v_{indexation})_j\right)_{1\leq j\leq 30}$$

Once this indexed amount obtained, the indexing ratio $rate_k^{indexation} \in \mathbb{R}^{30}$ at each time step is calculated:

$$rate_k^{indexation} = \left(\frac{\sum_{m=1}^{j} PP_m}{\sum_{m=1}^{j} (PP_{indexed})_m}\right)_{1\leq j\leq 30}$$

This ratio corresponds to the ratio between the non-indexed and indexed cumulative payment pattern. It is then used to compute a retention amount vector and an indexed limit vector. $R_k^{indexed} \in \mathbb{R}^{30}$ and $L_k^{indexed} \in \mathbb{R}^{30}$ for each claim, by multiplying the retention R and the limit L with the vector $rate_k^{indexation}$. If the treaty does not include an indexation clause, the vector $rate_k^{indexation}$ is taken to be equal to $(1)_{1\leq j\leq 30} \in \mathbb{R}^{30}$: retention and limit remain unchanged. The indexation clause, therefore, is primarily dependent on the payment pattern. By calibrating the payment pattern, one can apply the effects of the indexation clause and incorporate it into the calculation of recoveries.

Since the considered treaty does not have an indexation clause, let us consider an XL treaty with the following parameters:

Table 27: Indexation clause parameters

Quantity	Value
RETENTION	300 000
LIMIT	3 700 000
INDEXATION_CLAUSE	1
INDEXATION_CLAUSE_THRESHOLD	20 %
INDEXATION_CLAUSE_INFLATION_RATE	3,63 %

The payment pattern of this XL treaty is shown in *Figure 24*.

To demonstrate the effect of the stabilization clause, the evolution of the retention and limit over time is displayed below.

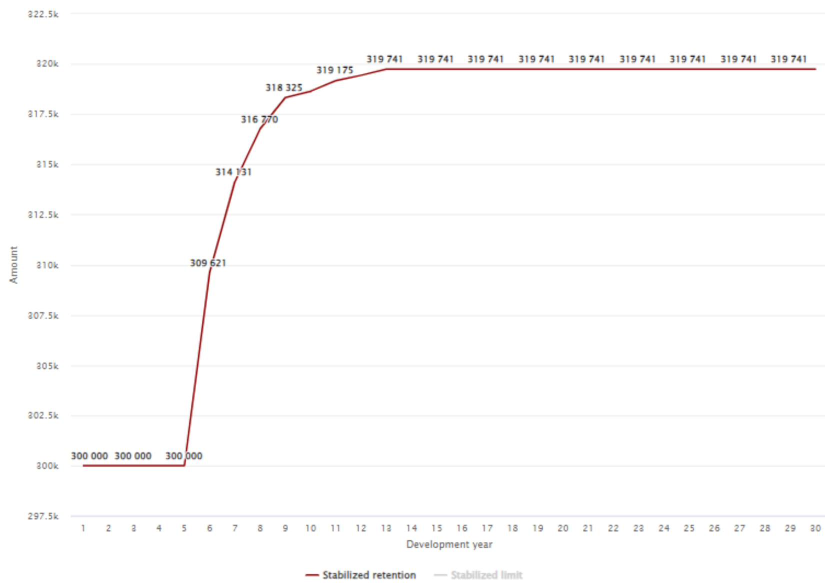

Figure 49: Evolution of retention under a stabilization clause

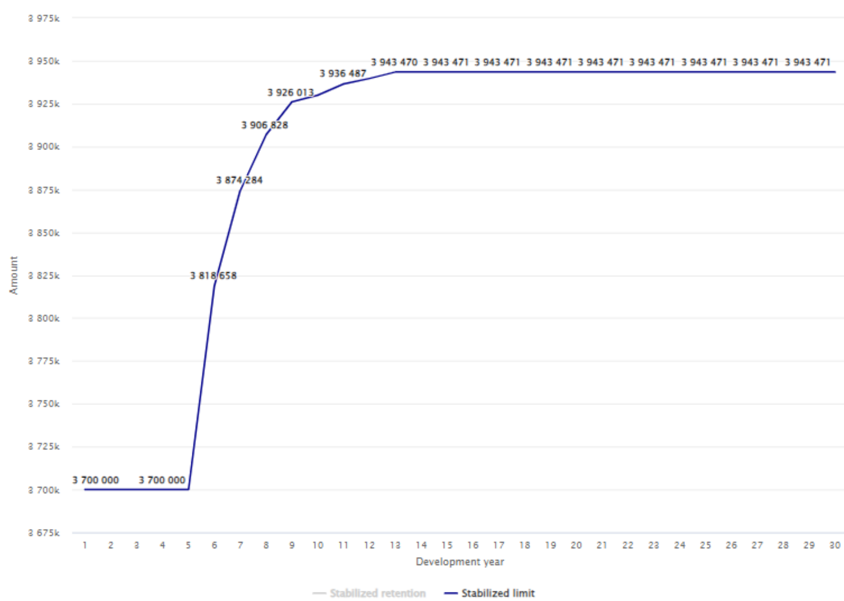

Figure 50: Evolution of limit under a stabilization clause

Before 5 years, the retention and limit amounts remain unchanged. This is due to the indexation clause, which sets a threshold of 20%. With an index evolution of approximately 3.6% per year, the index threshold will only exceed 0.2 after five years. Since the treaty covers a long-tail LoB, there are still payments occurring after 5 years, which triggers the application of the indexation clause. From year 5 onward, the retention and limit continue to evolve as claims are still being paid according to the payment pattern. A stabilization of the limit and retention occurs around year 13 since the claims are fully paid by this point.

This clause is particularly useful for the reinsurer, especially for long-tail business lines where claims are paid out over several years, where economic conditions may change.

5.1.3.2 Application of treaty T&C

Once the values of $R_{k,indexed}$ and $L_{k,indexed}$ obtained, the XL treaty conditions can be applied. In order to get the recovery amount for the loss experience year i and claim k, each individual amount X_k^i is projected over time using the payment pattern $PP = (PP_j)_{1 \le j \le 30}$ calibrated in section 4.3 to get a vector of cumulated individual payments made by the cedent:

$$X^{cedent}_{i,k,cumulative} = \left(\sum_{t=1}^{j} PP_t * X_k^i \right)_{1 \le j \le 30}$$

The use of the payment pattern will enable to grasp actualization considerations in the calculation of recoveries. For each component of the vector $X_{i,k,cumulative}$, the conditions of the XL treaty are applicated to compute the vector of individual recoveries below for $k \in \{1, \dots, N_i\}$:

$$X^{reinsurer}_{i,k,cumulative} = \left(\min\left(\left((X^{cedent}_{i,k,cumulative})_j - (R_{k,stabilized})_j \right)_+ , (L_{k,stabilized})_j \right) \right)_{1 \le j \le 30}$$

The resulting vector represents the cumulative amount ceded to the reinsurer over the next 30 years for a single claim. The figure below illustrates the evolution of the amounts ceded to reinsurance for the XL treaty examined in this section, with a simulated ultimate claim amount of 1 489 563 €, projected using the payment pattern. The reinsurer starts receiving recoveries from development year 2, as the amount in the first year is too low to exceed the retention. A sharp increase in recoveries follows, which aligns with the payment pattern and the characteristics of the short-tail line of business covered by the reinsurance treaty. The final amount of recoveries is known starting from year 11, which is consistent with the payment pattern's expected progression.

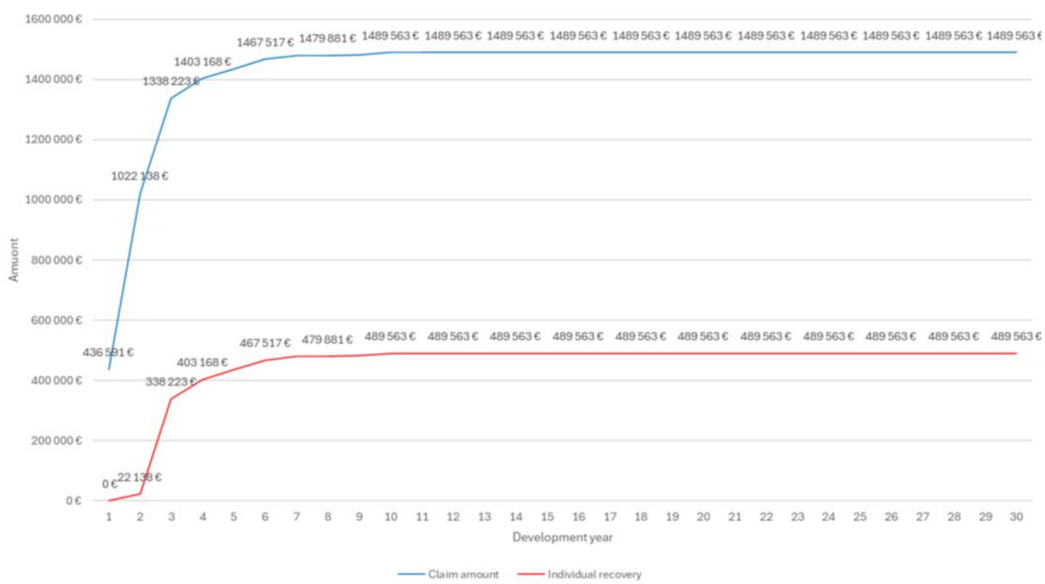

Figure 51: Cumulated individual claim amount/recovery – XL treaty number 1

After applying the retention and limit, only the AAD and the AAL remain to be considered. We make the simplifying assumption that for a given year $j \in \{1, \dots, 30\}$, all individual amounts are declared in the middle of the year. This assumption allows us not to consider the claims individually. Additionally, since AGCRe reinsurance treaties do not include temporal clauses regarding the reinstatement premiums, this assumption does not restrict the calculation of the reinstatement premium.

For each year i simulated, the vector of cumulative annual recoveries is:

$$S_{i,cumulative}^{reinsurer\ (without\ AAD\ or\ AAL)} = \left(\sum_{k=1}^{N_i} \left(X_{i,k,cumulative}^{reinsurer} \right)_j \right)_{1 \leq j \leq 30}$$

If multiple generators have been calibrated, the previous vector is derived by aggregating all claims from the different generators with the same simulated year and frequency/severity rankings. Once this vector has been calculated for each of the simulations, AAL and AAD are applied to it:

$$S_{i,cumulative}^{reinsurer} = \left(\min\left(\left(\left(S_{i,cumulative}^{reinsurer\ (without\ AAD\ or\ AAL)} \right)_j - AAD \right)_+, AAL \right) \right)_{1 \leq j \leq 30}$$

Afterwards, the final vector of cumulative recoveries per simulation i is used to calculate the cash flow vectors of recoveries for each simulation year:

$$S_{i,incremental}^{reinsurer} = \left(\left(S_{i,cumulative}^{reinsurer} \right)_j - \left(S_{i,cumulative}^{reinsurer} \right)_{j-1} 1_{j \geq 2} \right)_{1 \leq j \leq 30}$$

These future cash flows are then discounted by multiplying the elements of the vector $S_{i,incremental}^{reinsurer\ (undiscounted)}$ by those of the vector $r = \left(\frac{1}{(1+r_{discount})^{j-0.5}} \right)_{1 \leq j \leq 30}$. We have assumed, in the development of this reinsurance pricing tool, a constant discount rate set at 2,5 %. This assumption, along with the discrete time modeling approach, may impact the discounted values since all claims are assumed to be reported at the same time of the year. Unfortunately, the granularity of the payment pattern calculation does not allow us to account for these variations within our current estimation of recoveries. Moreover, the calibration of specific discount curves related to different LoBs could represent an area for improvement. By incorporating more granular discount rates, one would be able to refine the recovery estimates and better reflect the time value of money, especially for long-tail treaties where payments are spread over extended periods.

Finally, one computes, for each simulation i, a discounted vector comprising all the annual discounted cash flows of recoveries noted $S_{i,incremental}^{reinsurer\ (discounted)}$ and the total present value of recoveries noted $S_i^{reinsurer\ (discounted)}$ described by the following formulas :

$$S_{i,incremental}^{reinsurer\ (discounted)} = \left(\frac{\left(S_{i,incremental}^{reinsurer} \right)_j}{(1 + r_{discount})^{j-0.5}} \right)_{1 \leq j \leq 30} \in \mathbb{R}^{30}$$

$$S_i^{reinsurer\ (discounted)} = \sum_{j=1}^{30} \left(S_{i,incremental}^{reinsurer\ (discounted)} \right)_j \in \mathbb{R}$$

Over all simulated years $i = 1, \dots, N_{simul}$, one has simulated a vector of recoveries $\left(S_i^{reinsurer\ (discounted)} \right)_{1 \leq i \leq N_{simul}}$ corresponding to the amount ceded to the reinsurer under the reinsurance contract.

The figure below details the distribution of recoveries for each of the four distributions. As expected, the ME Pareto and ME GPD distributions have heavier tails, while the Gamma distribution exhibits the thinnest tail. However, most of the recovery amounts are zero, which is consistent with the low probability of exceeding the retention.

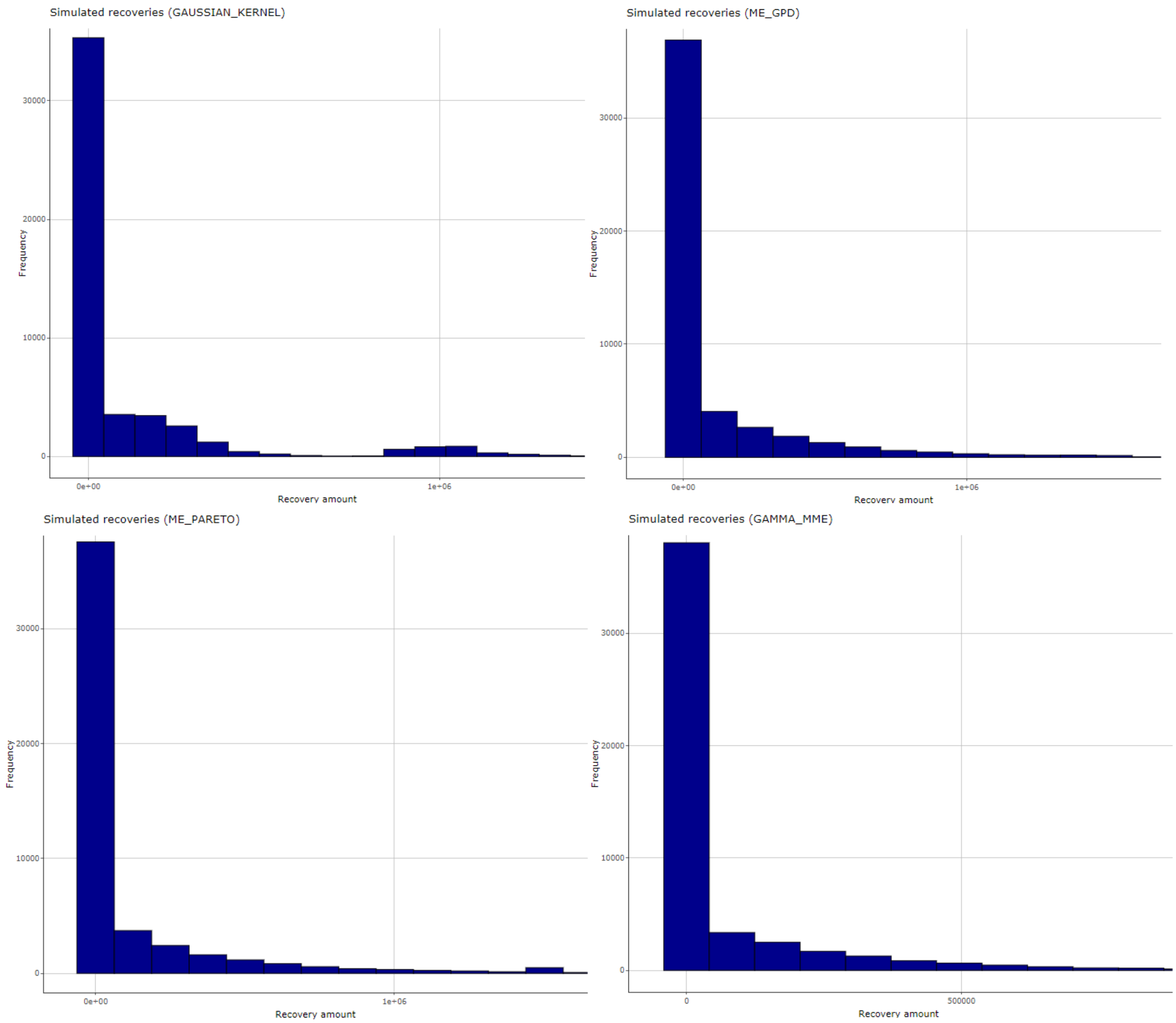

Figure 52: Distribution of recoveries – XL treaty number 1

To sum up, the following figure details the underlying claims modeling process, defined as “P9” in *Figure 31*.

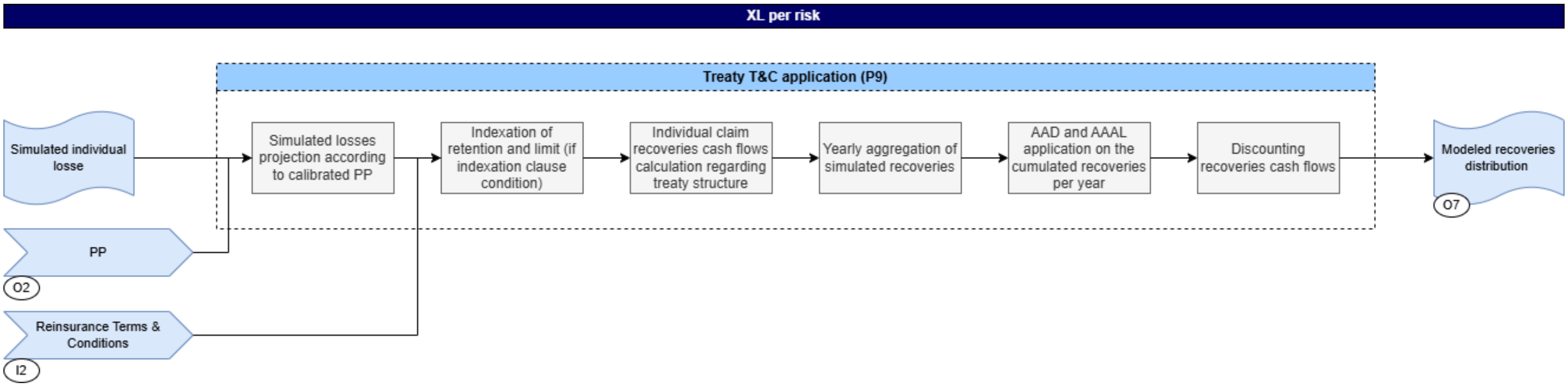

Figure 53: Treaty T&C application process - XL per risk treaties

5.2 Modeling losses per exposure (XL per Risk and SP)

The frequency-severity approach developed earlier offers numerous advantages, particularly in its flexibility regarding the choice of distributions and estimation methods. However, this method has certain limitations, as it relies heavily on the availability of sufficient underlying claims data and it does not consider portfolio exposure evolution, i.e. the latest portfolio configuration.

To address these issues, the exposure-based method has been implemented as an alternative. This approach circumvents the need for extensive claims history and can also be applied in cases where frequency-severity struggles, such as high-excess layers with no historical claims large enough to generate recoveries. Furthermore, it takes into account the latest covered insurance portfolio structure and composition and thus gives rise to an assessment that better reflects the actual covered exposure.

In this actuarial project, the exposure-based methodology has been developed mainly for XL per risk and SP property treaties (which are not modeled within the internal model), owing to the characteristics of this LoB. For property treaties, the exposure methodology relies on exposure curves from which loss degree distributions can be derived. One can thus model loss amounts based on a destruction rate and a sum insured. This approach is based on the main hypothesis that policies within the same risk group have a similar destruction rate distribution.

Future improvements could involve extending the methodology to other LoBs, such as liability[1] and marine, where exposure pricing could also be relevant. However, the exposure methodology presents some drawbacks. Indeed, the exposure methodology requires knowledge of the loss ratio per exposure range which might be difficult to obtain, especially for bands with few insurance policies. Additionally, the limited number of risk profiles available does not yet allow for generalizing this method to all treaties. To maximize the number of treaties that can be accurately priced, it will be necessary to gather more data from entities in the future. In addition, exposure curve calibration requires a large amount of data, which was not available in this project. It is therefore necessary to rely on benchmark exposure curves, which may reduce the accuracy of the modeling process. Consequently, for XL per risk, the exposure methodology is not the primary focus, and the frequency-severity approach remains preferred due to its capacity to assess more treaties. For SP treaties, the exposure curve is used thanks to its capacity to integrate destruction rate and sum insured considerations. The next section will detail the methodology developed in the pricing tool to model recoveries.

5.2.1 Exposure pricing theory

Let us consider an XL per risk treaty L xs R. First, for a risk p, a destruction rate $x_p \in [0,1]$ is defined as $x_p = \frac{X_p}{Q_p}$ where:

- X_p is a random variable corresponding to the gross loss of an individual claim related to risk p
- Q_p represents the sum insured of the risk p

For property treaties, the destruction rate CDF F is supposed not to depend on i for risks belonging to the same risk group. Under this assumption, the cumulative distribution function of the random variable x_p is noted F for all i. We further define the normalized retention r_p as $r_p = \frac{R}{Q_p}$, the normalized limit as l_p as $l_p = \frac{L}{Q_p}$ and the limited expected value function as $L(t) = \mathbb{E}(\min(t, x_p))$.

By decomposing the random gross loss X_p into the part retained by the cedent and the one ceded to the reinsurer as: $X_p = X_p^{cedent} + X_p^{reinsurer}$, we can express the expected value of both the retained and the reinsured amounts as follows:

$$\mathbb{E}\left(X_p^{reinsurer}\right) = \left(L\left(r_p + l_p\right) - L\left(r_p\right)\right) * Q_p$$

[1] *Concerning the liability LoB, a first approach has been implemented as an exploration*

$$\mathbb{E}(X_p^{cedent}) = L(1) * Q_p - \mathbb{E}(X_p^{reinsurer})$$

The exposure curve function can then be defined as the percentage of losses retained by the cedent which is given by the function $G(t) = \frac{L(t)}{L(1)}$, also known as exposure curve function. It is also expressed as:

$$G(t) = \frac{L(t)}{L(1)} = \frac{\int_0^t (1 - F(z))dz}{\int_0^1 (1 - F(z))dz} = \frac{\int_0^t (1 - F(z))dz}{\mathbb{E}(x_i)}$$

The exposure curves are concave and increasing functions on the interval $[0; 1]$ with $G(0) = 0$ and $G(1) = 1$. Furthermore, the expected destruction rate is $\mathbb{E}(x_p) = \frac{1}{G'(0)}$ and the probability of total loss is defined as $p = \mathbb{P}(x_i = 1) = \frac{G'(1)}{G'(0)}$. One can also deduce the destruction rate CDF F from the exposure curve G:

$$F(t) = 1 - \frac{G'(t)}{G'(0)} 1_{0 \le t \le 1}$$

The following section focuses on presenting the exposure curves that can be used and the ones we have selected.

5.2.2 Exposure pricing curves

5.2.2.1 Empirical exposure curves

For an XL per risk treaty within the property LoB the assumption that risks within the same risk group share the same destruction rate distribution allows for the calibration of empirical distribution curves. These are based on the historical destruction rates $(x_p)_{1 \le p \le n}$ of historical applicable claims.

To calibrate an empirical exposure curve, the first step is to compute the empirical CDF $\widehat{F}_x(t) = \frac{1}{n}\sum_{p=1}^n 1_{x_p \le t}$ and the empirical mean $\widehat{\mathbb{E}}(x) = \frac{1}{n}\sum_{p=1}^n x_p$ of the destruction rate distribution. It is then necessary to make the empirical distribution function continuous to enable its integration for calculating the exposure curve function. Several methods are available: spline smoothing, local smoothing, or piecewise linear interpolation. Afterwards, for a scalar $t \in [0; 1]$, one can construct an empirical estimate of the exposure curve with the function $\widehat{G}$:

$$\widehat{G}(t) = \frac{\int_0^t 1 - \widehat{F}_x(z)dz}{\widehat{\mathbb{E}}(x)}$$

The function $\widehat{G}$ corresponds to an empirical estimate of the exposure curve.

Unfortunately, this method was not implemented during this study due to the unavailability of destruction rate data. Establishing contact with the AXA entities and determining whether such data is accessible at their level could enable the calibration of exposure curves specific to each treaty. This approach could act as a basis for further improvements.

5.2.2.2 MBBEFD Distribution Approach

One can also set an exposure curve for a given XL per risk treaty by assuming a probability distribution for destruction rates. (Bernegger, 1997) proposed to implement MBBEFD (Maxwell-Boltzman-Bose-Einstein-Femi- Dirac) for modeling destruction rates on the interval [0,1]. The MBBEFD distribution is a continuous distribution defined by two parameters a and b with the following CDF:

$$F_{MBBEFD(a,b)}(t) = \begin{cases} 1, & t = 1 \\ \dfrac{1-(a+1)b^t}{a+b^t}, & 0 \le t \le 1 \end{cases}$$

The exposure curve related to this distribution is:

$$G_{MBBEFD(a,b)}(t) = \frac{\ln(a+b^t) - \ln(a+1)}{\ln(a+b) - \ln(a+1)}$$

This distribution can be parametrized with a parameter $g = \frac{a+b}{(a+1)b}$ instead of a (and thus $a = \frac{(g-1)b}{1-gb}$). The parameter g corresponds to the inverse of the probability $p = \mathbb{P}(x = 1)$. For the probability p to be within the range $[0,1]$ and $G_{MBBEFD(g,b)}$ to be well defined, we restrain the parameters to $b \ge 0$ and $g \ge 1$. The corresponding exposure defined by (Bernegger, 1997) is:

$$G_{MBBEFD(g,b)}(t) = \begin{cases} t, & g = 1 \vee b = 0 \\ \dfrac{\ln(1+(g-1)t)}{\ln(g)}, & b = 1 \wedge g > 1 \\ \dfrac{1-b^t}{1-b}, & bg = 1 \wedge g > 1 \\ \dfrac{\ln(\frac{(g-1)b+(1-gb)b^t}{1-b})}{\ln(gb)}, & b > 0 \wedge b \neq 1 \wedge bg \neq 1 \wedge g > 1 \end{cases}$$

The notation $g = 1 \vee b = 0$ signifies "$g = 1$ or $b = 0$", while the notation $b = 1 \wedge g > 1$ means "$b = 1$ and $g > 1$".

Based on the MBBEFD exposure curve, one can compute the corresponding CDF of the destruction rate $F_{MBBFED(g,b)}$:

$$F_{MBBEFD(g,b)}(t) = \begin{cases} 1, & t = 1 \\ 0, & t < 1 \wedge (g = 1 \vee b = 0) \\ 1 - \dfrac{1}{1+(g-1)t}, & t < 1 \wedge b = 1 \wedge g > 1 \\ 1 - b^t, & t < 1 \wedge bg = 1 \wedge g > 1 \\ 1 - \dfrac{1-b}{(g-1)b^{1-t}+(1-gb)}, & t < 1 \wedge b > 0 \wedge b \neq 1 \wedge bg \neq 1 \wedge g > 1 \end{cases}$$

If the destruction rates are assumed to follow an MBBEFD distribution, one needs to estimate the parameters g and b of the distribution. The methods of moments and the maximum likelihood estimation can be used to estimate the g and b parameters.

For the method of moments, (Bernegger, 1997) showed that for every pair p and $\mathbb{E}(x)$ satisfying the condition $0 \le p \le \mathbb{E}(x) \le 1$, there exists only one distribution belonging to the MBBEFD class. The parameter g is estimated by $\hat{g} = \frac{1}{\mathbb{P}(x=1)}$ and the parameter $\hat{b}$ is the solution of the equation $\hat{\mathbb{E}}(x) = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$. One can also estimate the parameters g and p with the MLE method, but it requires the use of numerical methods such as the Newton-Raphson method. This method iteratively refines the estimates of the parameters by using the first and second derivatives of the likelihood function until convergence is achieved.

Like the empirical destruction curves, this method could not be applied in the context of this study due to the lack of data on destruction rates. Still, the tool allows the user to select its own MBBEFD parameters and apply this methodology.

5.2.2.3 Swiss Re exposure curves

Instead of calibrating exposure curves for each reinsurance treaty, one can use benchmark exposure curves such as the Swiss Re exposure curves. The original Swiss Re curves, also called Gasser curves, were developed by Peter Gasser based on the data of "Fire statistics of the Swiss Association of Cantonal Fire Insurance Institutions" for the years 1959-1967. These curves were parameterized with MBBEFD distributions. A new sub-class of the one-parameter MBBEFD exposure curves was then defined as:

$$G_c = G_{MBBEFD(g(c),b(c))}$$

With $b(c) = \exp(3.1 - 0.15(1+c)c)$ and $g(c) = \exp((0.78 + 0.12c)c)$.

Each Swiss Re exposure curve has a defined parameter c. The table below describes the Swiss Re curves and recommendation for their use within the LoB property.

Table 28: Swiss Re exposure curves

Exposure Curve	*c*	*b*	*g*	Scope of application
Swiss 1	1.5	12.648	4.221	Personal lines
Swiss 2	2.0	9.025	7.691	Commercial lines (small scale)
Swiss 3	3.0	3.669	30.569	Commercial lines (medium scale)
Swiss 4	4.0	1.105	154.470	Industrial and large commercial

Due to the lack of data on destruction rates, these exposure curves will be used.

5.2.2.4 AXA Group exposure curves

AXA has also developed and calibrated distinct exposure curves for Industrial, Commercial, and Residential sub-LoBs for some entities within the property LoB. These exposure curves are based on hyperbolic MBBEFD distributions. This distribution is a limit case of the two-parameter MBBEFD for which $b \rightarrow 1$ and $g = \frac{1}{p}$. The advantage of this one-parameter family is the easiness to fit due to a one-parameter assessment. Consider $1 > m > 0$. The properties of a random variable x which follows a hyperbolic MBBEFD distribution of parameter m are listed below:

- Density function: For $t \geq 0$, $f(t) = \frac{1}{m(1+\frac{t}{m})} 1_{0 \leq t < 1} + \frac{m}{1+m} 1_{t=1}$
- Mean: $\mathbb{E}(x) = \mathrm{m} * \ln(1 + \frac{1}{m})$
- Cumulative Distribution Function (CDF): $F(t) = \begin{cases} \left(1 - \frac{1}{1+\frac{t}{m}}\right) 1_{0<t<1} + 1_{t\geq 1}, \ t \geq 0 \\ 0, \ t < 0 \end{cases}$

The parameter m of a hyperbolic MBBEFD can be linked to the parameter g of an MBBEFD thanks to the probability $\mathbb{P}(x = 1)$, leading to $m = \frac{1}{g-1}$.

The AXA exposure curves are also based on the hypothesis that the destruction rate of risks within the same group follows a hyper-MBBEFD distribution of parameter m, where m is the median of the destruction rate within the group. Since the median of the destruction rate per group is not always available, a regression analysis which allows to determine the median of the destruction rate m knowing the line of business and the average sum insured of the considered portfolio layer has been developed internally. A linear relationship between the mean of the Sums Inured Q_{mean} and the estimated parameter $\widehat{m}$ per sub-Lob is then available:

$$\log(\widehat{m}) = A * \log(Q_{mean}) + D$$

Where A and D are regression parameters depending on the considered entity and LoB. In total, the AXA curves cover 5 entities across 3 sub-LoBs within the property LoB.

As opposed to the other curves, the AXA curves depend on the range of the risk profile thanks to the relationship between the parameter of the exposure curve and the mean of the Sum Insured within the risk profile. It thus takes into account the evolution of the destruction rate as a function of the sum insured. Intuitively, the higher the sum insured, the lower the destruction rate is likely to be. Indeed, a small property is more likely to be completely destroyed in the event of a claim than a castle. With AXA exposure curves, an increase of Q_{mean} leads to an increase of m, which lowers the average destruction rate, thus illustrating this effect.

After presenting the different exposure curves, we will now detail how these curves are used in the recoveries modeling for an XL treaty, using the risk profile database defined in 3.5.

5.2.3 Pure loss modeling (XL per Risk)

If the risk profile of an entity is known and an exposure curve G_{curve} has been selected to model recoveries, it becomes possible to quickly estimate the reinsurer's average loss for an XL per risk treaty L xs R.

After receiving the risk profile and choosing an exposure function G_{curve}, the assessment of the reinsurer's average losses can be conducted through the following steps:

1. Setting the mean individual exposure per range Q_h
2. Setting the mean normalized retention r_h and limit l_h for each band h as: $r_h = \min(\frac{R}{Q_h}, 1)$ and $l_h = \min(\frac{L+R}{Q_h}, 1)$
3. The mean gross loss per range is then estimated with the gross premiums GP_h per band provided. The total loss per range shall be calculated as: $\widehat{\mathbb{E}}(Total\ Loss_h) = LR_h * GP_h$ where LR_h is the budgeted loss ratio for h-th band.
4. An appropriate exposure curve for each band is chosen.
5. For each range, the value of exposure curve function $G_{curve}(r_h)$ and $G_{curve}(l_h)$ are calculated.
6. The reinsurer loss for each band is then estimated with the formula:
$$\widehat{\mathbb{E}}\left(S_h^{reinsurer}\right) = \left(G_{curve}(l_h) - G_{curve}(r_h)\right) * \widehat{\mathbb{E}}(Total\ Loss_h)$$
7. The estimated mean total loss of the treaty for the reinsurer $\widehat{\mathbb{E}}(S^{reinsurer})$ can be expressed as:
$$\widehat{\mathbb{E}}(S^{reinsurer})^{Pure\ Loss} = \sum_{h=1}^{H} \widehat{\mathbb{E}}\left(S_h^{reinsurer}\right)$$
For all ranges $h \in \{1, \dots, H\}$

In this actuarial work, this estimation will be provided for indicative purposes only. It represents a simplified assessment, as many treaty parameters are not accounted for in this model. Specifically, no payment pattern can be applied, and only the pure premium of the treaty is computed. Furthermore, the pure premium is particularly dependent on the budgeted loss ratios per band, which are provided in the risk profile and are not necessarily correctly estimated. Moreover, a precise estimate of the loss ratio per band is not always available, and loss ratios can be approximated by a loss ratio on a wider scale. The effects of reinstatements, AAD, AAL or indexation clauses are also not measured. Moreover, this methodology is highly dependent on the risk profile, and the choice of the exposure curve. The latter can be subjective and largely influenced by the user's judgment if Swiss Re curves are chosen. Additionally, at this stage, the same exposure curve is applied across all ranges of the risk profile. A more refined estimation could involve selecting different exposure curves for various ranges, which is the case for the AXA exposure curves. However, this was not implemented within the tool when using MBBEFD exposure curves to simplify automation, though it could serve as a potential area for future improvements.

To illustrate the methodology, let us consider the following XL treaty, which covers a single cedent within the Property – Commercial LoB:

- Limit: 2 000 000 €.
- Retention: 3 000 000 €.
- Reinstatement: 2 reinstatements available, each at 100% of the original premium.
- Stabilization Clause: None.

- Loss Basis: Loss Occurring (losses are covered based on when they occur, regardless of the policy's inception date).
- AAD/AAL: No Annual Aggregate Deductible (AAD) and an Annual Aggregate Limit (AAL) of 6 000 000 €.

Given the LoB (Commercial Property), the Swiss Re 4 exposure curve has been selected for this example. A more realistic approach would be to consider different Swiss Re curves for the first exposure bands. However, since only the higher exposure bands will impact the recovery amount, the choice of the exposure curve for the lower bands is not particularly important when estimating the pure loss of an XL treaty. This would not be the case when modeling recovery distribution or SP treaties since the entire risk profile is relevant in this case. The risk profile considered is displayed in the following table. However, the values in the dataset have been adjusted for confidentiality purposes. The actual results derived from the real risk profile will be shared as indicative figures.

Table 29: Risk profile example – XL exposure pricing

Range Min	Range Max	Number of risks	Sum Insured	Gross Written Premium	Budgeted LR	Mean Sum Insured
0 €	500 000 €	358003	74 314 971 558 €	54 847 761 €	84%	207 582 €
500 001 €	1 000 000 €	49059	33 871 144 329 €	19 556 137 €	68%	690 417 €
1 000 001 €	2 000 000 €	27926	39 416 005 433 €	17 896 638 €	70%	1 411 445 €
2 000 001 €	3 000 000 €	9644	23 536 417 536 €	9 970 029 €	69%	2 440 524 €
3 000 001 €	4 000 000 €	4386	15 131 484 262 €	6 304 789 €	78%	3 449 951 €
4 000 001 €	5 000 000 €	2258	10 085 109 923 €	4 244 733 €	80%	4 466 391 €
5 000 001 €	6 000 000 €	1150	6 316 189 232 €	2 688 083 €	69%	5 492 338 €
6 000 001 €	8 000 000 €	1174	8 135 715 868 €	3 401 340 €	70%	6 929 911 €
8 000 001 €	10 000 000 €	547	4 927 973 682 €	1 957 181 €	67%	9 009 093 €
10 000 001 €	15 000 000 €	533	6 370 926 686 €	2 499 792 €	70%	11 952 958 €
15 000 001 €	20 000 000 €	116	2 010 733 793 €	670 134 €	75%	17 333 912 €
20 000 001 €	30 000 000 €	83	2 029 779 486 €	713 290 €	76%	24 455 175 €
30 000 001 €	40 000 000 €	28	939 074 553 €	282 152 €	73%	33 538 377 €
40 000 001 €	50 000 000 €	9	399 697 027 €	95 993 €	71%	44 410 781 €
50 000 001 €	60 000 000 €	8	432 269 500 €	159 145 €	65%	54 033 688 €
60 000 001 €	75 000 000 €	2	141 500 000 €	53 614 €	67%	70 750 000 €

The table below presents the estimated quantities calculated during the pure loss modeling part and the estimation of the reinsurer's average losses.

Table 30: Pure premium calculation

Range Min	Range Max	r_h	l_h	$(G_{curve}(l_h) - G_{curve}(r_h))$	$\widehat{\mathbb{E}}(Total\ Loss_h)$	$\widehat{\mathbb{E}}(S_h^{reinsurer})$
0 €	500 000 €	1	1	0,00%	46 072 119 €	0 €
500 001 €	1 000 000 €	1	1	0,00%	13 298 173 €	0 €
1 000 001 €	2 000 000 €	1	1	0,00%	12 527 647 €	0 €
2 000 001 €	3 000 000 €	1	1	0,00%	6 879 320 €	0 €
3 000 001 €	4 000 000 €	0,87	1	2,82%	4 917 735 €	138 676 €
4 000 001 €	5 000 000 €	0,67	1	8,06%	3 395 786 €	273 631 €
5 000 001 €	6 000 000 €	0,55	0,91	10,06%	1 854 777 €	186 679 €
6 000 001 €	8 000 000 €	0,43	0,72	10,20%	2 380 938 €	242 896 €
8 000 001 €	10 000 000 €	0,33	0,55	10,01%	1 311 311 €	131 260 €
10 000 001 €	15 000 000 €	0,25	0,42	10,07%	1 749 854 €	176 177 €
15 000 001 €	20 000 000 €	0,17	0,29	10,22%	502 601 €	51 379 €
20 000 001 €	30 000 000 €	0,12	0,20	9,63%	542 100 €	52 205 €
30 000 001 €	40 000 000 €	0,09	0,15	9,49%	205 971 €	19 544 €
40 000 001 €	50 000 000 €	0,07	0,11	8,25%	68 155 €	5 622 €
50 000 001 €	60 000 000 €	0,06	0,09	7,30%	103 444 €	7 552 €
60 000 001 €	75 000 000 €	0,04	0,07	9,76%	35 921 €	3 507 €

The reinsurer's average loss is estimated at 1 289 128 € based on this modified risk profile. For the actual estimation, pure premium calculations have been provided for each Swiss Re curve corresponding to the commercial line of business. The estimated average loss decreases when higher-numbered Swiss Re curves are used. This outcome aligns with the fact that the average destruction rate associated with Swiss Re Curve 4 is lower than the one of Swiss Re Curves 2 and 3, resulting in lower recovery amounts for the reinsurer.

Table 31: Pure premium comparison

Exposure curve	$\widehat{\mathbb{E}}(S^{reinsurer})^{Pure\ Loss}$
SwissRe 2	769 161 €
SwissRe 3	614 893 €
SwissRe 4	454 834 €

5.2.4 Loss distribution modeling

The previously developed method provides an estimate of the average recoveries for an XL treaty. However, this approach does not yield a distribution of recoveries or allow for the application of various treaty conditions, leading to a loss of information during the modeling process. Nevertheless, it is possible to use exposure curves to generate a recovery distribution, which would not only work as a basis for comparison but also provide an alternative when the number of claims is too low. The following section outlines the methodology adopted in this study. The SP modeling will also be based on the following methodology.

5.2.4.1 Modeling sum insured

To be able to model a given XL treaty's recoveries distribution or ceded losses under a SP treaty, it is first necessary to model the covered portfolio risks' random Sum Insured Q arranged within a set of ranges. If one has access to the detailed sum insured per policy within an exposure band, one could fit a bounded nonparametric distribution on Q for the considered band. In this project, since the distribution of sum insured within the exposure band is not available, the Beta distribution has been selected to model sums insured due to its bounded nature. Indeed, it takes values within the interval $[0; 1]$, which is convenient since the sums insured are modeled within a range. Furthermore, the Beta distribution can take a wide variety of shapes, making it an ideal candidate for modeling the sums insured across all ranges. Additionally, the parameters of the beta distribution are easily estimable using the method of moments.

Consider $\alpha > 0$ and $\beta > 0$. Considering the underlying risk profile and a given exposure band, the random variable Q corresponding to the sum insured is assumed to follow a Beta distribution of parameter (α, β):

- Density function: For $1 > t > 0$, $f(t) = \frac{1}{B(\alpha,\beta)} t^{\alpha-1}(1-t)^{\beta-1}$ where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$
- Mean: $\mathbb{E}(Q) = \frac{\alpha}{\alpha+\beta}$
- Variance: $Var(Q) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
- Cumulative Distribution Function (CDF): For $t \geq 0$

$$F(t) = \begin{cases} \frac{\int_0^t u^{\alpha-1}(1-u)^{\beta-1}du}{B(\alpha,\beta)}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Here, we consider a range of the risk profile, bounded by Q_{max} and Q_{min}, a mean sum insured per range Q_{mean} and the variance $Q_{variance}$ per range. Since the beta distribution is defined on the interval $[0; 1]$, one needs to normalize the mean and variance of the sum insured within the range. The normalized mean and variance of sum insured are given by the following formulas:

$$q_{mean} = \frac{Q_{mean} - Q_{min}}{Q_{max} - Q_{min}}$$

$$q_{variance} = \frac{Q_{variance}}{(Q_{max} - Q_{min})^2}$$

The estimation of the beta distribution parameters is performed with the method of moments. The estimators of the beta parameters α and β are thus:

- $\hat{\alpha} = q_{mean}\left(\frac{q_{mean}(1-q_{mean})}{q_{variance}} - 1\right)$
- $\hat{\beta} = (1 - q_{mean})(\frac{q_{mean}(1-q_{mean})}{q_{variance}} - 1)$

Whenever the variance of the sum insured distribution within a range is not known, as it is the case in the current AGCRe's risk profile database, the variance of the sum insured within the range must still be estimated. To do so, we will use the principle of entropy to determine the value of $q_{variance}$.

In information theory, the entropy of a random variable quantifies the average level of uncertainty or information associated with the variable's potential outcomes. To estimate the variance, the parameter which maximizes this entropy function has been selected. Indeed, given the lack of sufficient information about the distribution, maximizing entropy allows us to select the Beta distribution that contains the least information, and thus the least arbitrary among all distributions one could use. Maximizing the entropy of a parametric distribution family is also equivalent to seeking the distribution that is closest to the uniform distribution. The Nelder-Mead algorithm is thus used to maximize the following entropy function of the beta distribution:

$$\underset{\alpha,\beta>0}{\operatorname{argmax}} \mathbb{E}(-ln\left(f(Q)\right) = \underset{\alpha,\beta>0}{\operatorname{argmax}} \ln(B(\alpha,\beta)) - (\alpha-1)\,\psi(\alpha) - (\beta-1)\,\psi(\beta) + (\alpha+\beta-2)\psi(\alpha+\ \beta)$$

Where ψ is the digamma function: $\psi = \frac{\Gamma\prime}{\Gamma}$.

Since one knows the mean of the beta distribution $q_{mean} = \frac{\alpha}{\alpha+\beta}$, the two parameters optimization problem can be reduced to a one parameter problem by replacing β by $\beta(\alpha) = \alpha \frac{1-q_{mean}}{q_{mean}}$:

$$\underset{\alpha>0}{\operatorname{argmax}} \ln(B(\alpha,\beta(\alpha))) - (\alpha-1)\,\psi(\alpha) - (\beta(\alpha)-1)\,\psi(\beta(\alpha)) + (\alpha+\beta(\alpha)-2)\psi(\alpha+\beta(\alpha))$$

The parameters $\hat{\alpha}$ and $\hat{\beta}$ can then be estimated from the previous equations.

Sums insured within a risk profile range can be generated by simulating realizations q of a Beta distribution with parameters $\hat{\alpha}$ and $\hat{\beta}$ and derive a distribution of sums insured Q by applying the following transformation:

$$Q = (Q_{max} - Q_{min}) * q + Q_{min}.$$

For example, and to illustrate the methodology, the XL treaty defined in 5.2.3 with the risk profile defined in *Table 29* will be studied. Let us consider the range between 500 001 € and 1 000 000 €. The mean of the sum insured is 690 417 €. By rescaling this range between 0 and 1, we get the parameter $q_{mean} = 0.38$, which corresponds to the mean of the beta distribution between 0 and 1. Since the variance of the sum insured for this type of treaty is unknown, the parameters of the Beta distribution are assessed using the Nelder-Mead algorithm. After performing the estimation, the following parameters for the Beta distribution were obtained: $\hat{\alpha} = 0.86$ and $\widehat{\beta} = 1.40$.

The figure below illustrates the distribution of sums insured within the given range. One can notice the similarity to a uniform distribution, although there is a slight concentration of values towards the lower part of the range. This outcome was expected, as the entropy maximization approach tends to bring the distribution closer to the uniform distribution. Additionally, since the mean sum insured is closer to the lower end of the range, it is consistent for the distribution to show a higher concentration of values in this lower segment.

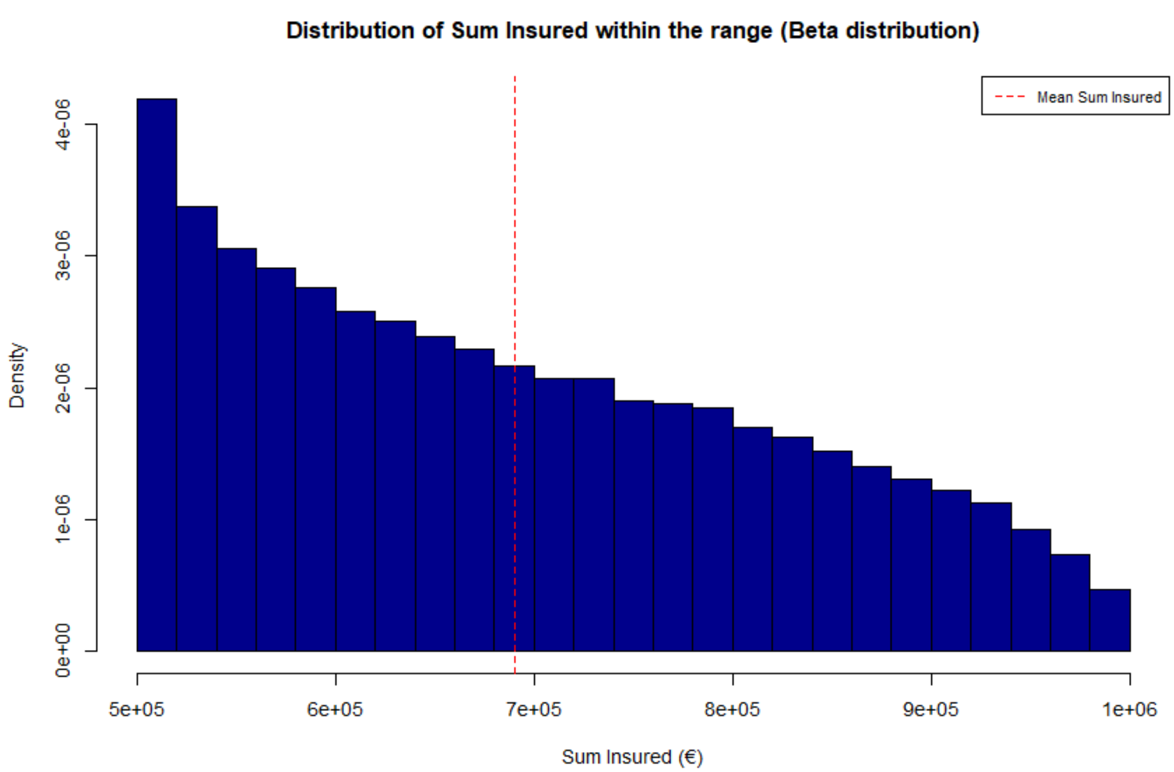

Figure 54: Sum Insured distribution – Range: 500 001 € - 1 000 000 €

The same methodology is applied to the others ranges of the risk profile:

Table 32: Sum Insured distribution parameters

Range Min	Range Max	Mean Sum Insured	q_{mean}	α	β
0 €	500 000 €	207 582 €	0,42	0,89	1.22
500 001 €	1 000 000 €	690 417 €	0,38	0,86	1,40
1 000 001 €	2 000 000 €	1 411 445 €	0,41	0,88	1,26
2 000 001 €	3 000 000 €	2 440 524 €	0,44	0,91	1,16
3 000 001 €	4 000 000 €	3 449 951 €	0,45	0,92	1,12
4 000 001 €	5 000 000 €	4 466 391 €	0,47	0,95	1,07
5 000 001 €	6 000 000 €	5 492 338 €	0,49	0,98	1,02
6 000 001 €	8 000 000 €	6 929 911 €	0,46	0,93	1,09
8 000 001 €	10 000 000 €	9 009 093 €	0,50	1	1
10 000 001 €	15 000 000 €	11 952 958 €	0,39	0,87	1,35
15 000 001 €	20 000 000 €	17 333 912 €	0,47	0,95	1,07
20 000 001 €	30 000 000 €	24 455 175 €	0,45	0,92	1,12
30 000 001 €	40 000 000 €	33 538 377 €	0,35	0,85	1,57
40 000 001 €	50 000 000 €	44 410 781 €	0,44	0,91	1,16
50 000 001 €	60 000 000 €	54 033 688 €	0,40	0,87	1,31
60 000 001 €	75 000 000 €	70 750 000 €	0,72	2.17	0,84

5.2.4.2 Individual claim severity modeling

Once the distribution of sums insured has been calibrated, the next step is to determine a distribution for the destruction rate. Given that the distribution F of the destruction rate x is supposed to be identical for risks within the same risk group, a specific distribution for each range of the risk profile can be parametrized.

It is important to recall that the exposure curve is derived from the underlying distribution of destruction rates. It implies that the CDF F of the destruction rates can therefore be deduced from the exposure curve. In this pricing tool development, we exclusively use the MBBFED (Swiss Re exposure curves, based on the MBBFED distribution) or the Hyperbolic MBBFED distributions (AXA exposure curves).

Destruction rates can be generated for each range depending on the type of exposure curve picked:

- **MBBFED exposure curve:**
  If, within the range, a MBFFED distribution with parameters b and g is settled on, destruction rates can be simulated directly from this distribution. The mean of the destruction rate x within the range is thus $\mathbb{E}(x) = \frac{\ln(gb)(1-b)}{\ln(b)(1-gb)}$.
- **Hyperbolic MBBFED exposure curve (AXA Exposure Curves):**
  If an AXA exposure curve based on a hyperbolic MBBFED distribution is selected, with parameter m (estimated using the mean sum insured of the range), destruction rates can be generated using the following Monte Carlo technique: simulate realizations u of a uniform distribution on $(0,1)$ and deduce a distribution of destruction rates x with the transformation $x = \max(0, \min(1, \frac{mu}{1-u}))$. This transformation is deduced from the CDF of an Hyperbolic MBBFED distribution. The mean of the destruction rate within the range is thus $\mathbb{E}(x) = \text{m} * \ln(1 + \frac{1}{m})$

To simulate individual loss amounts, the destruction rate x_k is multiplied by the corresponding sum insured Q_k generated in the previous step. This assumes the following relationship for individual claim amounts in property insurance: $X_k = x_k * Q_k$.

This methodology relies on the strong assumption of independence between destruction rates and sums insured. However, in practice, this assumption does not always hold - higher sums insured tend to correspond to lower destruction rates compared to smaller sums insured. The AXA exposure curves address this issue by calibrating exposure curves based on the mean sum insured for each range, making the approach more realistic. On the other hand, with Swiss Re exposure curves, the same exposure curve is used across all ranges, which ignores the dependency between destruction rates and sums insured. This is a limitation of the Swiss Re approach and suggests that results derived using these curves should be interpreted with caution.

5.2.4.3 Frequency modeling

The sums insured and destruction rates within an exposure range can now be generated, thereby enabling the simulation of individual claim amounts. However, one needs to determine the expected number of claims per year. Based on the distribution of individual claims amounts and the risk profile, the expected number of claims per attachment year can be deduced using assumptions from the collective model.

Focusing on a specific range h, we model the total loss within this range using the collective risk model previously introduced. Recall that the total loss S_h can be expressed as:

$$S_h = \sum_{k=1}^{N_h} X_{k,h}$$

Where N_h is a discrete random variable corresponding to the number of losses per the year and $\left(X_{k,h}\right)_{1 \le k \le N_h}$ is a sequence of independent and identically distributed random variables corresponding to a claim k. Furthermore, it is assumed that for all k, N_h and $X_{k,h}$ are independent. Under these assumptions, the Wald formula applies:

$$\mathbb{E}(S_h) = \mathbb{E}(N_h) * \mathbb{E}(X_h)$$

Since the distribution of the destruction rate x_h is assumed to be independent from the distribution of the sum insured Q_h, one has:

$$\mathbb{E}(X_h) = \mathbb{E}(x_h) * \mathbb{E}(Q_h)$$

Furthermore, the total loss per range h of the portfolio risk profile can be estimated with the budgeted loss ratio LR_h and the gross premium [1]GP_h of the current year:

$$\widehat{\mathbb{E}}(S_h) = LR_h * GP_h$$

Given that the distributions x_h and Q_h for the range h have been derived, $\mathbb{E}(x_h)$ and $\mathbb{E}(Q_h)$ can thus be estimated. The total loss amount is then divided by the individual average loss to estimate of the mean number of losses for the current year according to the Wald Formula:

$$\widehat{\lambda_h} = \widehat{\mathbb{E}}(N_h) = \frac{\widehat{\mathbb{E}}(S_h)}{\widehat{\mathbb{E}}(x_h) * \widehat{\mathbb{E}}(Q_h)}$$

The number of losses within the range N_h is supposed to follow a Poisson distribution with mean $\widehat{\lambda_h}$. This choice is motivated by the fact that Poisson distributions are commonly used to model the number of claims per year. Moreover, since only the expected number of claims per year is known, the Poisson distribution stands out as one of the most suitable candidates for modeling the annual claim count. This approach heavily relies on the budgeted loss ratio for each band and does not directly account for the underlying distribution. This reliance represents an additional limitation compared to the previously developed frequency-severity approach. An inaccurate estimation of the loss ratio will inevitably lead to a flawed modeling of the underlying claims experience.

Let us consider the example of the previously discussed XL treaty. We remind below the parameters b and g for the Swiss Re exposure curves:

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[1] *For a Loss occurring treaty, the Gross Premium corresponds to the Gross Earned Premium, whereas for a Risk Attaching treaty, the Gross Premium corresponds to the Gross Written Premium.*

Table 33: Swiss Re exposure curves parameters

Exposure Curve	b	g	$\mathbb{E}(x)$
Swiss 1	12.65	4.22	34.85 %
Swiss 2	9.03	7.70	22.61 %
Swiss 3	3.67	30.57	8.72 %
Swiss 4	1.11	154.47	3.19 %

Keeping the previous range, with a budgeted loss ratio of 68 % and a gross premium of 19 556 136 €, one has, with Swiss Re 4:

$$\widehat{\lambda_h} = \widehat{\mathbb{E}}(N_h) = \frac{\widehat{\mathbb{E}}(Total\ Loss_h)}{\widehat{\mathbb{E}}(x_h) * \widehat{\mathbb{E}}(Q_h)} = 602{,}23$$

The following table represents the estimated parameters for the rest of the range with Swiss Re 4 exposure curve.

Table 34: Frequency parameters

Range Min	Range Max	$\widehat{\mathbb{E}}(S_h)$	$\widehat{\mathbb{E}}(X_h)$	$\widehat{\lambda_h}$
0 €	500 000 €	46 176 184 €	6 622 €	6973,29
500 001 €	1 000 000 €	13 263 701 €	22 024 €	602,23
1 000 001 €	2 000 000 €	12 443 435 €	45 025 €	276,37
2 000 001 €	3 000 000 €	6 850 861 €	77 853 €	88,00
3 000 001 €	4 000 000 €	4 895 628 €	110 053 €	44,48
4 000 001 €	5 000 000 €	3 385 287 €	142 478 €	23,76
5 000 001 €	6 000 000 €	1 857 600 €	175 206 €	10,60
6 000 001 €	8 000 000 €	2 392 917 €	221 064 €	10,82
8 000 001 €	10 000 000 €	1 302 932 €	287 390 €	4,53
10 000 001 €	15 000 000 €	1 748 736 €	381 299 €	4,59
15 000 001 €	20 000 000 €	505 356 €	552 952 €	0,91
20 000 001 €	30 000 000 €	538 748 €	780 120 €	0,69
30 000 001 €	40 000 000 €	207 037 €	1 069 874 €	0,19
40 000 001 €	50 000 000 €	68 076 €	1 416 704 €	0,05
50 000 001 €	60 000 000 €	103 454 €	1 723 675 €	0,06
60 000 001 €	75 000 000 €	35 728 €	2 256 925 €	0,02

5.2.4.4 Obtaining recoveries

The aim here is to simulate a distribution of recoveries (respectively ceded losses) for XL treaties (respectively SP treaties) covering the property LoB. Let us consider a simulated loss year i and an exposure range h of the risk pofile. Let N_h^i be the number of simulated losses and $\left(X_{k,h}^i\right)_{1\le k\le N_h^i}$ the individual loss amount for the considered simulation year i and range h. The amounts $\left(X_{k,h}^i\right)_{1\le k\le N_h^i}$ are generated by multiplying simulated destruction rates $\left(x_{k,h}^i\right)_{1\le k\le N_h^i}$ and sum insured $\left(Q_{k,h}^i\right)_{1\le k\le N_h^i}$. By repeating this process across all ranges, one gets a distribution of individual loss amounts $\left(X_k^i\right)_{1\le k\le N_i}$ where $N_i = \sum_{h=1}^{H} N_h^i$ corresponds to the total number of claims simulated for the loss year i.

5.2.4.4.1 XL per risk

To derive a distribution of recoveries, the approach mirrors the one developed earlier in the frequency-severity framework. Regarding the payment pattern, a single pattern is calibrated using data from the historical individual claims database. Once this payment pattern is established, the recovery calculation process is applied as previously described. This results in a distribution of recoveries, denoted as $\left(S_i^{reinsurer\ (discounted)}\right)_{1 \leq i \leq N_{simul}}$, representing the amounts ceded to the reinsurer under the terms of the reinsurance contract. The following figure summarizes the implemented process to model recoveries using exposure pricing for XL treaties, which corresponds to process "P8" in *Figure 31*.

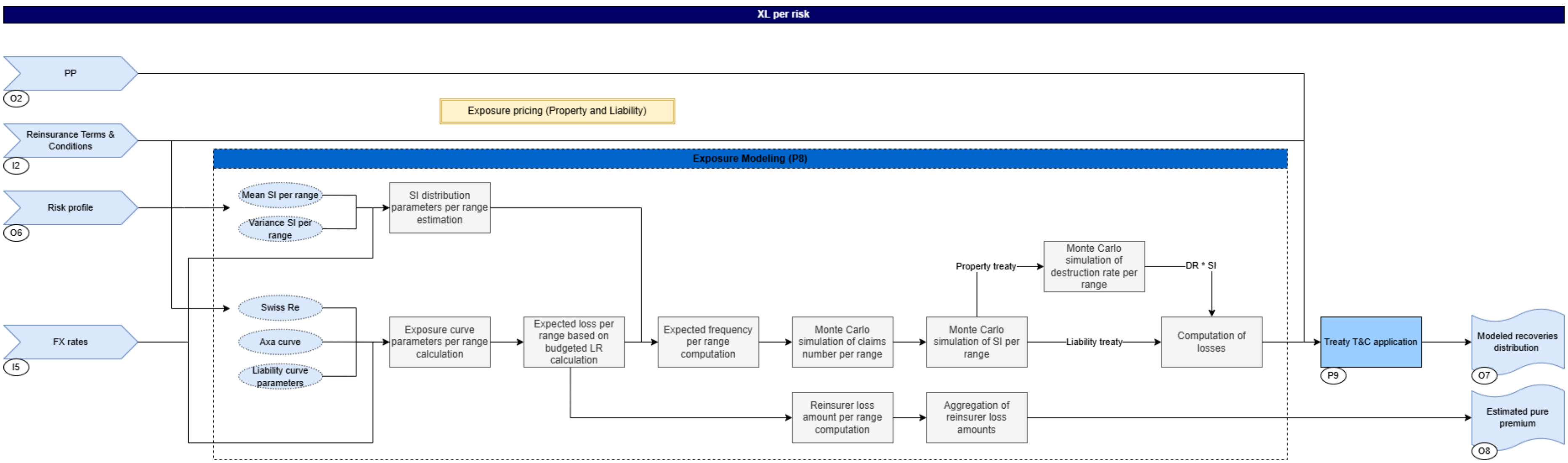


Figure 55: Exposure modeling process – XL per risk treaties

5.2.4.4.2 Surplus

For SP treaties, once distributions $\left(X_k^i\right)_{1\le k\le N_i}$ and $\left(Q_k^i\right)_{1\le k\le N_i}$ are simulated, it is possible to obtain losses ceded to reinsurance. A SP treaty being a proportional form of reinsurance, the individual cession rate must first be calculated. We remind that, for SP treaties, the treaty variables used to calculate the cession rate are the retention R and the capacity C. For each claim X_k^i simulated, the individual cession rate c_k^i is deduced as follows:

$$c_k^i = \frac{\min\left(\max\left(0, Q_k^i - R\right), C\right)}{Q_k^i}$$

From this result, one can compute the distribution of losses ceded to reinsurance $\left(X_k^i * c_k^i\right)_{1\le k\le N_i}$ for a simulation year i.

To apply the surplus T&C, which in the context of this project are the same as the conditions of QS treaties, we are more interested in the annual underlying claim amount represented through ultimate loss ratios. First, for a given simulation, the cedent's annual loss ratio LR_i^{cedent} is calculated as follows:

$$LR_i^{cedent} = \frac{\sum_{k=1}^{N_i} X_k^i}{Premium_{subject}}$$

Where $Premium_{subject}$ corresponds to the covered portfolio GEP (respectively GWP) if the treaty is Loss Occurring (respectively Risk Attaching). It can be calculated by summing the gross premium per range GP_h of the risk profile. Hence, a distribution of ultimate cedent's loss ratios is computed $\left(LR_i^{cedent}\right)_{1\le i\le N_{simul}}$.

Regarding reinsurer's losses, for each simulation i, the amounts $\left(X_k^i * c_k^i\right)_{1\le k\le N_i}$ are aggregated into an annual ceded loss amount $Ceded\ Losses_i$. One can then determine the distribution of losses ceded to reinsurance $\left(S_i^{reinsurer}\right)_{1\le i\le N_{simul}}$. To compute the reinsurer's loss ratio, the ceded premium must be determined. Since the total distribution of sum insured and individual is not available, the ceded premium is approximated by summing ceded premium across all bands of the risk profile. For a range h, the ceded rate c_h is calculated: $c_h = \frac{\min\left(\max\left(0, Q_{mean,h} - R\right), C\right)}{Q_{mean,h}}$ where $Q_{mean,h}$ is the mean sum insured of the corresponding range. The ceded premium of the considered range GP_h^{ceded} is deduced: $GP_h^{ceded} = c_h * GP_h$. The ceded premium $Premium_{ceded}$ is ascertained by aggregating the ceded premiums of the risk profile. The reinsurer's loss ratio is then estimated as follows:

$$LR_i^{reinsurer} = \frac{S_i^{reinsurer}}{Premium_{ceded}}$$

To conclude, using the exposure methodology, one has simulated a distribution of cedent's loss ratio $\left(LR_i^{cedent}\right)_{1\le i\le N_{simul}}$, a distribution of reinsurer's loss ratio $\left(LR_i^{reinsurer}\right)_{1\le i\le N_{simul}}$ and ceded losses $\left(S_i^{reinsurer}\right)_{1\le i\le N_{simul}}$. The ceded premium $Premium_{ceded}$ has also been calculated. Theses quantities will be used in part 5.3.2.1 to calculate the reinsurer's commission.

For now, the figure below describes the process for obtaining losses ceded to reinsurance for SP treaties. The process "P15" will be described in part 5.3.2.1.

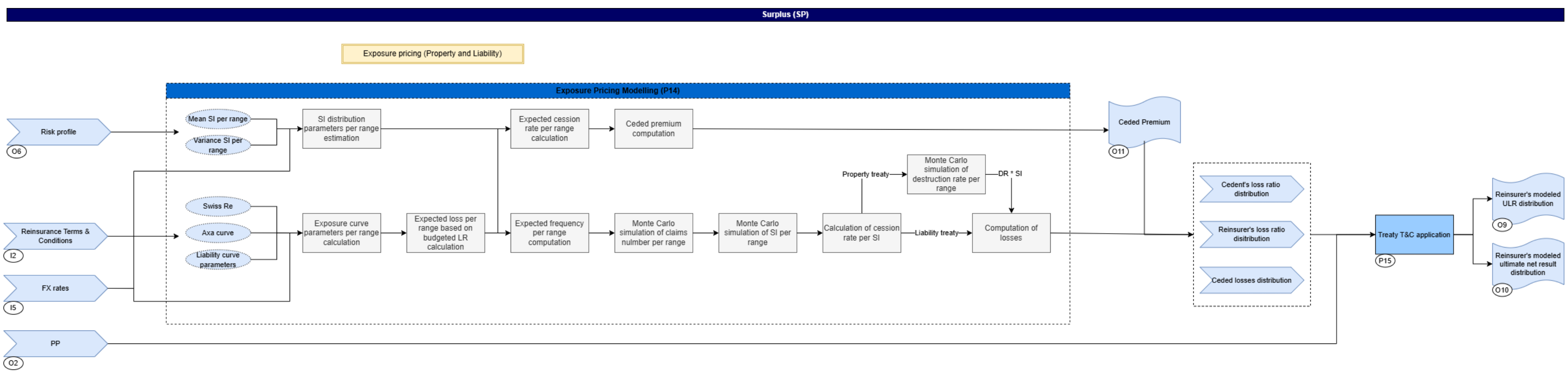


Figure 56: Modeling process - SP

5.2.4.5 Comparison with experience pricing (XL per risk)

As a comparison, we will evaluate the differences between the two modeling methods for a reinsurance program, where the first layer corresponds to the treaty discussed in this section. The program consists of three layers, where the remaining two layers have the following properties:

- Limit: 5 000 000 € (layer 2), 10 000 000 € (layer 3)
- Retention: 5 000 000 € (layer 2), 10 000 000 € (layer 3)
- Reinstatement: 2 reinstatements available, each at 100% of the original premium (layer 2), 1 reinstatement available, each at 100% of the original premium (layer 3)
- Indexation Clause: None.
- Loss Basis: Loss Occurring
- AAD/AAL: No Annual Aggregate Deductible (AAD) and an Annual Aggregate Limit (AAL) of 15 000 000 € (layer 2) and 20 000 000 € (layer 3)

For the frequency distribution, a negative binomial distribution with parameters $\hat{n} = 4.34$ and $\hat{p} = 0.38$ was fitted. Regarding the severity distribution, the top four distributions identified using the previously developed method, ranked in order, are the Gaussian Kernel, ME Pareto, ME GPD and GPD (MLE). The fit statistics and the CDF functions are detailed in appendix C.

Based on the following QQ plot, one can notice that the frequency-severity approach struggles to accurately model the underlying claims behavior. The quantiles of the fitted distributions deviate significantly from the bisector, indicating a mismatch between the model and the observed data. Additionally, no historical claim has ever reached the retention of the highest layer, making the recovery estimation of the final layer challenging.

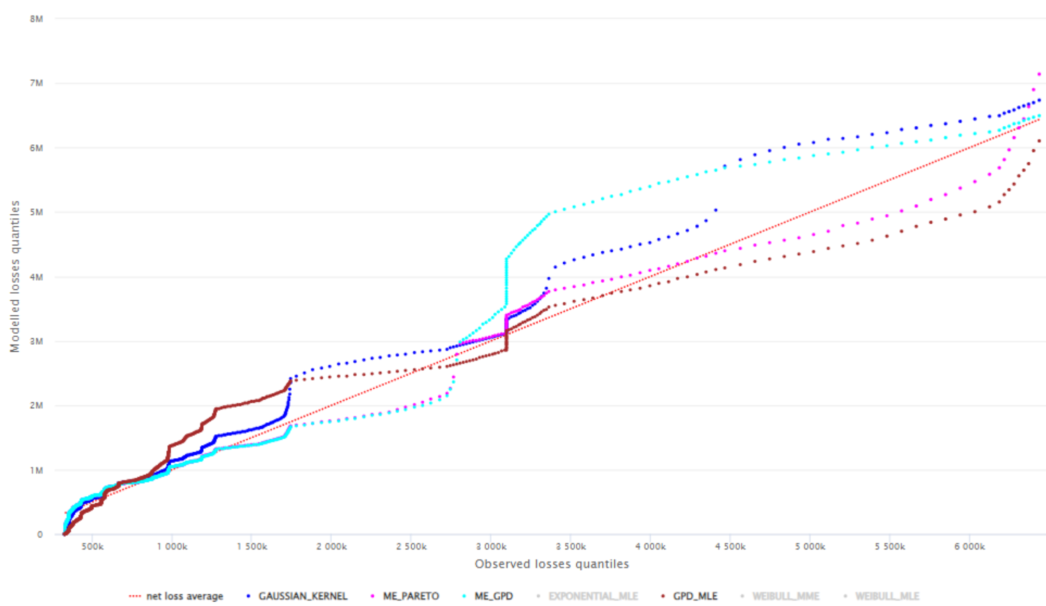


Figure 57: QQ plot – XL exposure pricing

Considering these challenges, the exposure method can serve as an alternative approach to estimate risks indicators, including the premium of the treaty. For premium calculation, the average recoveries will, for now act as an approximation of the premium.

Regarding the exposure method, the details of the simulated claim frequency and severity distributions are provided in the following graphs, using the Swiss Re 4 exposure curve. The simulations yield an average of 619 claims per year, with an individual claim cost averaging approximately 22 000 €.

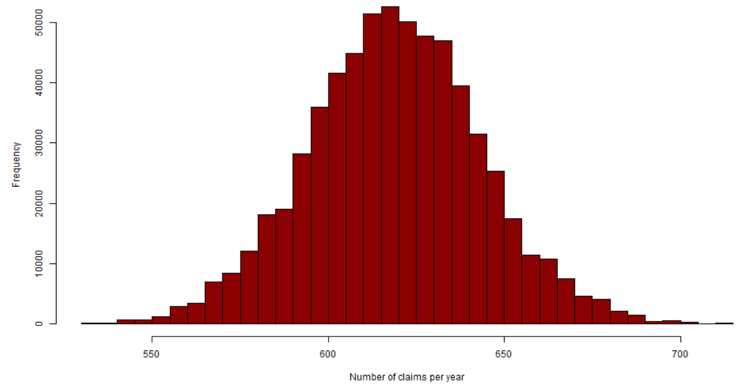


Figure 58: Distribution of claim number per year – exposure modeling

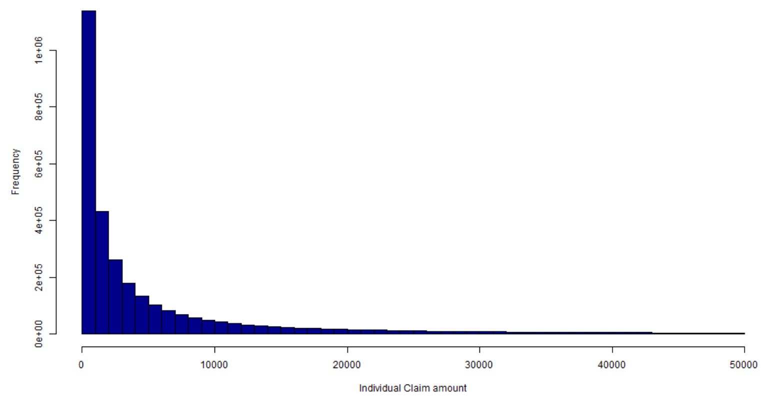


Figure 59: Distribution of individual claims amounts – exposure modeling

To calculate the average recoveries, 50,000 simulations were conducted for each of the Swiss Re exposure curves applicable to commercial LoB, namely Swiss Re 2, 3, and 4. Similarly, 50,000 simulations were performed for the frequency-severity approach using the four best-performing severity distributions. Additionally, a pure recovery estimation method was also computed (see 5.2.3). The tables below summarize the results for the average recovery estimation:

Table 35: Recovery mean – frequency/severity

Layer	Frequency distribution	Severity distribution	$\widehat{\mathbb{E}}(S^{reinsurer})$
1	Negative binomial	Gaussian Kernel	759 625 €
1	Negative binomial	ME - Pareto	732 504 €
1	Negative binomial	ME - GPD	748 036 €
1	Negative binomial	GPD (MLE)	627 251 €
2	Negative binomial	Gaussian Kernel	384 780 €
2	Negative binomial	ME - Pareto	351 481 €
2	Negative binomial	ME - GPD	362 108 €
2	Negative binomial	GPD (MLE)	212 356 €
3	Negative binomial	Gaussian Kernel	0 €
3	Negative binomial	ME - Pareto	10 976 €
3	Negative binomial	ME - GPD	14 916 €
3	Negative binomial	GPD (MLE)	0 €

Table 36: Recovery mean – exposure pricing

Layer	Exposure curve	$\widehat{\mathbb{E}}(S^{reinsurer})^{Pure\ Loss}$	$\widehat{\mathbb{E}}(S^{reinsurer})$
1	SwissRe2	769 161 €	744 112 €
1	SwissRe3	614 894 €	582 312 €
1	SwissRe4	454 835 €	441 022 €
2	SwissRe2	867 365 €	885 422 €
2	SwissRe3	658 635 €	638 624 €
2	SwissRe4	457 874 €	444 018 €
3	SwissRe2	657 368 €	655 255 €
3	SwissRe3	458 724 €	449 981 €
3	SwissRe4	281 205 €	284 796 €

The frequency-severity method produces comparable results for the first and second layers across the top three severity distributions. However, the GPD distribution reflects more divergent results, which was anticipated due to its higher Kolmogorov-Smirnov and Cramér-von Mises distances, as well as the rejection of the null hypothesis for this distribution. For the third layer, the estimated recovery amounts are low across all four distributions, reflecting the model's challenges in estimating extreme values for higher layers.

In contrast, the exposure method reveals significant variations based on the selected exposure curve. The Swiss Re 2 curve generates the highest recovery amounts due to its higher destruction rate, which increases the likelihood of claims reaching retentions compared to Swiss Re 3 and 4 curves. Additionally, the recovery means are more heterogeneous when using the Swiss Re curves, underscoring the influence of curve selection and highlighting the dependence with the choice of the exposure curve. The exposure method also predicts higher recovery amounts for the second and third layers compared to the frequency-severity method. Furthermore, the impact of the discounting rate using the exposure method is low for each exposure curve. This outcome can be attributed to the characteristics of the property line, where short payment patterns minimize the temporal impact of discounting. Additionally, the absence of a stabilization clause further reduces the differences between the estimated recovery amount for the exposure curve method.

To provide additional context, market premiums for these treaties are presented below, offering a reference point for the calculated recoveries.

Table 37: Market premiums

Layer	Market premium
1	569 155 €
2	474 295 €
3	284 577 €

The frequency-severity method struggles to accurately match the market premium in this case, showing a deviation of approximately +28% for layer 1 using the ME Pareto and ME GPD distributions, and -25% for layer 2. For layer 3, the frequency-severity method appears unsuitable, as it predicts recovery amounts close to zero.

In contrast, the exposure curve approach delivers better results. For instance, the Swiss Re 4 curve, initially selected to model the destruction rate, provides relatively accurate recovery estimates for the treaty, with deviations of less than 10% for layers 2 and 3. However, for layer 1, the Swiss Re 3 exposure curve performs better. Indeed, the Swiss Re 3 curve is more commonly used for medium-scale commercial property claims, which are more likely to impact layer 1 and fall within the treaty's scope. Conversely, higher layers are more influenced by large-scale claims, making the Swiss Re 4 curve more suitable for layers 2 and 3. This highlights the importance of selecting an appropriate exposure curve and suggests the potential benefit of combining different curves within the same risk profile, an approach not yet implemented in the tool.

This analysis illustrates an alternative method for estimating recoveries under an XL treaty. However, this method is limited to property and liability treaties to treaties liability to a lesser extent and should be used cautiously due to its reliance on the exposure curve/ budgeted loss ratios within the risk profile. As a conclusion, the exposure method could be used as a mean of comparison and could be particularly valuable for higher layers where retentions are high.

5.2.5 Liability treaties

In liability insurance, the sum insured is not equal to the maximum amount of a possible third-party claim but is just a limit of indemnity chosen by the insured up to which he wants to be reimbursed by the insurer in case of any third-party claims. Therefore, the assumption used for property treaties that the distribution of $x_i = \frac{X_i}{Q_i}$ does not depend on i is not realistic here because risks with identical claims exposure may have greatly varying sums insured. Nevertheless, we propose a first approach based on (Mack & Fackler, 2003) for modeling liability treaties using exposure curves. However, this approach is still exploratory and suffers from several shortcomings but could lay the foundations for future improvements.

To use exposure curve for liability insurance, we make the following assumptions based on (Mack & Fackler, 2003). Let $P_i(Q_i)$ be the net premium for a risk with limit of indemnity Q_i and for a risk i facing the claim severity X. Let z be the percentage of premium increasing if the limit of indemnity doubles, i.e., $P_i(2Q_i) = (1+z)P_i(Q_i)$. Here, Q_i acts only as limit of indemnity and not as a measure of the size of the risk like in property insurance. One can then obtain the Riebesell's formula:

$$\mathbb{E}(\min(X, Q)) = P_i(Q) = P_i(Q_i)(\frac{Q}{Q_i})^{ld(1+z)}$$

Where Q represents any positive limit of indemnity. Then, if the risk i with limit indemnity Q_i is facing the claim severity X, its exposure curve G_i is, for $R \leq Q_i$:

$$G_i(R) = \frac{\mathbb{E}(\min(X, R))}{\mathbb{E}(\min(X, Q_i))} = \frac{P_i(R)}{P_i(Q_i)} = (\frac{R}{Q_i})^{ld(1+z)}$$

If $R > Q_i$, $G_i(R) = 1$.

Thus, knowing the parameter z, one could use risk profiles to approximate the pure Loss of XL treaties following the methodology detailed in part 5.2.3. However, modeling a distribution of losses is more challenging, since only $\mathbb{E}(\min(X, Q))$ can be calculated for a given limit of indemnity Q. Furthermore, it would be necessary to fix a standard premium and a standard sum insured related to z, which would require further studies. Assuming the parameters z, $P_i(Q_i)$ and Q_i are determined, we could apply the previously developed methodology to derive a distribution of recoveries (for XL treaties) or losses (for SP treaties). This would be based on the formula $P_i(Q_i)(\frac{Q}{Q_i})^{ld(1+z)}$ to estimate the expected individual loss.

This methodology represents an initial approach, heavily dependent on the parameters z, $P_i(Q_i)$ and Q_i, in addition to other factors within the risk profile. Accurately estimating these quantities at a granular level for the risk profile is challenging and was not made in this project. However, within the tool, these values can be fixed by the user and allow generating loss distribution. Modeling liability treaties through the exposure methodology will require further development and should be considered as an exploratory step towards generalizing the exposure curve methodology to other lines of business.

5.3 Modeling losses based on loss ratios (QS, AGG and SP)

Previously, we examined how recoveries for XL treaties were modeled and we detailed how ceded losses/reinsurer's LR are simulated for SP treaties in this actuarial work. The following part will focus on modeling underlying losses for QS and AGG treaties, which are treaties that are not currently modeled within the internal model. We will eventually detail how the treaty's TC are applied for QS/SP and AGG treaties.

To do so, the annual loss ratios underlying these QS and AGG treaties will be modeled. This direct modeling approach was retained for several reasons. While it would have been possible to consider the set of claims related to the treaties and adopt a frequency-severity approach - like the method used for XL treaties - this option was not retained due to operational and computational challenges. One would need to model attritional, atypical and CAT claims separately and through different methods to model properly AGG or QS. Collecting such granular data for all cedents and parametrize each distribution would thus be complex. Consequently, cedents are instead asked to directly provide aggregated loss triangles, simplifying data collection across all cedents. Additionally, for computational efficiency, modeling loss ratios is more practical for QS and AGG treaties, as these cover aggregated portfolios. Simulating individual claims would require significant computational resources due to the potentially high claim volumes. However, a lack of sufficient historical data can sometimes complicate the modeling process, especially for newer lines of business with limited historical experience (e.g., QS treaties covering the Cyber LoB).

Furthermore, the currently available aggregated historical claims data cannot be used directly, as it does not include all claims incurred by the entities. To address this limitation, exhaustive data must be collected from the entities. Throughout this section, the examples provided will rely on simulated data to illustrate the methodology developed within the tool.

5.3.1 ULR modeling (QS and AGG)

For both AGG and QS treaties, losses will be modeled based on the underlying modeling of the ultimate loss ratio (ULR) (see 4.2). Let us denote the vector of historical ULRs for the treaty under consideration as $(ULR_i)_{1\le i\le n}$. To model the ultimate loss ratio, an array of continuous distributions - both parametric and non-parametric - are employed to estimate the ULR distribution.

5.3.1.1 Continuous distributions

The continuous distributions calibrated based on all observations $(ULR_i)_{1\le i\le n}$ are listed below:

- Exponential distribution
- Log Normal distribution
- Gamma distribution
- Generalized Pareto distribution
- Weibull distribution
- Fréchet distribution
- Gaussian Kernel

The parameters of these distributions were described in detail in the previous section. Loss ratios modeling does not require the complexity of spliced distributions, as the available data may not always be sufficient to justify their use. Moreover, loss ratios are rarely extreme compared to individual claims, primarily due to the diversification effect across the considerable number of insurance policies covered. This reduces the likelihood of exceedingly high loss ratios and avoids the heavy-tailed behavior typically observed in claim severity distributions.

5.3.1.2 Parameter estimation

For each of the distributions considered, two statistical methods will be used to evaluate their parameters, as in the frequency-severity approach for XL treaties:

- Maximum likelihood estimation
- Moment matching method

Estimation using the Gaussian Kernel is obtained directly from empirical data.

5.3.1.3 Selection of loss ratio distribution

Regarding the selection of the most appropriate distribution, the same method used for modeling severity will be applied (see 5.1.2.4). A similar ranking process is employed to evaluate and compare the fitted distributions, allowing us to identify the best fits.

5.3.2 Recoveries modeling

Once the parameters of the ULR distribution have been established and the appropriate distribution selected, the next step in the Monte Carlo estimation process is to obtain a distribution of amounts ceded to reinsurance. To achieve this, 50 000 years of loss experience will be simulated ($N_{simul} = 50000$). Since treaty conditions differ between QS/SP and AGG treaties, it results in distinct methods for calculating recoveries. These methods will be detailed separately in the following sections.

5.3.2.1 Quota Share and Surplus

Once a distribution of loss ratios has been simulated, which accounts for the gross loss ratio of the insurer, the treaty conditions should be applied to determine the reinsurer's losses. QS and SP treaties can have several clauses and parameters:

- A subject premium $Premium_{subject}$, corresponding to the covered portfolio GEP (respectively GWP) if the treaty is Loss Occurring (respectively Risk Attaching)
- A cession rate c (only for QS treaties)
- A ceded $Premium_{ceded}$. For QS treaties, the ceded premium corresponds to $c * Premium_{subject}$ while for SP treaties the ceded premium corresponds to the sum of individual ceded premiums. Its calculation has been detailed in part 5.2.4.4.2.
- A commission rate: r_{CF} if it is fixed, $r_{CSS}(.)$ if it is calculated on a sliding scale
- A loss corridor (only for QS treaties in this project)
- A profit sharing depending on $r_{profit\ sharing}$ (the profit-sharing rate) and $r_{target\ margin}$ (margin the reinsurer wishes to retain on the treaty)

These various clauses have an impact on the loss ratio borne by the reinsurer and must be considered in the calculation of the reinsurer's final obligations. In a nutshell, these clauses will enable us to compute the following distributions:

- $\left(S_i^{reinsurer}\right)_{1 \leq i \leq N_{simul}}$ and $\left(LR_i^{reinsurer}\right)_{1 \leq i \leq N_{simul}}$ which corresponds to the distribution of ceded losses and loss ratios ceded to the reinsurer (net of loss corridor effect for QS treaties)
- $(Ceding\ Commission_i)_{1 \leq i \leq N_{simul}}$ which is the corresponding commission amount paid by the reinsurer
- $\left(S_i^{reinsurer\ (incl.commission)}\right)_{1 \leq i \leq N_{simul}}$ and $\left(LR_i^{reinsurer\ (incl.\ commission)}\right)_{1 \leq i \leq N_{simul}}$ which denotes the reinsurer's losses and loss ratios, considering the commission amount
- $(Experience\ Account_i)_{1 \leq i \leq N_{simul}}$ which is the reinsurer's result without the profit-sharing clause

- $(Profit\ Sharing_i)_{1 \le i \le N_{simul}}$ which is an optional commission paid by the reinsurer, depending on the experience account
- $\left(Net\ Loss_i^{reinsurer\ (discounted)}\right)_{1 \le i \le N_{simul}}$ and $\left(Net\ Loss_i^{reinsurer\ (undiscounted)}\right)_{1 \le i \le N_{simul}}$, with $\left(Net\ LR_i^{reinsurer\ (discounted)}\right)_{1 \le i \le N_{simul}}$ and $\left(Net\ LR_i^{reinsurer\ (undiscounted)}\right)_{1 \le i \le N_{simul}}$ which are the net reinsurer's losses and loss ratios, without any tax considerations

The following part will detail how these distributions are calculated within the reinsurance pricing tool developed and their corresponding formulas.

Let $\left(LR_i^{cedent}\right)_{1 \le i \le N_{simul}}$ represents the simulated distribution of ultimate cedent's loss ratio. It corresponds to the ratio between the applicable losses and the subject premium. Let $PP = \left(PP_j\right)_{1 \le j \le 30}$ be the payment pattern calibrated for the QS/SP treaty to be priced.

Each LR_i^{cedent} is then projected over time to obtain a vector of cumulative loss ratios:

$$LR_{i,cumulative}^{cedent} = \left(\sum_{k=1}^{j} PP_k * LR_i^{cedent}\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

For SP treaties, each $LR_i^{reinsurer}$ is also projected accordingly:

$$LR_{i,cumulative}^{reinsurer} = \left(\sum_{k=1}^{j} PP_k * LR_i^{reinsurer}\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

Afterwards, the treaty conditions are applied to get the distribution of loss ratios and the loss amounts borne by the reinsurer. These distributions will be used to calculate various indicators for pricing analysis.

5.3.2.1.1 Loss corridor (QS)

A loss corridor is a clause where the cedent retains full liability for losses within a specified corridor, meaning that no claims or premiums are ceded to the reinsurer for loss ratios falling within this range. Loss corridors are generally set at high thresholds, with the objective of encouraging insurers to price their contracts accurately and ensuring that the reinsurer is shielded from excessively high losses. In this project, loss corridor clauses has been considered only for QS treaties.

A Loss corridor clause includes two variables:

- $LR_{lower\ bound}^{LC}$ which represent the lower bound of the loss corridor
- $LR_{upper\ bound}^{LC}$ which represents the upper bound of the loss corridor

When the gross loss ratio is between these two limits, the reinsurer does not intervene, i.e., no claims are ceded to the reinsurer and the gross loss ratio is not impacted. Let us define the function $f_{net\ loss\ corridor}$ such that:

$$f_{loss\ corridor}(LR^{cedent}) = LR^{cedent} - \min\left((0, LR^{cedent} - LR_{lower\ bound}^{LC})_+, LR_{upper\ bound}^{LC} - LR_{lower\ bound}^{LC}\right)$$

To obtain the ceded losses of a given simulation year i, the ceded premium is needed. For QS treaties, it corresponds to the product of the subject premium $Premium_{subject}$ of the treaty by the cession rate of the treaty:

$$Premium_{ceded} = c * Premium_{subject}$$

Then, the cumulated ceded losses and the reinsurer's LR considering the loss corridor effect can be derived as follows:

$$S_{i,cumulative}^{reinsurer} = \left(f_{loss\ corridor}\left(\left(LR_{i,cumulative}^{cedent}\right)_j\right) * Premium_{ceded}\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

$$LR_{i,cumulative}^{reinsurer} = \frac{S_{i,cumulative}^{reinsurer}}{Premium_{ceded}} \in \mathbb{R}^{30}$$

One can get the scalar value of these quantities as follows:

$$S_{i}^{reinsurer} = \left(S_{i,cmulative}^{reinsurer}\right)_{j=30} \in \mathbb{R}$$

$$LR_{i}^{reinsurer} = \left(LR_{i,cumulative}^{reinsurer}\right)_{j=30} \in \mathbb{R}$$

Consider, for example, a simulated loss ratio of 80%, within the scope of a QS treaty with a loss corridor ranging from 50% to 75%. The figure below illustrates the evolution of the gross loss ratio for both the cedent and the reinsurer.

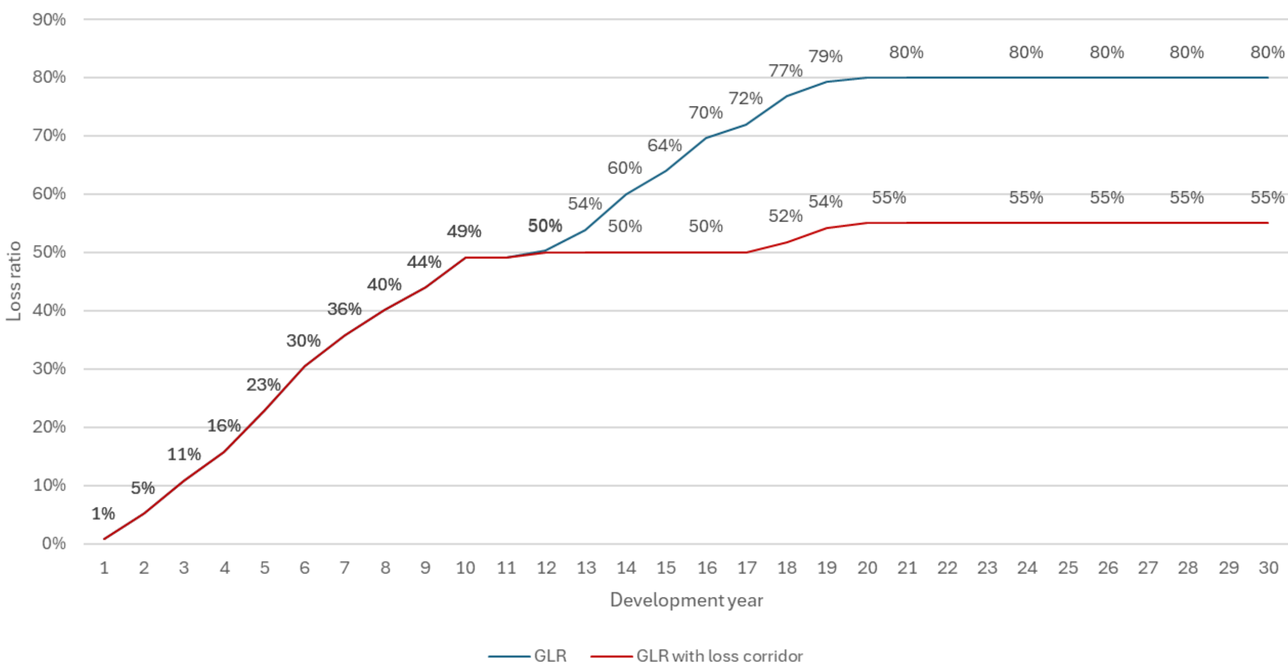


Figure 60: Impact of a loss corridor on a GLR

The long-term evolution of the loss ratio reflects the long-tail nature of the portfolio covered by the treaty. Furthermore, one can notice the loss corridor effect during the development years 12 to 17. During these years, the cedent's loss ratio falls within the boundaries of the loss corridor, meaning the reinsurer does not assume any liability for losses in this range. As a result, the reinsurer's loss ratio remains unaffected and only starts to rise from year 18 onwards, when the cedent's loss ratio exceeds the corridor thresholds. This loss corridor has allowed the reinsurer to bear fewer losses than the cedent, limiting its exposure.

5.3.2.1.2 *Commission application (fixed and sliding scale)*

For QS and SP treaties, the premium amount earned by the reinsurer is directly known and no premium can be fixed by the reinsurer. To influence the "price" of the QS or SP treaty, the reinsurer generally pays a commission to the cedent. This commission can either be fixed or, depending on the actual loss experience of the ceding company, calculated on a sliding scale.

If the commission is fixed, a percentage r_{CF} is indicated which is then transformed into an amount by multiplying the ceded premium with this percentage. The projected cumulative ceding commissions (in percent) simply corresponds to:

$$Ceding\ Commission_{i,cumulative}^{\%} = (r_{CF})_{1 \le j \le 30}$$

If the commission is based on a sliding scale, the calculation becomes more complex. A sliding scale commission is a percentage of the ceded premium, which varies according to the cedent's loss ratio. In such a case, the following information must be provided:

- A minimum loss ratio $LR^{CSS}_{lower\ bound}$ and a maximum loss ratio $LR^{CSS}_{upper\ bound}$
- The commission rate for the lower limit $r^{CSS}_{lower\ bound}$ and the upper limit $r^{CSS}_{upper\ bound}$

There are diverse types of sliding scale commissions. In this report, only the most used sliding scale commission has been calibrated, which corresponds to a linear commission. The commission payable by the reinsurer to the ceding company is calculated as follows: if the loss ratio (after deduction of the loss corridor) is less than $LR^{CSS}_{lower\ bound}$ (resp. greater than $LR^{CSS}_{upper\ bound}$), the commission rate is equal to $r^{CSS}_{lower\ bound}$ (resp. $r^{CSS}_{upper\ bound}$). If the loss ratio is between $LR^{CSS}_{lower\ bound}$ and $LR^{CSS}_{upper\ bound}$, the commission is linearly interpolated.

Let us define the function r_{CSS} such that for a variable $LR \in \mathbb{R}$:

$$r_{CSS}(LR) = \begin{cases} r^{CSS}_{lower\ bound} & LR \leq LR^{CSS}_{lower\ bound} \\ r^{CSS}_{lower\ bound} + a_{CSS} * (LR - LR^{CSS}_{lower\ bound}) & LR^{CSS}_{lower\ bound} \leq LR \leq LR^{CSS}_{upper\ bound} \\ r^{CSS}_{upper\ bound} & LR^{CSS}_{upper\ bound} \leq LR \end{cases}$$

With $a_{CSS} = \frac{r^{CSS}_{upper\ bound} - r^{CSS}_{lower\ bound}}{LR^{CSS}_{upper\ bound} - LR^{CSS}_{lower\ bound}}$.

To illustrate the evolution of the sliding scale commission r_{CSS}, the figure below displays the evolution of a sliding scale defined between 60% and 80%, with a 30% commission at the lower bound (60%) and 10% at the upper bound (80%).

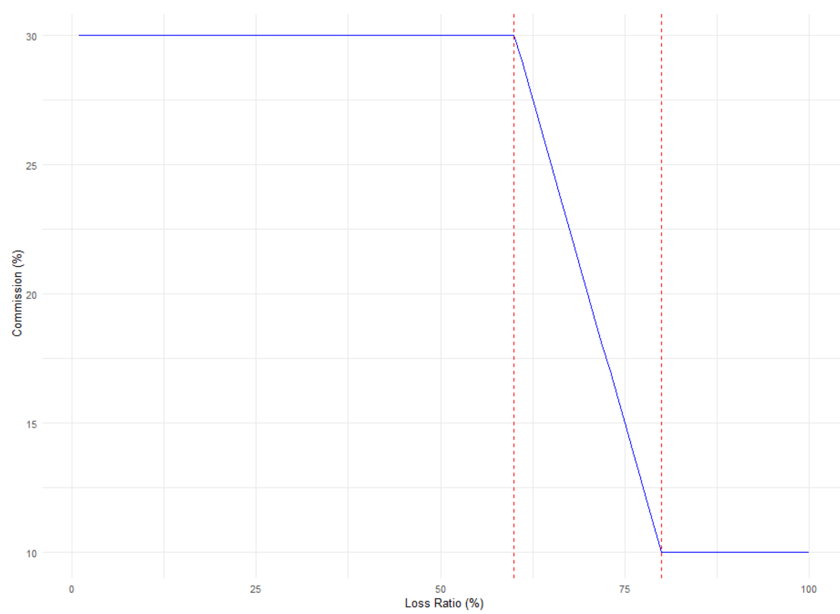


Figure 61: Effect of a sliding scale commission

In our study, the r_{CSS} function will be applied to the vector $LR^{cedent}_{i,cumulative}$ to calculate the commission amount at each time step, resulting in the vector $Ceding\ Commission^{\%}_{i,cumulative}$:

$$Ceding\ Commission^{\%}_{i,cumulative} = \left(r_{CSS}\left(\left(LR^{cedent}_{i,cumulative} \right)_j \right) \right)_{1 \leq j \leq 30}$$

One can also calculate the financial cumulative ceding commission and the total:

$$Ceding\ Commission_{i,cumulative} = Ceding\ Commission^{\%}_{i,cumulative} * Premium_{ceded} \in \mathbb{R}^{30}$$

$$Ceding\ Commission_i = \left(Ceding\ Commission^{\%}_{i,cumulative}\right)_{j=30} * Premium_{ceded} \in \mathbb{R}$$

5.3.2.1.3 Profit-sharing mechanism

QS and SP treaties may also include profit-sharing clauses. Profit sharing is an incentive mechanism where the reinsurer shares a portion of the treaty's profitability with the ceding company.

The profit sharing is calculated based on the experience account vector (expressed as a percentage of the ceded premium) which is computed as follows:

$$Experience\ Account^{\%}_{i,cumulative} = \left(1 - \left(LR^{reinsurer\ (incl.\ commission\ and\ expenses)}_{i,cumulative}\right)_j - r_{target\ margin}\right)_{1 \le j \le 30}$$

Where:

$$LR^{reinsurer\ (incl.\ commission\ and\ expenses)}_{i,cumulative} = LR^{reinsurer}_{i,cumulative} + Ceding\ Commission^{\%}_{i,cumulative} + r_{expenses}$$

And $r_{expenses}$ denotes the rate used to calculate treaty's allocated expenses

It is then possible to determine the vector $Profit\ Sharing^{\%}_{i,cumulative}$ and the scalar $Profit\ Sharing_i$ as follows:

$$Profit\ Sharing^{\%}_{i,cumulative} = \left(r_{profit\ sharing} * \left(\left(Experience\ Account^{\%}_{i,cumulative}\right)_j\right)_+\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

$$Profit\ Sharing_i = \left(Profit\ Sharing^{\%}_{i,cumulative}\right)_{j=30} * Premium_{ceded} \in \mathbb{R}$$

5.3.2.1.4 Obtaining recoveries

The cumulated reinsurer's net loss ratio for a considered simulation i is thus:

$$Net\ LR^{reinsurer}_{i,cumulative} = LR^{reinsurer\ (incl.\ commission\ and\ expenses)}_{i,cumulative} + Profit\ Sharing^{\%}_{i,cumulative} \in \mathbb{R}^{30}$$

By multiplying the vector $Net\ LR^{reinsurer}_{i,cumulative}$ with the expected ceded premium of the treaty $Premium_{ceded}$, one can measure the cumulative and scalar reinsurer's net loss over the years:

$$Net\ Loss^{reinsurer}_{i,cumulative} = Net\ LR^{reinsurer}_{i,cumulative} * \ Premium_{ceded} \in \mathbb{R}^{30}$$

$$Net\ Loss^{reinsurer\ (undiscounted)}_{i} = \left(Net\ Loss^{reinsurer}_{i,cumulative}\right)_{j=30} \in \mathbb{R}$$

As in the previous part, one can derive the incremental cash flows and discount them in order to obtain the vector $Net\ Loss^{reinsurer\ (discounted)}_{i,incremental} \in \mathbb{R}^{30}$ and the scalar $Net\ Loss^{reinsurer\ (discounted)}_{i}$ (see 5.1.3.2).

Over all simulated years $i = 1, \dots, N_{simul}$, a vector of reinsurer's net losses $\left(Net\ Loss^{reinsurer\ (discounted)}_{i}\right)_{1 \le i \le N_{simul}}$ is simulated.

The different steps of the QS modeling process developed and implemented in this paper are summarized in the following figure.

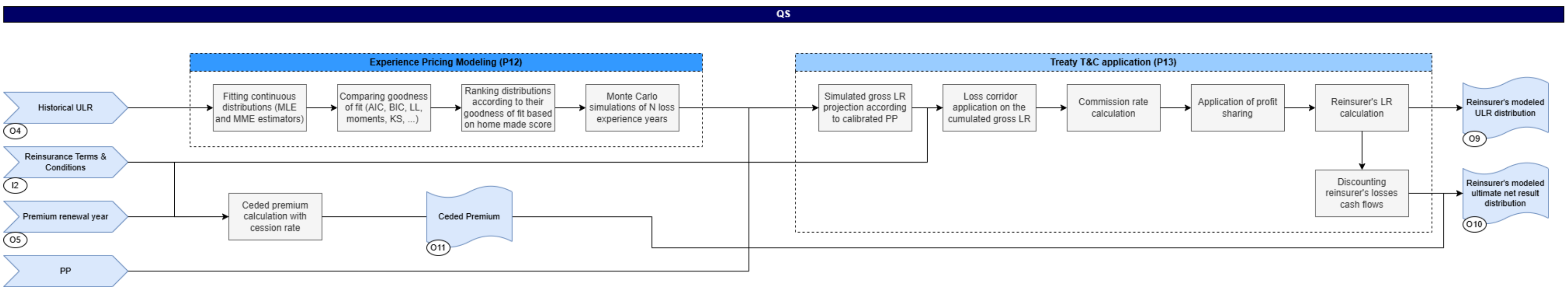


Figure 62: Modeling process – QS treaties

5.3.2.2 Aggregate

An AGG treaty being a non-proportional treaty, the method implemented for determining its recoveries is similar to the one developed for XL treaties in this work.

One is reminded that the information available for an AGG treaty includes its limit, denoted as $L_{\%}$ and its retention, denoted as $R_{\%}$ and expressed as a percentage of the covered portfolio total premium volume (the subject premium). To align with the previous framework, the treaty parameters are converted into amounts by multiplying these percentages with the subject premium $Premium_{subject}$. The resulting amounts are given by $R = R_{\%} * Premium_{subject}$ and $L = L_{\%} * Premium_{subject}$

Once a distribution of gross loss ratios $\left(LR_i^{cedent}\right)_{1 \le i \le N_{simul}}$ simulated, each gross loss ratio LR_i^{cedent} is then multiplied by the amount $Premium_{subject}$ to obtain the distribution $(S_i)_{1 \le i \le N_{simul}}$ which corresponds to the aggregated annual loss. Each amount X_i is then projected over time to obtain a cumulative aggregate loss vector $S_{i,cumulative}^{cedent} = \left(\sum_{t=1}^{j} PP_t * X_i\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$ where $PP = \left(PP_j\right)_{1 \le j \le 30}$ is the calibrated payment pattern.

Once the vector $S_{i,cumulative}^{cedent}$ has been computed for each year, the treaty conditions are applied to get recovery amounts:

$$S_{i,cumulative}^{reinsurer} = \left(\min(\left(\left(S_{i,cumulative}^{cedent}\right)_j - R\right)_+, L)\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

As in the previous part, one can obtain the cash flows of recoveries and the discounted cash flows in order to obtain the vector $S_{i,incremental}^{reinsurer\ (discounted)} \in \mathbb{R}^{30}$ and the scalar $S_i^{reinsurer\ (discounted)}$ (see 5.1.3.2).

Over all simulated years $i = 1, \dots, N_{simul}$, a vector of recoveries $\left(S_i^{reinsurer\ (discounted)}\right)_{1 \le i \le N_{simul}}$ corresponding to the amount ceded to the reinsurer under the AGG treaty is modeled.

The different steps of the implemented AGG modeling process are summarized in the following figure.

In conclusion of this section, we proposed a method for generating loss distributions for XL per risk treaties through an automated frequency-severity model based on the entity own experience as well as an approach based on exposure curves for property treaties. This approach was implemented to model SP treaties within the property LoB, which were not assessed through the internal model and thus constitutes a first attempt to address part of the considered problematic, which aimed at increasing the number of treaties that can be priced by AGCRe using their internal data. The same applies to AGG and QS treaties, for which we suggested a method for simulating cedent's losses based on automated fitting and ranking of candidate distributions leading to the generation of loss ratios.

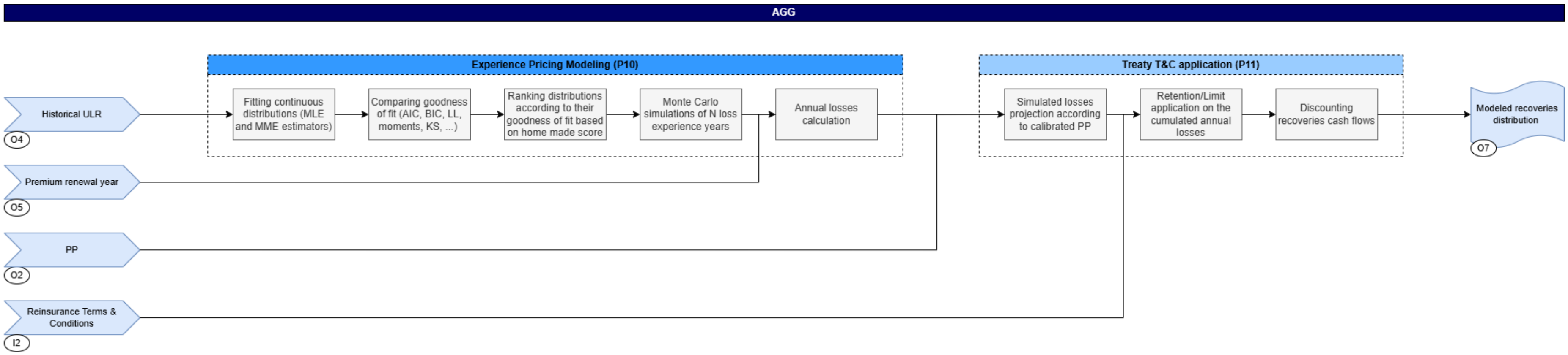


Figure 63: Modeling process – AGG treaties

6. Pricing indicators

After detailing the retained methods for modeling the amounts ceded to reinsurance, we now reach the final phase of the actuarial thesis, which involves generating indicators used for the overall risk evaluation of the treaty in a centralized manner, which is the final objective of the problem at hand. This section highlights the advantage of choosing Monte Carlo simulations, as they allow us to estimate the reinsurer's losses and derive a wide range of quantitative metrics, such as risk and profitability indicators. Since the treaty's characteristics differ, a distinction will be made based on whether the treaty is proportional or non-proportional, and the indicators generated will not be the same for each type.

The following section will therefore focus on outlining the selected indicators and their estimation. The following figure presents the pricing process leading to the estimated indicators.

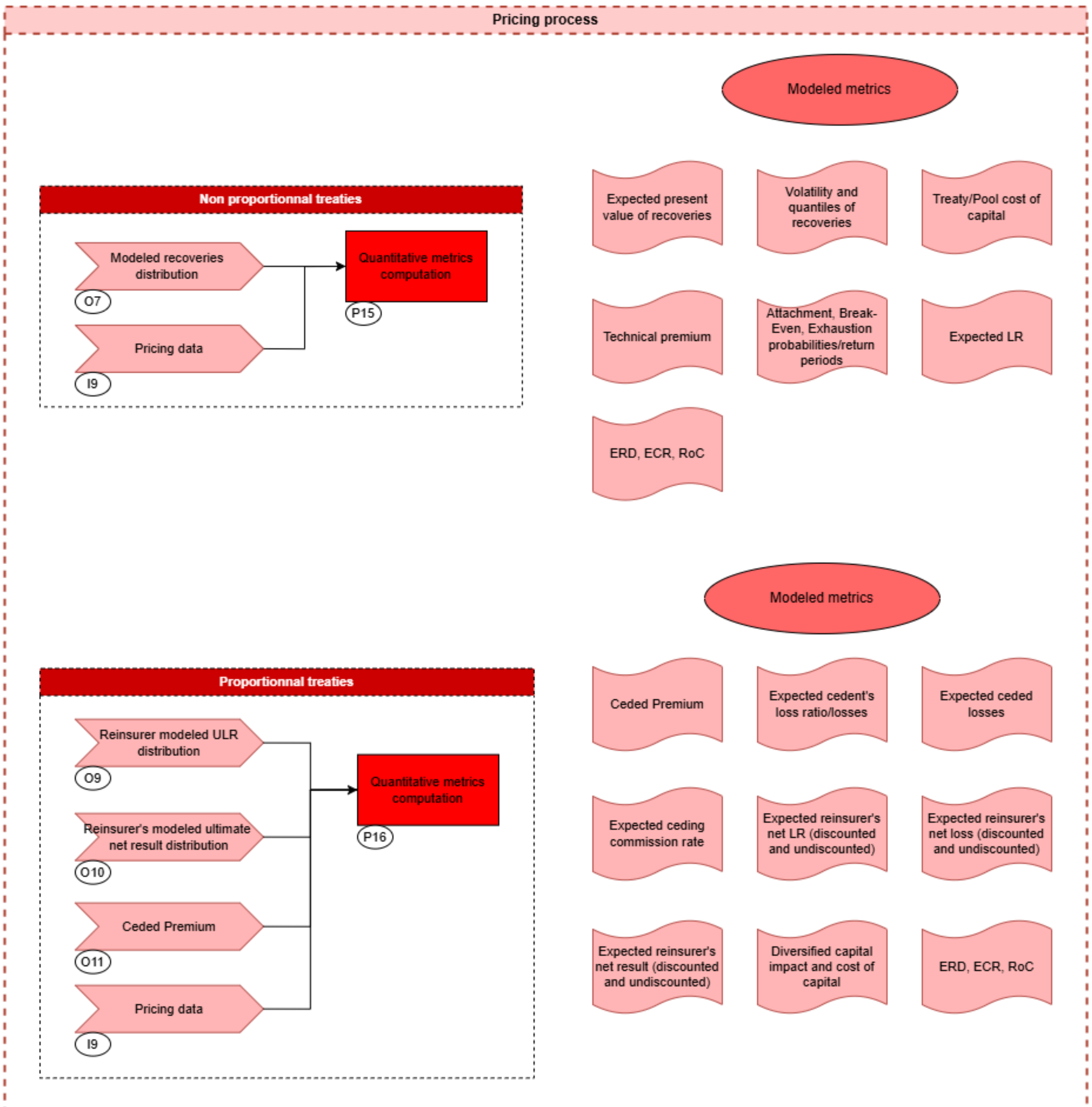


Figure 64: Pricing process

6.1 Non-proportional treaties

Throughout this chapter, **let us resort to the classical notation *R* to designate the random recoveries under an XL per risk or an AGG treaty**. The previous chapter has been dedicated to approximating the probability distribution of that random variable through the Monte Carlo method.

6.1.1 Technical premium

The most important task when evaluating a non-proportional reinsurance treaty is determining the appropriate premium the cedent must pay to the reinsurer. Similar to premiums in primary insurance markets, an actuarial reinsurance premium is made up of the expected reinsured loss from the underlying risk plus a safety loading. Additionally, the final reinsurance premium must include a margin to cover various costs such as administrative expenses, participation in acquisition expenses, runoff costs, taxes, asset management fees, and the reinsurer's profit.

The technical premium for a non-proportional treaty can therefore be defined as follows:

$$TP = \frac{\mathbb{E}(R) + SL(R) + AE + Tax\ Effects}{\left(1 + Reinst_{factor}\right)}$$

Where:

- $\mathbb{E}(R)$ corresponds to the pure premium. The pure premium is the portion of the technical premium that allows the reinsurer to cover the losses incurred by the cedent, assuming zero management expenses and without accounting for the reinstatement premium.
- AE denotes the allocated expenses. The latter does not depend on the underlying distribution of recoveries but is influenced by the number of layers within the reinsurance program and the associated underwriting and run-offs expenses, among others according to AGCRe's practices.
- SL correspond to the safety loadings. This factor depends on the underlying distribution of recoveries and will be the subject of a detailed analysis in this section.
- $Tax\ Effects$ refers to the tax effects on benefits related to the reinsurance transaction net result.
- $\left(1 + Reinst_{factor}\right)$ refers to the impact of the reinstatement premium on the calculation of the technical premium. In the case of an aggregate treaty, this factor is equal to 1, as no reinstatement premium has been modeled.

The challenge in estimating the pure premium lies in accurately determining the safety loading, as it depends on multiple factors, which do not necessarily rely on the underlying distribution of recoveries. These factors could depend on the cedent financial stability or on management decisions regarding the acceptance of risks or demands for higher margins.

In this study, we have opted to primarily focus on the traditional actuarial approach to pricing, using the distribution of recoveries obtained by Monte Carlo.

6.1.1.1 Classical approach for modeling safety loadings

Let us first focus on the various premium principles commonly used in actuarial literature for calculating safety loadings. Some of the safety loading principles implemented in this project are listed below:

- **Expected value principle:**

This premium calculation principle is based on a safety loading proportional to the expected loss:

$$SL(R) = \theta * \mathbb{E}(R)$$

- **Variance principle:**

If information on the variance of R is available, then another common principle is to choose the safety loading proportional to that variance:

$$SL(R) = \alpha_{Var} * Var(R)$$

- **Standard deviation principle:**

By replacing the variance with the standard deviation, one gets the standard deviation principle:

$$SL(R) = \alpha_{\sigma} * \sigma(R)$$

This approach has the advantage of having a safety loading expressed in the same unit as the pure premium.

- **Exponential principle:**

By determining a risk aversion coefficient a, one can compute a safety loading according to the exponential principle:

$$SL(R) = \frac{1}{a}\log(\mathbb{E}(e^{aR})) - \mathbb{E}(R)$$

- **Utility principle:**

Based on utility theory, under which the premium is determined as the solution of the equation $u(w) = \mathbb{E}(u(w + P - R))$, where $u(x)$[1] is the utility for the reinsurer of having capital x, whereas w is the current surplus and P is the premium for which this entity is indifferent about entering the contract in the sense of expected utility. If $Var(R)$ is small, one can approximate P by $\mathbb{E}(R) + \frac{|u''(w)|}{2u'(w)} * Var(R)$, leading to the following safety loading:

$$SL(R) = \frac{|u''(w)|}{2u'(w)} * Var(R)$$

- **Esscher premium principle:**

Another approach is to distort the distribution of recoveries by transforming its probability mass. If this distortion is done by an exponential function, one obtains the Esscher premium safety loading:

$$SL(R) = \frac{\mathbb{E}(Re^{hR})}{\mathbb{E}(e^{hR})} - \mathbb{E}(R)$$

These different premium principles, even if they are risk measures, expose us to various criteria. For these safety loading principles, different parameters need to be estimated. Specifically, for the variance, expected value, and standard deviation principles, a multiplicative factor must be assessed. This factor can either be chosen arbitrarily or modeled, for instance, by simulating recoveries for different treaties and performing an estimation through linear regression using market premiums. Unfortunately, this analysis was not conducted in the study, leaving the choice of the multiplicative parameters to the reinsurance pricing tool user's discretion. The following section will, however, outline the method adopted for estimating these parameters for the standard deviation and variance principles. For other principles, parameter estimation is more complex, as no direct relationship can be established with market premiums, and thus largely depends on user preferences, especially in the selection of utility functions and risk-aversion coefficients. Nevertheless, these premiums will be calculated for information, based on the parameters set by the user.

[1] *The CCRA utility function has been used to mode the reinsurer's utility*

6.1.1.2 Cost of capital approach for modeling safety loadings

Another significant limitation of the previously defined premium principles lies in addressing solvency considerations. An insurer's solvency is its ability to fulfill its commitments to policyholders using the resources at its disposal, such as capital and investments. When underwriting reinsurance treaties, the reinsurer agrees to assume more risk, and the solvency regulatory framework requires them to hold additional capital to cover this additional risk. This ensures that the reinsurer maintains a sufficiently high level of capital, allowing the business to remain solvent even during exceptionally adverse years or events. This solvency consideration gives rise to two principles for calculating the treaty premium: the first one is based on a more traditional and historical approach, while the second is aligned with many regulatory frameworks. To illustrate these principles, the aggregated premium will be considered rather than individual premiums. The methods used to derive individual premiums will be based on this approach.

Historically, solvency was assessed using the ruin probability, which measures the likelihood of a reinsurer's surplus process $C(t)$ becoming negative over a given period. The classical model for $C(t)$ is the Cramér-Lundberg model, which defines $C(t) = w + ct - \sum_{i=1}^{N(t)} X_i$, where the premiums are assumed to be collected over time at a constant rate c, w denotes the initial insurer's capital and $N(t)$ is a Poisson process. This method relies on a purely mathematical approach, overlooking insurance dynamics such as delayed claims settlement, inflation, portfolio size fluctuations, risk dependencies, investment performance, and capital adjustments like dividends. Additionally, modeling the surplus stochastic process with sufficient accuracy to reflect a company's real situation can be extremely complex, introducing significant model uncertainty. This approach also demands extensive data and requires input from several departments, further complicating its practical application. These challenges have driven a shift toward more practical solvency frameworks, such as those employed in regulatory systems like Solvency II.

In this approach, a time horizon of one year is considered. Let $Z(t)$ be the available capital at time t, which corresponds to the assets minus the liabilities of an insurance company. To remain solvent, the criterion is to hold sufficient capital according to some risk measure ρ. Concretely, if $L(1) = Z(1) - Z(0)$ denotes the loss during the upcoming year, then the capital requirement is $\rho(L(1))$. For Solvency II, the risk measure determining the required capital is the Value-at-Risk at level 99,5%: $\rho\big(L(1)\big) = VaR_{99,5}(L(1))$, which is the 99,5% quantile of the loss distribution over a one-year horizon. In practice, modeling $L(1)$ is complicated since many risks can influence its distribution such as market risk, counterparty risk, insurance risk, etc. Within the Solvency II framework, these risks are assessed individually and then aggregated thanks to correlation matrices.

In the purpose of pricing P&C reinsurance treaties, one can make the simplifying assumption that the loss $L(1)$ is determined by the subtraction of the aggregate reinsurance loss $S(1)$ and the premium $P(1)$ received for the respective treaties, i.e. $L(1) = S(1) - P(1)$. This formulation implies a regulatory solvency capital requirement $SCR = \rho(L(1))$, which represents the capital the reinsurance company must hold to ensure solvency. The SCR is generally funded by the company's shareholders or external investors, who require a return on their investment at a rate RoC_{target}, resulting in a cost of $RoC_{target} * SCR$ for the reinsurer. One can now interpret $P(1)$ as the sum of the expected claim size $\mathbb{E}(S(1))$ and a safety loading $SL(S(1))$. From a regulatory perspective, the safety loading can be interpreted as the amount needed to finance the cost of capital requirement $CoC = RoC_{target} * SCR$. The safety loading in this framework is seen not as a buffer for long-term safety, as in classical ruin theory, but as a mechanism to meet regulatory capital requirements.

This methodology has been adopted for calculating the safety loadings within the technical premium estimation because of its consistency with regulatory measures and relative ease of implementation. The methodology selected for calculating safety loadings will be detailed in the following sections. However, the methodology differs whether the treaty is pooled within an AXA SA pools or not.

6.1.1.2.1 Pooled treaties

For a pooled treaty, the previously defined methodology can be used to calculate the safety loading at the individual level. By pooled treaty, we refer to treaties placed within the Marine, Liability, Property and Motor pools, the Cyber pool being only composed of QS treaties.

First, the global cost of capital CoC introduced earlier, which will be based on the AXA SA Non-life SCR, is computed. As in the Solvency II framework, the Value at Risk will be used as a risk measure to calculate the solvency capital requirement of the pool, which will be referred to as the standalone SCR. To calculate the pool standalone SCR, the internally simulated discounted pool results are used, and the SCR is obtained through the following formula, which is equivalent to the formula used for calculating the SCR in the previous section:

$$SCR_{standalone}^{Pool} = \widehat{\mathbb{E}}(Net\ Result_{Pool}) - \hat{q}_{0.05\%}(Net\ Result_{Pool})$$

Unfortunately, a direct use of the amount calculated previously would fail to account for various AGCRe specificities, particularly the effect of diversification within AXA SA non-life SCR modules, which dilutes each pool capital requirement and as a result decreases its cost of capital. To address this, transmission factors have been calibrated within AXA to account for this diversification effect. These factors, corresponding to different layers of diversification, can be merged into a single one for each pool, denoted as $TF_{pool \rightarrow AXA\ SA}$. Furthermore, out of caution, a coverage ratio $coverage\ ratio_{AXA\ SA}$ is considered within the cost of capital computation. This coverage ratio is calculated at a global level and defined as the ratio between the capital set aside by AXA SA and its SCR. If this ratio is superior to one, AXA SA will set aside more capital than what is actually required to support the risk it underwrites through reinsurance treaties, and the pool cost of capital will thus increase. By considering theses information, one can compute the pool cost of capital, which will be taken into account in the calculation of the safety loading inherent to the treaty.

$$CoC_{pool} = coverage\ ratio_{AXA\ SA} * RoC_{target} * TF_{pool \rightarrow AXA\ SA} * SCR_{standalone}^{Pool}$$

Once the cost of capital of the pool is computed, let us focus on how one can use the previously defined methodology at the aggregate level to deduce an individual safety loading. According to the hypothesis that the safety loading $SL(S)$ corresponds to the CoC_{pool}, the latter can be seen as the sum of the individual safety loadings of the pooled treaties:

$$CoC_{pool} = \sum_{j=1}^{n_{Pool}} SL(R_{treaty_j})$$

Where n_{pool} represents the number of pooled treaties and R_{treaty_j} is the recovery distribution of each pooled treaty.

The goal is thus to allocate a given pool global CoC to each treaty making up that pool, leading to express each $SL(R_{treaty_j})$ as a part of CoC:

$$SL\left(R_{treaty_j}\right) = \alpha_j * CoC_{pool}$$

Theoretically, the choice $\alpha_j = \frac{Cov(R_{treaty_j}, \sum_{j=1}^{n_{Pool}} R_{treaty_j})}{Var(\sum_{j=1}^{n_{Pool}} R_{treaty_j})}$ seems appropriate. However, knowledge about the covariance between a new treaty j and the aggregate risk is difficult to calibrate within the pricing tool. Within AGCRe, since pooled treaties of a same LoB are jointly modeled within the internal model, the coefficient α_j of a reinsurance treaty j is calculated as follows:

- Pool's results and treaty's recoveries are simulated using 50,000 simulations
- The 51 scenarios centered on the 250th worst global scenario are retrieved and used to compute the $SCR_{standalone}^{Pool}$ and the pool cost of capital CoC_{pool}

- Given those 51 identified scenarios, the weight α_j of each treaty within the pool is equal to the mean recoveries for the individual treaty divided by the sum of the mean recoveries over all treaties. By denoting A the set of 51 scenarios centered on the 250th worst global scenario, one has: $\alpha_j^{AGCRe} = \frac{\sum_{s\epsilon A} R_s^j}{\sum_{k=1}^{n_{Pool}} \sum_{s\epsilon A} R_s^k}$ where R_s^j denotes the recovery amount of the reinsurance treaty j for a scenario s

Inspired by the AGCRe methodology, an initial approximation based on the 99,5% quantiles of the pool's results/treaty's recoveries was implemented:

$$\alpha_q = \frac{\hat{q}_{99,5\%}(R)}{\hat{q}_{99,5\%}(Net\ Result_{Pool})}$$

Where R is the recovery distribution of the considered treaty. Unfortunately, it is challenging to incorporate a new treaty into the pool while considering its actual impact on the pool's result since the treaty is independently modeled in the pricing tool. The impact of the treaty is thus evaluated without considering its effect, which could lead to a higher rate α. Furthermore, in this approach, the sum of individual $SL\left(R_{treaty_j}\right)$ is not equal to the pool cost of capital CoC_{pool}. One approach would be to consider the fluctuations of each treaty independently, which simplifies the process of adding treaties to the pool and enables the equality between the pool cost of capital and the sum of individual safety loadings within the pool. One could thus evaluate the effect on the pool in terms of variance or standard deviation, leading to the following values:

$$\alpha_{Var} = \frac{\widehat{Var}(R)}{\sum_{j=1}^{n_{Pooled}} \widehat{Var}(R_{treaty_j})}$$

$$\alpha_{\sigma} = \frac{\hat{\sigma}(R)}{\sum_{j=1}^{n_{Pooled}} \hat{\sigma}(R_{treaty_j})}$$

The choice to model these coefficients in terms of variance or standard deviation is linked to the variance and standard deviation classical premium principles. Indeed, by using such coefficients, one could retrieve the premium principles developed previously. However, these methods have disadvantages, one of which being related to the composition of the reinsurance portfolio. In practice, the future composition of the reinsurance portfolio is unknown, so the coefficients α are calibrated based on the current portfolio composition. Furthermore, the assumption of independence between treaties may not always hold, especially when the treaties are layers of a reinsurance program. Nevertheless, these methods will be considered for estimating the safety loadings due to their alignment with solvency principles and their relative simplicity considering their implementation.

For information, the figure below presents the value of the coefficients α calculated across about forty pooled treaties within Liability, Property, and Marine pools. The contribution to the pool is higher when using the quantile method (referred to as SCR in the figure). This outcome was predictable, as for this type of treaty, the denominator considers the quantile of the global recovery distribution of the pool which results in a relatively lower denominator. Furthermore, α decrease for higher layers as top layers tend to generate fewer recoveries due to higher retention, meaning the impact of the treaty on the pool variance/standard deviation is lower. Therefore, calculation methods based on variance and standard deviation seem preferable.

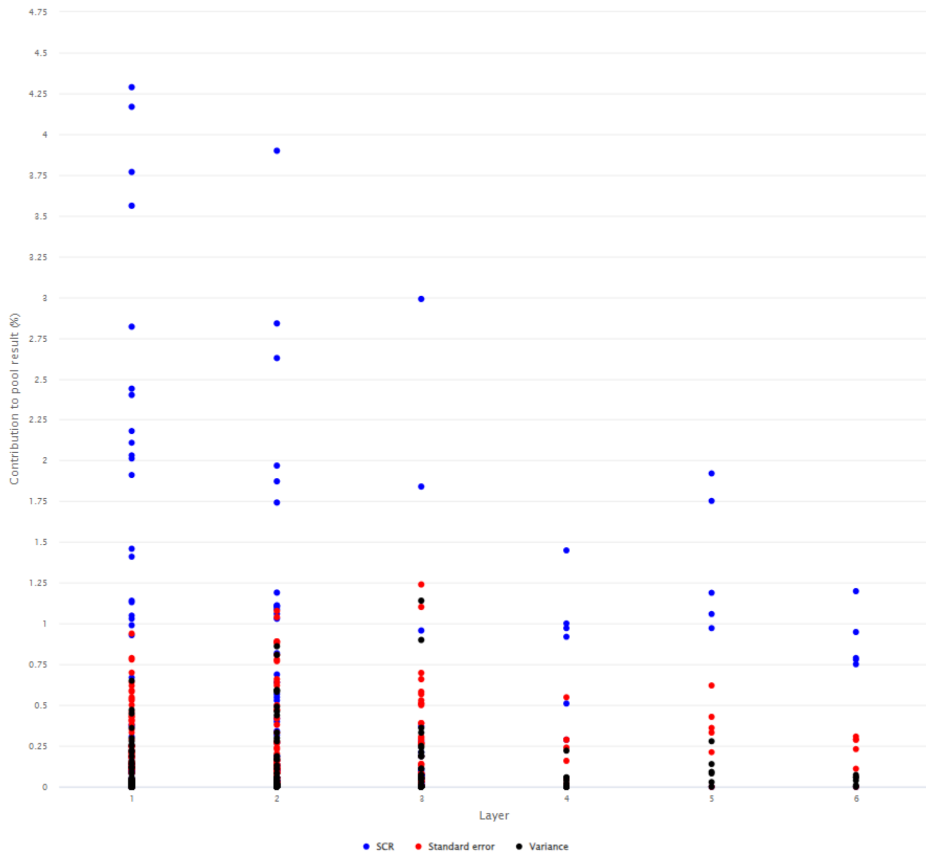


Figure 65: Comparison of safety loadings rates α

Once the different values of α have been evaluated, the safety loading of the considered treaty is:

$$SL(R) = \alpha * CoC_{pool} + RM_{additional}(R)$$

With: $RM_{additional}(R) = coverage\ ratio_{AXA\ SA} * RoC_{target} * \mathbb{E}(R) * (MI_{SCR\ reserve} + D * MI_{SCR\ operational})$

Where $MI_{SCR\ reserve\ |\ operational}$ denotes the marginal impact on the SCR reserve and operational. This additional risk margin aims to account for the impact on the SCR reserve within the Non-Life SCR of AXA SA. D represents the duration of the considered LoB.

6.1.1.2.2 Non-Pooled treaties

Let us now detail the methodology used for non-pooled treaties. The approach differs since it is not possible to simulate all reinsurance treaties underwritten by AXA SA. Instead, a Solvency II-based approach is employed, focusing on evaluating the marginal impact of the treaty on AXA SA's Non-Life SCR. Solvency II is a prudential regime for European insurers and reinsurers which came into force in 2016. It aims to ensure that companies' capital levels are adequate for the risks they face. Through one of its pillars, Solvency II defines a capital indicator known as the SCR (Solvency Capital Requirement), which corresponds to the level of capital required to limit the probability of ruin over a 1-year horizon to 0.5%. In other words, the SCR represents the capital necessary to absorb the shock of a catastrophic event expected to occur once every 200 years. Mathematically, the SCR is modeled as the 99.5% Value-at-Risk of the loss distribution. Since the global distribution of losses is difficult to model, the SCR is calculated through the aggregation of various risk modules. The table below details the different risk modules involved in the calculation of the SCR.

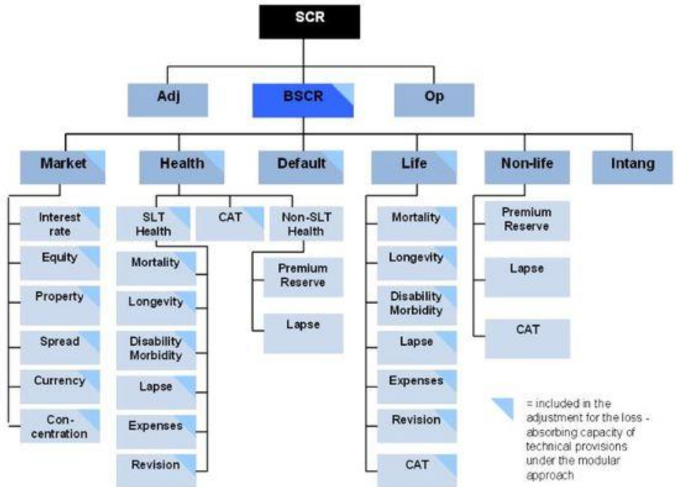


Figure 66: SCR risk modules

In this project, we focus exclusively on the non-life SCR, which consists of three sub-modules: premium/reserve, lapse, and CAT. However, the lapse SCR is not considered in this project. Concerning the risk for premium and reserve, it is calculated as follows according to the Solvency II standard formula:

$$SCR_{premium\,|reserve} = 3 * CoV_{premium\,|\,reserve} * V_{premium\,|reserve}$$

Where:

- $CoV_{premium\,|reserve}$ represents the covariance of premium and reserve risk, which are calibrated and given by the Solvency II framework. For non-proportional treaties, the value of the $CoV_{premium}$ (respectively $CoV_{reserve}$) is set at 0,17 (respectively 0,2) for every LoB.
- $V_{premium\,|reserve}$ represents the volume measure for premium and reserve risk in non-life insurance. Let us consider a non-proportional treaty, with the recovery distribution R. For the treaty under consideration, this volume will be approximated by $\mathbb{E}(R)$ for the premium SCR and by $\mathbb{E}(R) * \max(1, D-1)$ for the reserve SCR where D represents the duration for the considered Line of Business calibrated internally.

This formula for the premium/reserve SCR has been established within the Solvency II framework as it provides a good approximation of the reserve and premium risks when the net result follows a Log-Normal distribution.

Concerning the capital requirement for the CAT module, it is set to 0 since CAT treaties are not considered in this work. One could now derive a vector corresponding to the standalone SCR of the treaty for each module of the non-life SCR:

$$SCR^{treaty} = \begin{pmatrix} SCR_{premium} \\ SCR_{reserve} \\ SCR_{CAT} \end{pmatrix} = \begin{pmatrix} 3 * CoV_{premium} * \mathbb{E}(R) \\ 3 * CoV_{reserve} * \mathbb{E}(R) * \max(1, D_{reserve} - 1) \\ 0 \end{pmatrix}$$

One needs to account for the diversification effect within AXA SA's underwritten reinsurance portfolio. To do so, a diversification matrix calibrated by the Risk Management team is used. This matrix, denoted as $\Sigma_{cedent,\ AXA\,SA}$ is a 2x2 matrix with the following structure:

Table 38: Diversification matrix

	Cedent	AXA SA
Cedent	1	0,5
AXA SA	0,5	1

The SCR accounting for the impact of the treaty underwriting is thus:

$$SCR_{standalone}^{AXA\ SA\ with\ treaty} = \begin{pmatrix} \sqrt{\left(SCR_{premium}^{treaty} \quad SCR_{premium}^{AXA\ SA}\right) * \Sigma_{AGCRe,\ \ AXA\ SA} * \left(SCR_{premium}^{treaty} \quad SCR_{premium}^{AXA\ SA}\right)^{T}} \\ \sqrt{\left(SCR_{reserve}^{treaty} \quad SCR_{reserve}^{AXA\ SA}\right) * \Sigma_{AGCRe,\ \ AXA\ SA} * \left(SCR_{reserve}^{treaty} \quad SCR_{reserve}^{AXA\ SA}\right)^{T}} \\ SCR_{CAT}^{AXA\ SA} \end{pmatrix}$$

The following step will enable us to account for the diversification between the SCR modules using the following correlation matrix:

$$\Sigma^{SCR\ P\&C} = \begin{pmatrix} 1 & 0.5 & 0.1 \\ 0.5 & 1 & 0 \\ 0.1 & 0 & 1 \end{pmatrix}$$

The calculation of the SCR, considering the underwriting of the treaty, is then given by the following formula:

$$SCR^{AXA\ SA\ with\ treaty} = \sqrt{\left(SCR_{standalone}^{AXA\ SA\ with\ treaty}\right)^{T} * \Sigma^{SCR\ P\&C} * \left(SCR_{standalone}^{AXA\ SA\ with\ treaty}\right)}$$

To quantify the treaty underwriting final impact on AXA SA's Non-Life SCR, one also needs to calculate the current SCR, i.e., the SCR without the underwriting of the treaty. It can be expressed as:

$$SCR^{AXA\ SA\ without\ treaty} = \sqrt{\begin{pmatrix} SCR_{premium}^{AXA\ SA} \\ SCR_{reserve}^{AXA\ SA} \\ SCR_{CAT}^{AXA\ SA} \end{pmatrix}^{T} * \Sigma^{SCR\ P\&C} * \begin{pmatrix} SCR_{premium}^{AXA\ SA} \\ SCR_{reserve}^{AXA\ SA} \\ SCR_{CAT}^{AXA\ SA} \end{pmatrix}}$$

The subtraction of $SCR^{AXA\ SA\ without\ treaty}$ from $SCR^{AXA\ SA\ with\ treaty}$ gives the marginal impact of the treaty on AXA SA's global P&C diversified required capital. In accordance with the methodology used for pooled treaties, the safety loading for a non-pooled treaty can be defined as follows:

$$SL(R) = coverage\ ratio_{AXA\ SA} * RoC_{target} * \left(SCR^{AXA\ SA\ with\ treaty} - SCR^{AXA\ SA\ without\ treaty}\right) + RM_{additional}(R)$$

Where $RM_{additional}(R) = coverage\ ratio_{AXA\ SA} * RoC_{target} * \mathbb{E}(R) * D * MI_{SCR\ operational}$ in order to consider the effect on the operational SCR.

The following table displays the amounts of safety loadings obtained for the distributions calibrated for the XL treaty number 1. Since this treaty is not pooled, the previously detailed method was applied. Mathematically, the safety loading amount is linked to the expected recoveries of the treaty. As such, their evolution is consistent here - higher expected recoveries naturally result in a higher safety loading.

Table 39: Comparison of safety loadings – XL treaty number 1

Distributions	$\mathbb{E}(R)$	MI[1]	$SL(R)$
GAUSSIAN_KERNEL	116 678 €	36 367 €	43 178 €
ME_GPD	113 154 €	35 267 €	41 873 €
ME_PARETO	118 070 €	36 801 €	43 694 €
GAMMA_MME	69 744 €	21 729 €	25 800 €

[1] $MI = cvrge\ ratio_{AXA\ SA} * r_{coc} * \left(SCR^{AXA\ SA\ with\ treaty} - SCR^{AXA\ SA\ without\ treaty}\right)$

The only remaining component to obtain the technical premium is the reinstatement factor, which accounts for the reinstatement premium. The reinstatement factor is a rate that measures the reinstatement premium paid by the cedent to the reinsurer for the restoration of the treaty's capacity. It is defined as:

$$Reinst_{factor} = \mathbb{E}(\sum_{k=1}^{nb_{rec}} rate_{rec,k} * \max(0, 1 + \min(\frac{R}{L} - k, 0)))$$

For example, let us consider an annual recovery amount of $R = 250\ M€$. The treaty's limit is set to $L = 100\ M€$ and the treaty has 2@100%, 1@50% reinstatements. Then, the reinstatement factor is: $Reinst_{factor} = 100\% * 1 + \ 100\% * 1 + 50\% * 0{,}5 = 2{,}25$. It means that the cedent will have to pay 2,25 times the technical premium to restore the treaty's capacity.

The reinstatement factor can be evaluated based on the recovery distribution $(R_i)_{1 \leq i \leq N_{simul}}$ obtained through Monte Carlo simulations. In this context, no time considerations are included in the reinstatement premium calculation since AXA treaties do not include a temporal clause for reinstatement premiums. The table below details the reinstatement factor of each distribution. The more the treaty's capacity will be consumed, the higher the reinstatement factor. This, in turn, will mechanically reduce the final technical premium.

Table 40: Reinstatement factor – XL treaty number 1

Distributions	$\mathbb{E}(R)$	$Reinst_{factor}$
GAUSSIAN_KERNEL	116 678 €	7,78 %
ME_GPD	113 154 €	7,57 %
ME_PARETO	118 070 €	7,80 %
GAMMA_MME	69 744 €	4,66 %

Finally, one can obtain the technical premium. The table below presents the technical premium amounts calculated for the treaty under study.

Table 41: Technical premium – XL treaty number 1

Distributions	TP
GAUSSIAN_KERNEL	216 899 €
ME_GPD	212 455 €
ME_PARETO	218 776 €
GAMMA_MME	156 762 €

The ME-Pareto distribution is the one that yields the highest technical premium. However, one can note that the first three distributions provide relatively close technical premium amounts, which may confirm our valid estimation. As expected, the gamma distribution forecasts a lower technical premium, which was anticipated given the thinner tail of the fitted gamma distribution.

As a point of reference, the technical premiums obtained using the classical premium principles have also been computed. The following figure presents a heatmap of the computed technical premiums. This graph is intended to provide an overview of the different technical premiums calibrated according to cost of capital/classical premium principles, for several ranked statistical models. The amounts obtained with classical premium principles are not yet highly relevant, as the parameters for each premium have not been calibrated. Since the coefficients values have been arbitrarily defined, they seem consequently poorly selected here except perhaps for the standard deviation, where the premium returned is close to the technical premium calculated using the cost of capital approach.

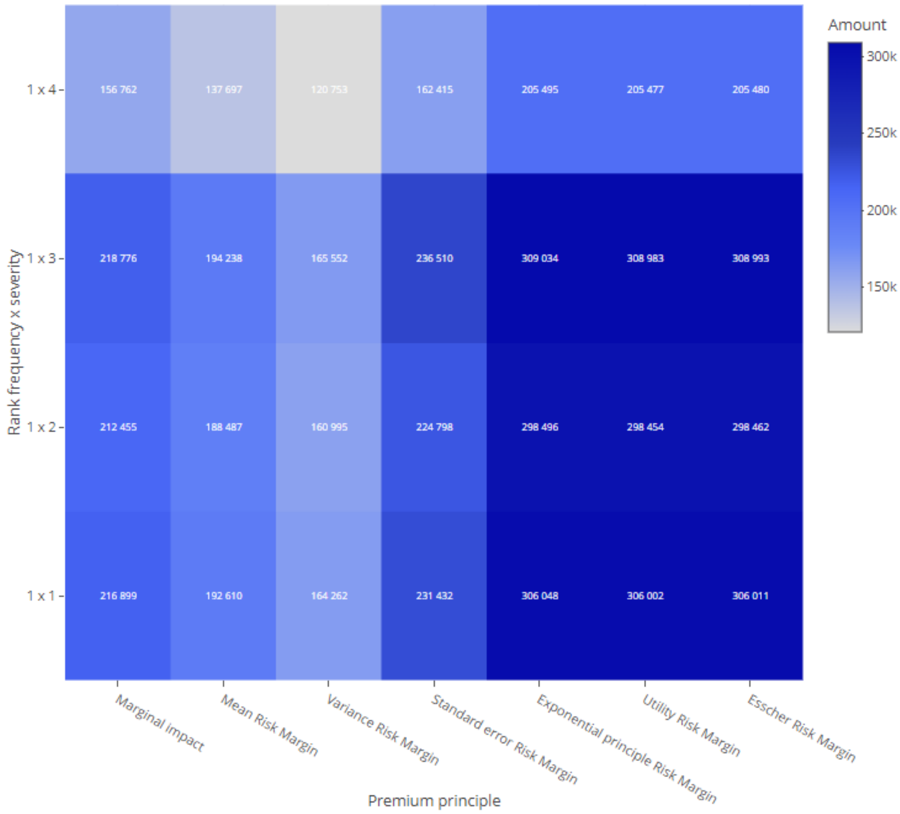


Figure 67: Technical premium heatmap – XL treaty number 1

To check if our model is consistent, it could be interesting to compare the calculated premium amounts with the market premium, if available. For the treaty under study, the market premium is 235 000 €. Thus, just like the premium amounts, a heatmap of the relative differences compared to the market premium could be provided, as shown in the following figure. In this figure, focusing on the first column, the technical premium is slightly lower than the market premium, with a deviation of less than 10% for the first three distributions. The market premium here serves as a reference to check if the treaty is appropriately priced, thus evaluating the robustness of the method, which seems to be the case here. However, one can observe a slight error in the ranking of the distributions, with the Pareto distribution being the one that most closely matches the actual treaty premium. This highlights the need to fit different statistical distributions to have several points of comparison in the calculation of the technical premium.

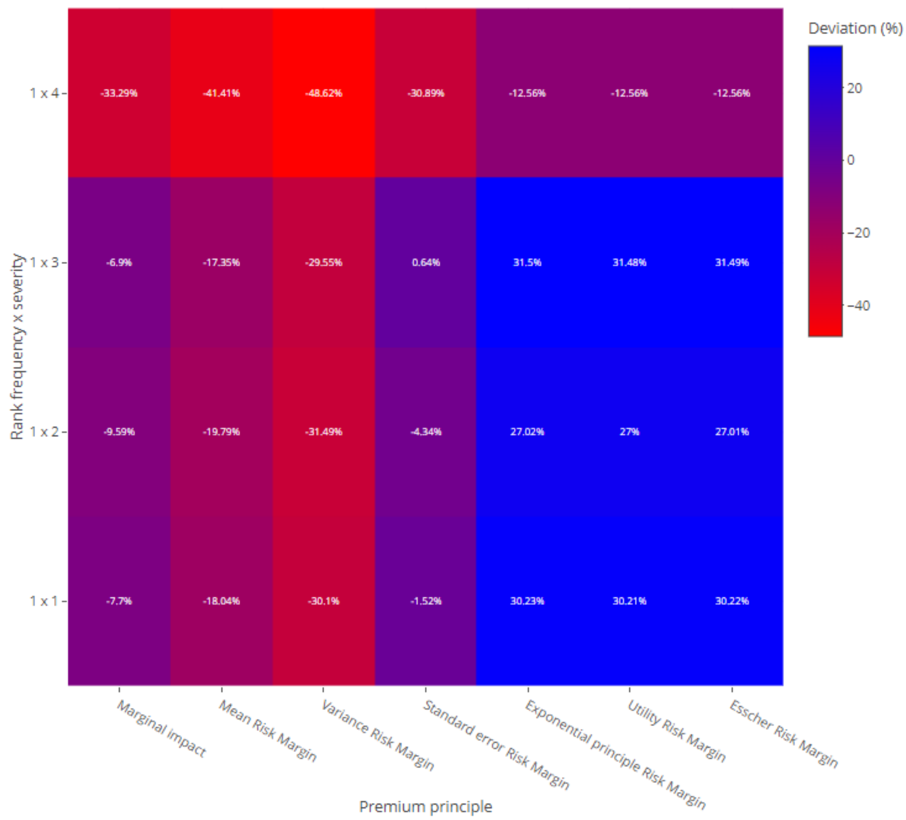


Figure 68: Technical premium comparison with market premium heatmap – XL treaty number 1

An interesting graphical approach to determine if the treaty is properly evaluated when no market premium is available would be to examine the position of the treaty within the LoB. To do so, the priced treaty is displayed along a normalized three-dimensional axis: $(Reserve;\ Limit;\ TP)$. The expected premium income of each cedent within the Lob will be used for normalization. One can eventually generate a scatter plot with every treaty within the LoB. By adding the priced treaty, one can visualize where the priced treaty lies within the LoB. If the pricing methodology is relevant, the point corresponding to the priced treaty should be close to the other points. Otherwise, it would suggest that our methodology produces an aberrant technical premium for this treaty.

For the considered treaty, the treaties within the LoB are represented in blue in the following figure, the remaining points corresponding to the priced treaty. This figure suggests that the XL treaty seems to be well-priced, as the points are relatively close to the blue scatter plot.

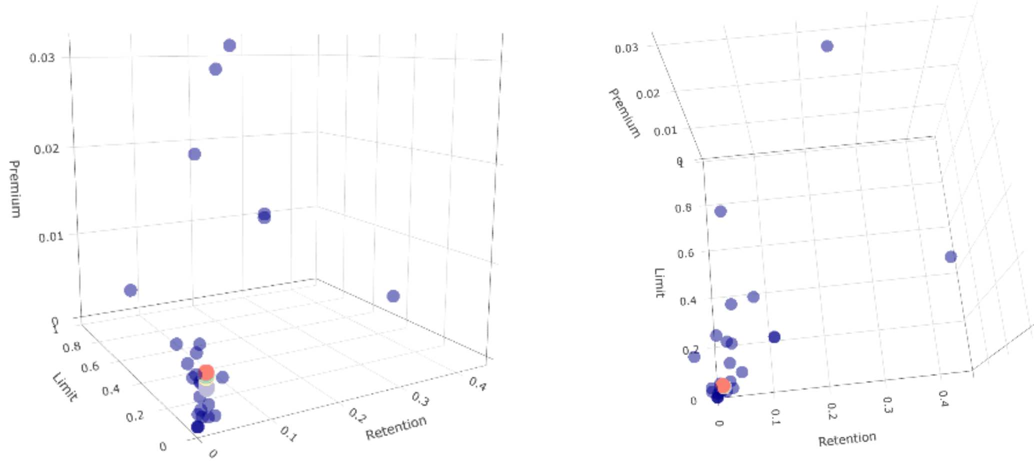


Figure 69: Technical premium within the LoB – XL treaty number 1

This graph can also be visualized in terms of surface, where the LoB surface is represented using interpolation techniques. It can be useful when the normalized values of the limit and retention are sparse, as it allows us to estimate a LoB reference in those areas. The treaty should therefore be close to the surface in order to be properly priced.

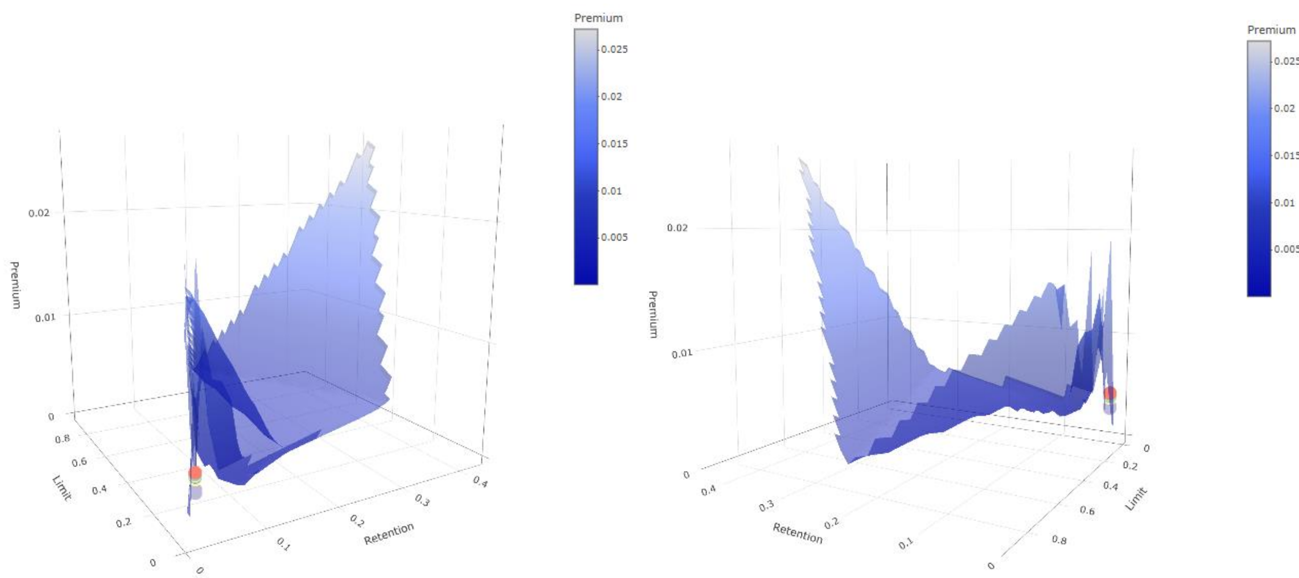


Figure 70: Technical premium and LoB surface – XL treaty number 1

These graphs can therefore be inspected for estimating whether the computed technical premium is aberrant or not. Let us now describe the risk indicators that have been retained.

6.1.2 Other indicators

Beyond the technical premium, other indicators can be of interest to properly quantify the risk and profitability of an underwritten treaty. These indicators are based on the distribution of recoveries obtained through Monte Carlo simulations. Let us describe the different indicators computed in this actuarial work.

6.1.2.1 SCR

The first indicator relates to what could be improperly called the treaty SCR. Mathematically, it can be expressed as: $q_{99.5\%}(R) - \mathbb{E}(R)$. This indicator helps assessing the reinsurer's extreme possible loss amounts. The following table exhibits the treaty SCR for XL treaty number 1.

Table 42: Treaty SCR – XL treaty number 1

Distributions	$q_{99.5\%}(R) - \mathbb{E}(R)$
GAUSSIAN_KERNEL	1 223 323 €
ME_GPD	1 348 134 €
ME_PARETO	1 438 635 €
GAMMA_MME	906 958 €

Regarding it, the ME-Pareto distribution has the highest treaty SCR, indicating that it could potentially generate the largest amount of losses, followed by the ME-GPD and the Gaussian Kernel distributions while the Gamma distribution has the lowest treaty SCR. These results are relatively consistent with the technical premium amounts calibrated. However, it is worth noting that the technical premium obtained with the Gaussian Kernel is higher than the one calculated with the ME-GPD, even though the latter has a higher SCR. This discrepancy arises because the technical premium considers only the mean of recoveries. This underscores the importance of calculating the treaty SCR as an indicator to gain insight into potential extreme loss amounts.

6.1.2.2 Exceedance probabilities

Exceedance Probabilities describe the probability that the annual recoveries exceed a threshold x. In this project, three exceedance probabilities have been considered:

- The attachment probability: This probability represents the likelihood of attachment, i.e., the probability that the annual recovery amount is non-zero. In other words, it is the probability that the reinsurer must indemnify the ceding company for claims. Mathematically, it corresponds to $\mathbb{P}(R > 0)$, where R denotes the recovery amount. This probability can be calculated empirically using the modeled distribution of recoveries.
- The exhaustion probability: This probability represents the likelihood that the treaty's capacity is fully exhausted, disregarding reinstatements. It is particularly useful in cases where there are multiple reinstatements, as it provides the likelihood of replenishing the treaty's capacity. Mathematically, it corresponds to $\mathbb{P}(R > L)$, where L is the treaty's limit.
- The break-even probability: This probability measures the likelihood that the reinsurance premium alone fails to cover the ceded losses. In this work, it corresponds to $\mathbb{P}(R > TP)$ where TP is the technical premium.

These probabilities have been calibrated to serve as clear risk indicators, reflecting among other things the impact of retention and limit. Indeed, they can be used to compare treaties covering the same cedent and risk but with varying limits and retentions, especially for attachment and exhaustion probabilities.

The figure below illustrates the exceedance probabilities for the treaty analyzed in this study.

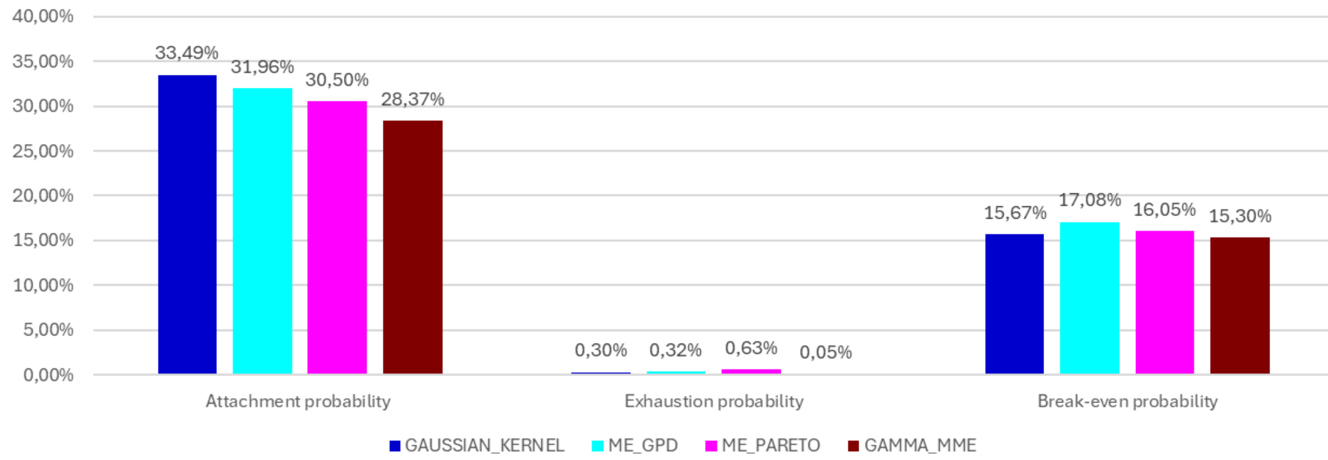


Figure 71: Exceedance probabilities – XL treaty number 1

The probabilities observed are relatively close, hovering around 30%. This indicates a 30% chance that the treaty's capacity will be consumed within a year. Regarding the exhaustion probabilities, these are extremely low. This aligns with the previously calculated VaR amounts, considering that the treaty's limit is set at 1.5 million. The likelihood of the capacity being fully exhausted is thus minimal. As for the break-even probability, it is approximately 15% across all treaties, with slightly higher values for the Pareto and GPD distributions, due to their heavier tails compared to the other two distributions. These tails increase the probability of extreme recoveries, which impacts the break-even probability.

6.1.2.3 Return periods

A Return Period is another way to express the annual exceedance probability and describes the estimated likelihood of a specific loss occurring within a given time frame. For example, a 50-year return period means that, on average, a given event or scenario is expected to occur once within a period of 50 years. The relationship between exceedance probability EP and return period RP is:

$$RP = \frac{1}{EP}$$

These indicators have been calibrated to provide an alternative representation of exceedance probabilities. The table below presents the return periods of the treaty under consideration.

Table 43: Exceedance return periods (in years) – XL treaty number 1

Distributions	Attachment Return Period	Exhaustion Return Period	Break-even Return Period
GAUSSIAN_KERNEL	2,99	333,33	6,38
ME_GPD	3,13	312,50	5,85
ME_PARETO	3,28	158,73	6,23
GAMMA_MME	3,52	2000,00	6,54

Regarding the return periods for the Gaussian Kernel distribution, a positive recovery amount occurs on average once every 3 years, while the technical premium fails to cover all recoveries once every 6 years and 5 months on average. This can be compared to the ME-GPD distribution, where claims are less likely to generate recoveries. However, they can lead to higher amounts, as the return period is longer for the ME-GPD compared to the Gaussian Kernel.

Another interesting point is to consider the amounts associated with specific return periods (e.g., the recovery amount associated with a 50-year return period). Mathematically, for a return period x, this corresponds to the quantile $q_{1-\frac{1}{x}}(R)$ of the recovery distribution. The following figure presents the amounts obtained for return periods of 10, 20, 100, and 200 years.

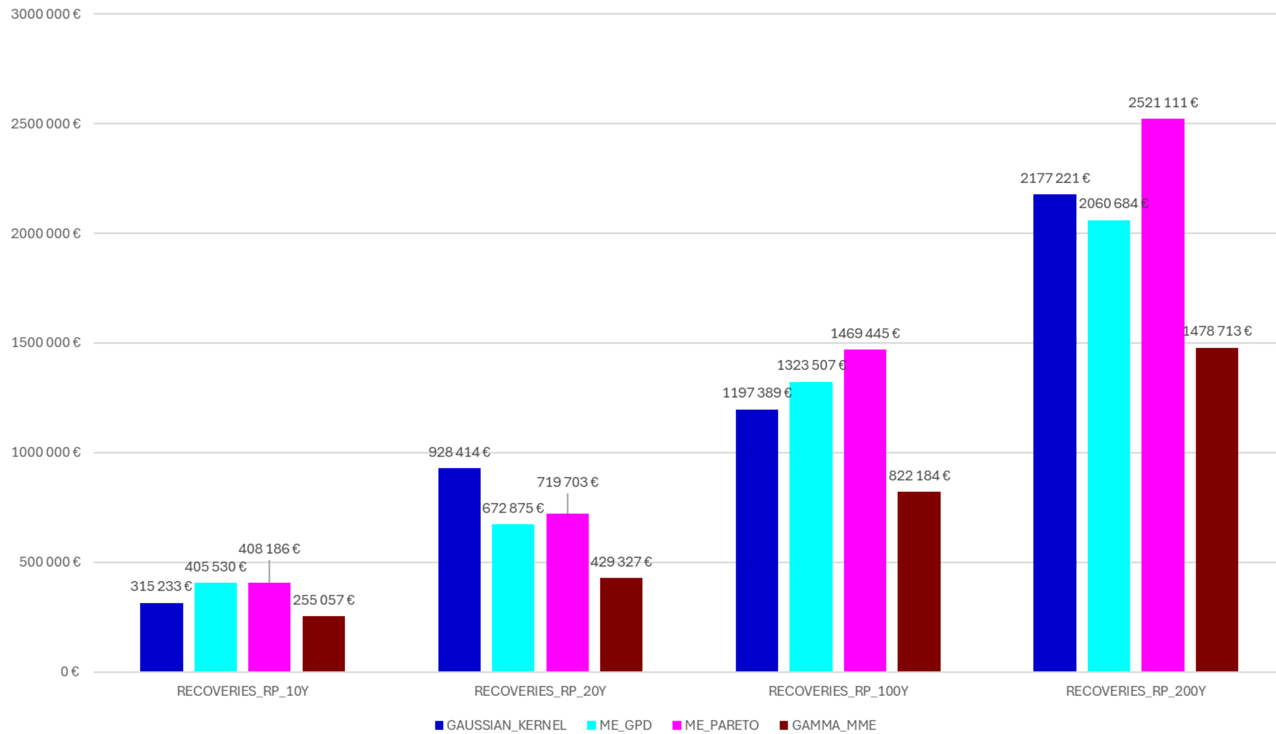


Figure 72: Recovery amounts for fixed return periods – XL treaty number 1

This graph clearly highlights the heavier tail of the Pareto distribution, which generates the highest recovery amounts for a 200-year return period, and the Gamma distribution, which has a much thinner tail. The ME-Pareto distribution is thus the distribution that generates the most severe extreme loss amounts.

6.1.2.4 Loss ratio

One can also be interested in looking at the treaty loss ratio. The loss ratio is generally defined as follows for a non-proportional treaty:

$$LR = \frac{\mathbb{E}(R)}{\left(1 + Reinst_{factor}\right) * TP}$$

Within AGCRe, the loss ratio was calibrated as follows:

$$LR = \frac{\mathbb{E}(R) - Reinst_{factor} * TP}{TP}$$

This change arises from the fact that the quantity $Reinst_{factor} * TP$, which corresponds to the reinstatement premium, is perceived as a loss reduction rather than a premium gain. Within the tool, the loss ratio has been calibrated accordingly to remain consistent with AGCRe's practices.

The table below presents the loss ratios calibrated for the treaty under study.

Table 44: Loss Ratios – XL treaty number 1

Distributions	***LR***
GAUSSIAN_KERNEL	46,01%
ME_GPD	45,69%
ME_PARETO	46,17%
GAMMA_MME	39,83%

The treaty is therefore profitable in our case, with an average loss ratio of approximately 46% for the three most suitable distributions.

6.1.2.5 ERD

A final useful indicator concerning the treaty is the Expected Reinsurer Deficit (ERD). It is generally defined as:

$$ERD = p * \frac{T}{P}$$

Where p is the probability of net income loss, T is the average of net economic loss when it occurs, and P the expected premium.

In this project, the ERD can be written as follows:

$$ERD = P(R > TP) * \frac{\mathbb{E}(R - TP \mid R > TP)}{TP}$$

The ERD is derived from the break-even probability. It is a method for testing reinsurance contracts to determine whether there is actual risk transfer. The ERD is an important indicator from a regulatory standpoint, as the ERD must be greater than 1% for the treaty to be considered as involving an actual risk transfer.

The table below presents the ERD values for the treaty under study.

Table 45: ERD – XL treaty number 1

Distributions	$\mathbb{E}(R \mid R > TP)$	ERD
GAUSSIAN_KERNEL	351 007 €	9,69%
ME_GPD	332 617 €	9,66%
ME_PARETO	433 268 €	15,74%
GAMMA_MME	211 624 €	5,35%

The highest ERD is obtained with the ME-Pareto distribution, and the lowest with the Gamma distribution. Modeling claims using the Pareto distribution is thus more pessimistic from the reinsurer’s point of view as it represents "riskier" underwritten reinsured risk. This is primarily due to the heavier tail of the Pareto distribution. In all cases, there is indeed a transfer of risk, and the treaties can be considered as valid reinsurance treaties.

6.1.2.6 ECR and RoC

One can also look at the Economic Combined Ratio (ECR). It evaluates the effectiveness of reinsurance by comparing incurred losses and expenses to premiums. Specifically, it is calculated by dividing the expected present value of losses, expenses, tax effects and cost of capital by the premium present value:

$$ECR = \frac{\mathbb{E}(PV(R + AE + Tax\ Effects + CoC))}{PV(Premium)}$$

An economic combined ratio below 100% signifies that the reinsurer is operating profitably, whereas a ratio above 100% indicates the reinsurer is operating at a loss. Within AGCRe, the ECR is calculated as the ratio between the technical and the market premium since the technical premium is designed to cover losses, expenses, tax effects and cost of capital associated with the reinsurance treaty. However, within the tool, the ECR is set to one since no market premium is available. As a result, the reinsurer is left in a neutral financial position - neither incurring a loss nor generating a profit. Consequently, the ECR is not a relevant metric for evaluating the treaty's profitability in this project.

Finally, the "ceded" return on capital is also one of the indicators assessed during underwriting committees at AGCRe. It represents the expected return on the treaty underwriting, reflecting the financial outcome of immobilizing a certain capital $Capital_{immobilized}$ to bear the risk resulting from the treaty underwriting. Its definition is thus:

$$RoC_{ceded} = \frac{\mathbb{E}\left(Net\ result^{reinsurer\ (discounted)}\right)}{Capital_{immobilized}}$$

However, within AGCRe, the net result of the treaty is based on the market premium which is not available in the pricing tool. Indeed, using the technical premium to compute the reinsurer's net result would be inconsistent since its calculation was designed to cover, among other things, the treaty cost of capital using the return on capital. Thus, the RoC_{ceded} is not a relevant indicator for non-proportional treaties since the market premium is not available.

To sum up, these different indicators detailed in this part offer a more precise evaluation than just the technical premium alone, especially with respect to the tail of the distribution and extreme events, which are particularly impactful for non-proportional reinsurance treaties. The following figure details the pricing process for non-proportional treaties, defined as "P15" in *Figure 64*.

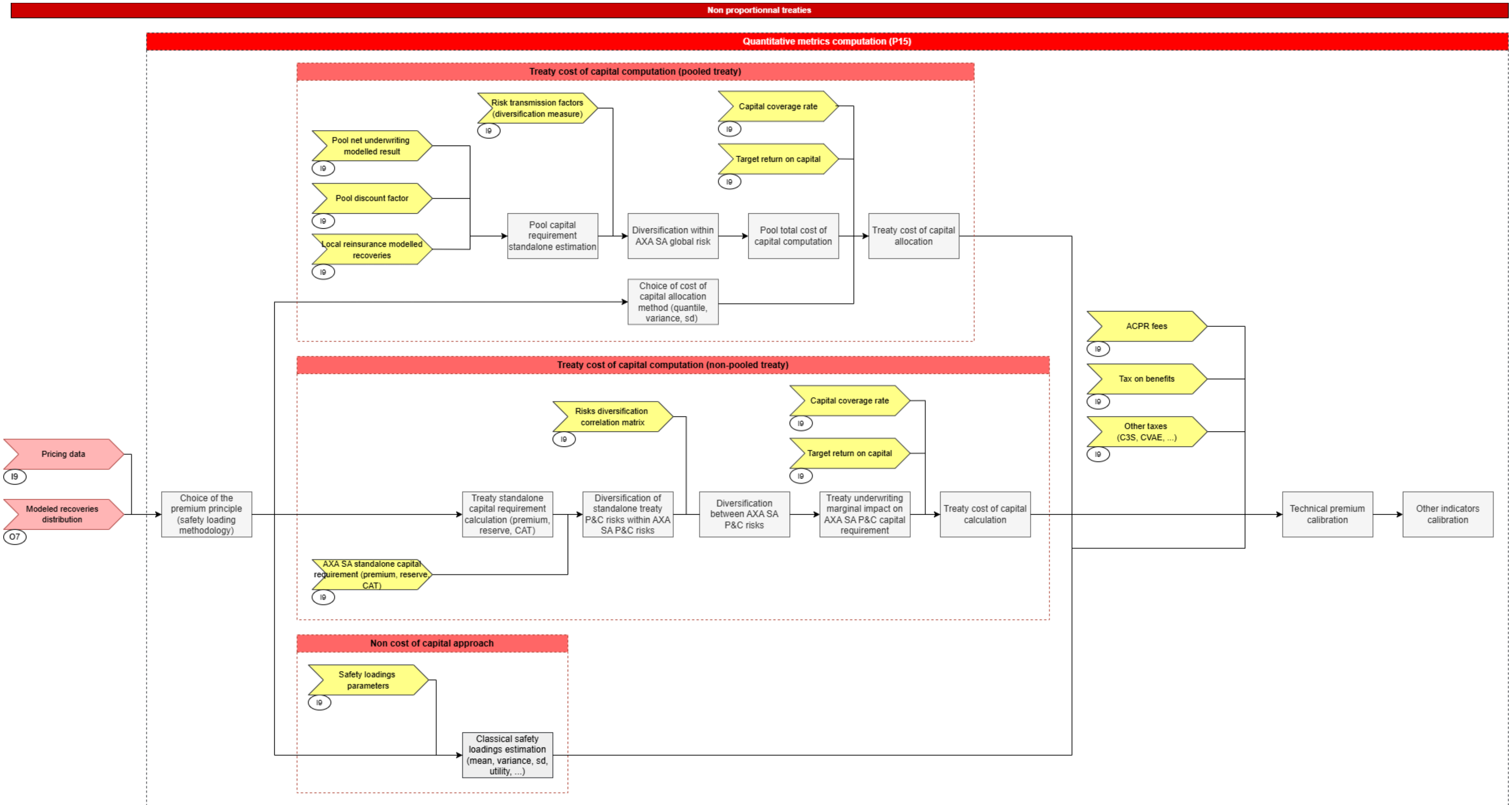


Figure 73: Pricing process – non-proportional treaties

6.2 Proportional treaties

Let us now consider proportional treaties. Due to their different nature, proportional treaties do not require premium calculation. For the evaluation of these treaties, the focus will be on comparing the margins and results obtained by the reinsurer when underwriting the treaty, along with the treaty cost of capital, using the theoretical elements developed in the previous section.

6.2.1 Results and margins

Let $Premium_{ceded}$ denote the premium acquired by the reinsurer, i.e., the ceded premium. We remind below the indicators that are calculated based on the simulations made in part 5.3.2.1:

- The expected gross loss ratio i.e., the cedent's loss ratio: $\mathbb{E}(LR^{cedent})$
- The expected cedent's loss: $\mathbb{E}(S^{cedent}) = \mathbb{E}(LR^{cedent}) * Premium_{ceded}$
- The expected reinsurer's loss ratio (net of loss corridor effect): $\mathbb{E}(LR^{reinsurer})$
- The expected ceded loss (net of loss corridor effect): $\mathbb{E}(S^{reinsurer})$
- The expected ceding commission: $\mathbb{E}(Ceding\ Commission)$
- The expected reinsurer's loss and loss ratio (including commission and expenses): $\mathbb{E}\left(S^{reinsurer\ (incl.\ commission\ and\ expenses)}\right)$ and $\mathbb{E}\left(LR^{reinsurer\ (incl.\ commission\ and\ expenses)}\right)$
- The expected experience account: $\mathbb{E}(Experience\ Account)$
- The expected profit sharing: $\mathbb{E}(Profit\ Sharing)$
- The expected reinsurer's net loss and loss ratio (un)discounted: $\mathbb{E}\left(Net\ Loss^{reinsurer\ (discounted)}\right)$ and $\mathbb{E}\left(Net\ Loss^{reinsurer\ (undiscounted)}\right)$, with $\mathbb{E}\left(Net\ LR^{reinsurer\ (discounted)}\right)$ and $\mathbb{E}\left(Net\ LR^{reinsurer\ (undiscounted)}\right)$

Additionally, based on the previous quantities, one can get the (un)discounted reinsurer's net result using the following formula:

$$\mathbb{E}\left(Net\ Result^{reinsurer\ (discounted)}\right) = Premium_{ceded} - \mathbb{E}\left(Net\ Loss^{reinsurer\ (discounted)}\right)$$

$$\mathbb{E}\left(Net\ Result^{reinsurer\ (undiscounted)}\right) = Premium_{ceded} - \mathbb{E}\left(Net\ Loss^{reinsurer\ (undiscounted)}\right)$$

One can add regulatory conditions by incorporating taxes. The formula becomes:

$$\mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer\ (undiscounted)}\right) = \mathbb{E}\left(Net\ Result^{reinsurer\ (undiscounted)}\right) - Tax$$

$$\mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer\ (discounted)}\right) = \mathbb{E}\left(Net\ Result^{reinsurer\ (discounted)}\right) - Tax$$

In this project, the Tax quantity is calculated as follows:

$$Tax = r_{income\ tax} * \mathbb{E}\left(Net\ Result^{reinsurer\ ((un)discounted)}\right) + Other\ Taxes$$

The $r_{income\ tax}$ corresponds to the tax rates that applies on the reinsurer's benefits (tax on companies, CVAE, etc), while the other taxes are composed of the C3S tax and other ACPR fees that apply to the ceded premium.

Let us consider the following treaty: a QS treaty with a 60% cession rate, leading to a ceded premium of 1 million euros. Let us assume the following distribution of simulated cedent's loss ratios derived from a log-normal distribution:

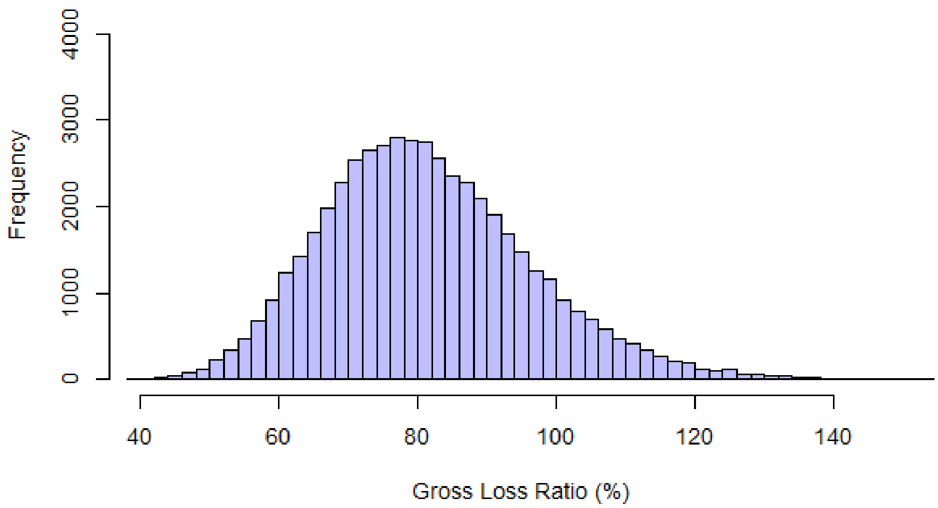


Figure 74: Distribution of cedents's loss ratios

The Quota Share treaty also has the following conditions:

- Commission Rate: A fixed commission of 15% is paid to the ceding company.
- Loss Corridor: A loss corridor is applied between 100% and 120%.

The figure below represents the distribution of the reinsurer's loss ratios. A +15% translation from the cedent's loss ratio is noticeable due to the fixed commission. The effect of the loss corridor is also apparent, with a higher number of loss ratios around 115% for the reinsurer, which corresponds to the range of cedent's loss ratios between 110% and 120%. This effect is clearly beneficial for the reinsurer, as it prevents the reinsurer from assuming excessive losses and thus diminishes the variance of its result.

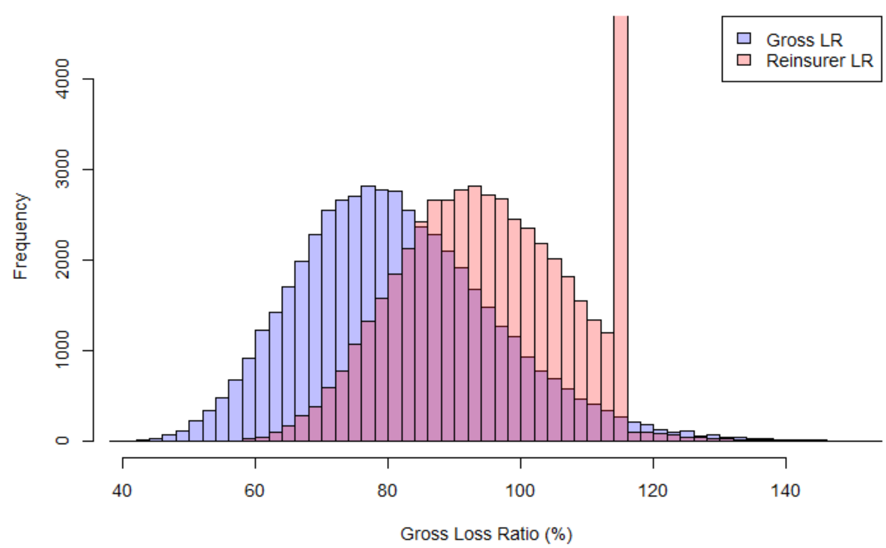


Figure 75: Comparison of cedent's LR and reinsurer's net LR

The table below summarizes some of the previously described quantities:

Table 46: Indicators for proportional treaties – Example

Quantity	Value
$\mathbb{E}(LR^{cedent})$	81,36 %
$\mathbb{E}(LR^{reinsurer})$	79,11 %
$\mathbb{E}(r_{reinsurer\ commision})$	15 %
$\mathbb{E}(Net\ Result^{reinsurer\ (undiscounted)})$	45 877 €
$\mathbb{E}(Net\ Result^{reinsurer\ (discounted)})$	61 005 €
$\mathbb{E}(Net\ Result^{reinsurer\ (discounted)}_{post\ tax})$	48 356 €

6.2.2 Cost of capital

In the same way as for non-proportional treaties, one can evaluate the treaty cost of capital and the impacts on AXA's SCR. To do so, the methodology used for non-proportional treaties will be applied, with a few differences.

First, it is necessary to calculate the $SCR_{premium}$ of the treaty:

$$SCR_{premium} = \hat{\mathbb{E}}(Net\ Result^{reinsurer\ (discounted)}) - \hat{q}_{0.05\%}(Net\ Result^{reinsurer\ (discounted)})$$

For the previously considered quota share, $SCR_{premium} = 278\ 186$ €

Now, one needs to calculate the $SCR_{reserve}$ of the treaty. To do so, let $\mathbb{E}(LR^{cedent})$ be the expected value of the cedent's loss ratio simulated, $PP_{cum} = \left(PP_{cum\,j}\right)_{1 \le j \le 30}$ the cumulated payment pattern calibrated for the corresponding quota share and $r_{stress\ reserve}$[1] the stress rate for reserves. The cumulated ultimate stressed loss ratio over time is:

$$LR^{cedent}_{stressed,cumulative} = \left(\mathbb{E}(LR^{cedent}) + r_{stress\ reserve} * \mathbb{E}(LR^{cedent}) * \left(1 - PP_{cum_j}\right)\right)_{1 \le j \le 30} \in \mathbb{R}^{30}$$

This vector represents the evolution of the ultimate loss ratio, with a reserve surplus which initially increases the loss ratio. Towards the end, the stressed loss ratio is equal to $\mathbb{E}(LR^{cedent})$.

Once this vector has been measured, the treaty terms and conditions are applied on $LR^{cedent}_{stressed}$ to obtain the reinsurer's stressed loss ratio: $Net\ LR^{reinsurer\ (discounted)}_{stressed,cumulative} \in \mathbb{R}^{30}$ The calculations are similar to the ones developed in the section 5.3.2.1. Finally, to obtain the $SCR_{reserve}$, one can apply the following calculation, which corresponds to the marginal impact of a reserve amounts increase:

$$SCR_{reserve} = \sum_{j=1}^{30} \left(\left(Net\ LR^{reinsurer\ (discounted)}_{stressed,cumulative}\right)_j - \left(Net\ LR^{reinsurer}_{i,cumulative}\right)_j\right)$$

For the considered treaty, $SCR_{reserve} = 640\ 115$ €. Unlike the SCR for premiums, the SCR for reserves is sensitive to the payment pattern: the more the payments are spread over time, the higher the SCR will be.

These quantities are then aggregated into a single vector of solvency capital requirement standalone named SCR_{treaty}:

$$SCR^{treaty} = \begin{pmatrix} SCR_{premium} \\ SCR_{reserve} \\ 0 \end{pmatrix}$$

[1] *The reserve stress rate is fixed at 32 % based on Risk Management recommendations.*

We find ourselves in the same situation as in the non-pooled treaty section (see 6.1.1.2.2), and the same quantities can be calculated, namely $SCR^{AXA\ SA\ with\ treaty}$ and $SCR^{AXA\ SA\ without\ treaty}$ from which the treaty cost of capital can be derived:

$$CoC_{treaty} = coverage\ ratio_{AXA\ SA} * RoC_{target} * (SCR^{AXA\ SA\ with\ treaty} - SCR^{AXA\ SA\ without\ treaty})$$

In our case, the treaty cost of capital is 26 821 €, with an impact on AXA P&C's overall SCR of 319 305 €. This final cost of capital can then be compared to the final margin $\mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer\ (discounted)}\right)$: if the cost of capital is greater than the reinsurer's margin, the treaty is not profitable.

Using the cost of capital, one can also compute the return on capital defined with the following formula for proportional treaties:

$$RoC_{ceded} = \frac{\mathbb{E}\left(Net\ Result^{reinsurer\ (discounted)}\right)}{Capital_{immobilized}} = \frac{\mathbb{E}\left(Net\ Result^{reinsurer\ (discounted)}\right)}{\frac{CoC_{treaty}}{RoC_{target}}}$$

In our case, the RoC_{ceded} is 13,65%.

Moreover, in the same way as for non-proportional treaties, the ERD and the ECR can also be measured. In our case, the ERD is 2.83%, with a probability of having a negative result of about 33% while the ECR is 97,85%. The low value of the ERD can be explained by the presence of the loss corridor, which in our case helps to limit extreme losses and thus makes the ERD lower. The ECR being inferior to 100%, the treaty is thus profitable.

The table below displays a summary of the various indicators calculated during the assessment of a proportional reinsurance treaty in this actuarial work. The amounts can be converted to rate by dividing them with the ceded premium.

Table 47: Overview of proportional treaty indicators

Name	Formula
Ceded Premium	$Premium_{ceded} = c * Premium_{subject}$ for QS treaties
Ceded Losses	$\mathbb{E}(S^{reinsurer}) = \mathbb{E}(f_{loss\ corridor}(LR^{cedent}) * Premium_{ceded})$ for QS treaties
Ceding Commission	$\mathbb{E}(Ceding\ Commission) = \begin{cases} r_{CF} * Premium_{ceded} \\ \mathbb{E}(r_{CSS}(LR^{reinsurer}{}^{1})) * Premium_{ceded} \end{cases}$
Expenses	$Expenses = r_{expenses} * Premium_{ceded}$
Experience Account	$\mathbb{E}(Experience\ Account) = Premium_{ceded} - \mathbb{E}(S^{reinsurer}) - \mathbb{E}(Ceding\ Commission) - Expenses - r_{target\ margin} * Premium_{ceded}$
Profit Sharing	$\mathbb{E}(Profit\ Sharing) = r_{profit\ sharing} * \mathbb{E}((Experience\ Account)_+)$
Net Loss (reinsurer)	$\mathbb{E}(Net\ Loss^{reinsurer}) = \mathbb{E}(S^{reinsurer}) + \mathbb{E}(Ceding\ Commission) + Expenses + \mathbb{E}(Profit\ Sharing)$
Net Result (reinsurer)	$\mathbb{E}(Net\ Result^{reinsurer}) = Premium_{ceded} - \mathbb{E}(Net\ Loss^{reinsurer})$
Net Result post tax (reinsurer)	$\mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer}\right) = \mathbb{E}(Net\ Result^{reinsurer}) - Tax$
Cost of Capital (treaty)	$CoC_{treaty} = coverage\ ratio_{AXA\ SA} * RoC_{target} * (SCR^{AXA\ SA\ with\ treaty} - SCR^{AXA\ SA\ without\ treaty})$
Return on Capital	$RoC_{ceded} = \frac{\mathbb{E}(Net\ result^{reinsurer})}{\frac{CoC_{treaty}}{RoC_{target}}}$
ERD	$ERD = \mathbb{P}\left(Net\ Result_{post\ tax}^{reinsurer} > 0\right) * \frac{\mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer} \middle\vert Net\ Result_{post\ tax}^{reinsurer} > 0\right)}{Premium_{ceded}}$
ECR	$ECR = \frac{\mathbb{E}(PV(R + AE + Tax\ Effects + CoC))}{PV(Premium)} = \frac{Premium_{ceded} - \mathbb{E}\left(Net\ Result_{post\ tax}^{reinsurer}\right) + CoC_{treaty}}{Premium_{ceded}}$

To conclude, this section has detailed the computation of risk and profitability indicators. These indicators, derived from simulated loss distributions, enable the Analytics and Pricing team to evaluate its reinsurance treaties for future underwriting decisions. The use of Monte Carlo simulations has proven to be an effective choice, as it allows for the calculation of treaty indicators for both

[1] $LR^{reinsurer} = \frac{S^{reinsurer}}{Premium_{ceded}}$

proportional and non-proportional treaties. The indicators displayed within the pricing tool are not exhaustive: they were chosen in collaboration with the team to identify those most relevant beyond traditional metrics based solely on loss distribution. This section could be further expanded with additional indicators and serves as an initial step, concluding the description of the methodology that forms the foundation for the tool's development. This has also enabled us to respond part of the problem, which was to develop a centralized approach for evaluating treaties by producing risk and profitability indicators for all types of treaties.

Furthermore, the following figure details the pricing process for proportional treaties, defined as “P16” in *Figure 64*.

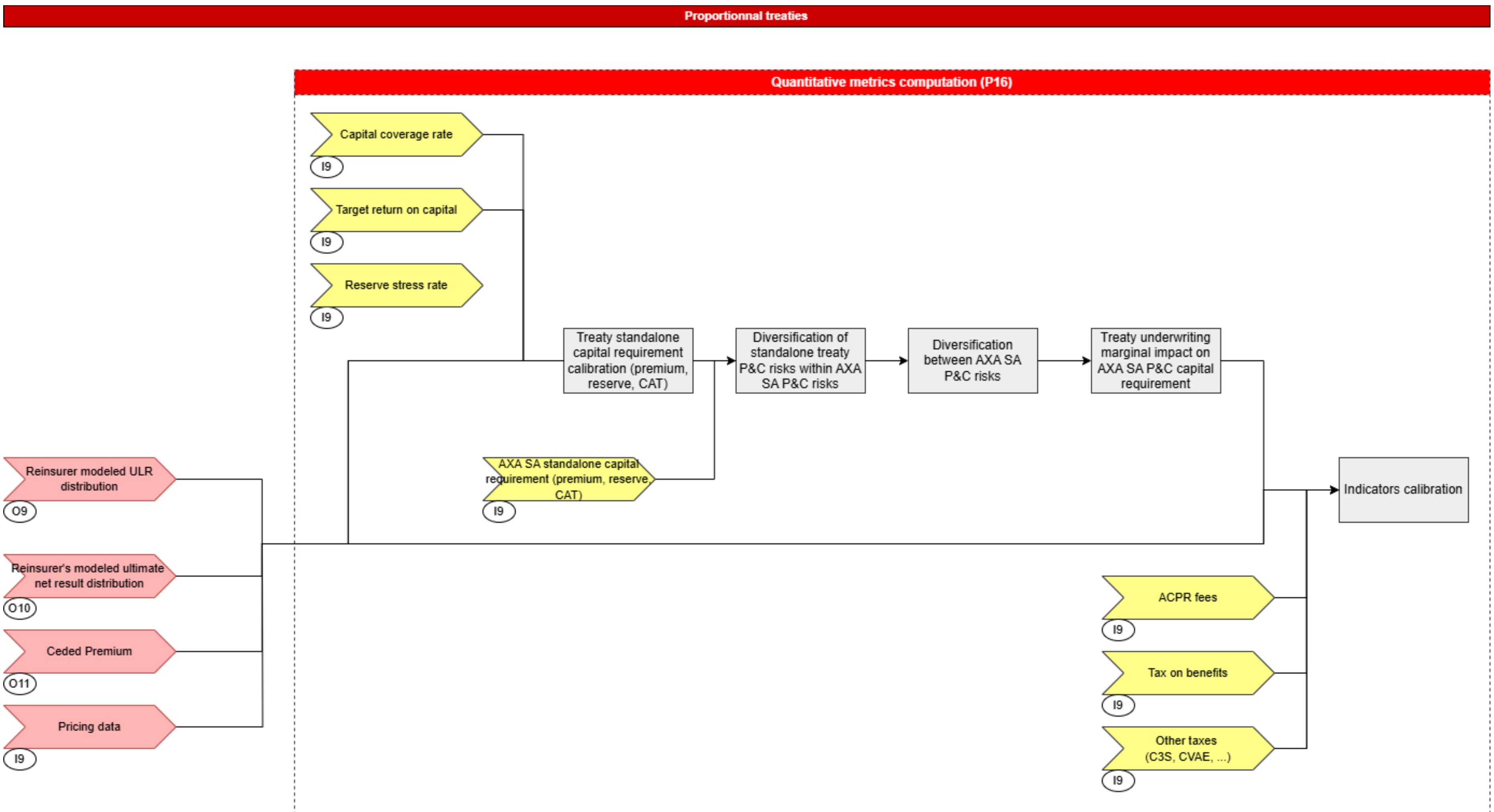


Figure 76: Pricing process – proportional treaties

7. Comparison with market premiums

To assess the results of our modeling approach, we will focus on the study of XL per risk treaties, which make up a large portion of AGCRe's portfolio, being also the type of treaties for which the data is the most reliable. One way to evaluate our pricing methodology is to compare the modeled technical premium with market premiums. This allows us to evaluate whether the developed methodology provides a reliable estimate of the technical premium. To do this, the technical premium obtained for both pooled and non-pooled treaties will be considered. Furthermore, only the frequency-severity cost modeling approach was used here, as it enables assessing more treaties.

7.1 Pooled treaties

In the case of pooled treaties, 30 treaties were selected from the marine, liability and property pools for which market premiums and enough historical claims (>150) are available. These treaties are part of about twenty different reinsurance programs.

The technical premiums were computed using 50 000 simulations for each treaty. The three methodologies for calculating safety loadings were applied (see 6.1.1.2.1). The technical premium amounts were then compared through linear regression, regressing the technical premiums against the market premium. The following figure presents the results of this analysis, showing the technical premiums calculated for the frequency and severity distribution with the highest range based on the established ranking methodology.

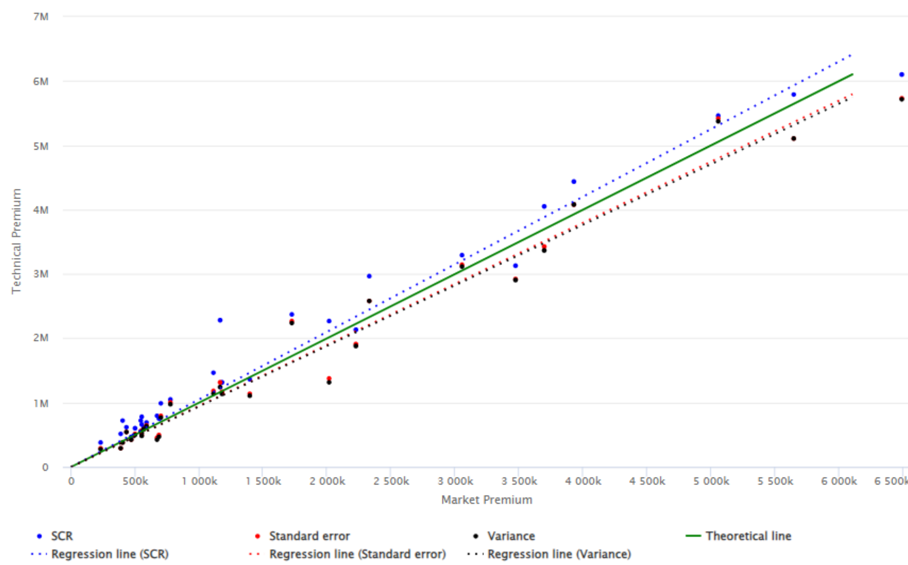


Figure 77: Comparison with market premium – Pooled treaties

The technical premiums calculated with a "variance" safety loading seems close to those based on a "standard error" safety loading, while the technical premium measured using the quantile method (referred to as SCR in the figure) is higher. This was expected, as the previously plotted graph showed that the safety loading coefficients were higher for the quantile method. The predicted amounts from the variance and standard deviation methodologies are also close to each other, which was also anticipated. Furthermore, the regression lines for each of the methodologies are close to the theoretical line, which corresponds is the line with a linear coefficient equal to 1. Indeed, the coefficients of the regression line are around 0.95 for variance and standard deviation methods, while for the quantile method, the coefficient is around 1.05. The average difference between the calculated technical premiums and market premiums is around +/- 5%, which indicates a comforting estimation of the technical premium. Additionally, the R^2 coefficients of the regression are above 98%, and the regression coefficients are statistically significant at the 5% level, confirming the relatively accurate estimation of premiums for the selected sample. The regression results are provided in the appendix.

For informational purposes, the most used severity distributions are the ME Pareto, GPD, and ME GPD (about 75% of the cases), while the negative binomial distribution is the most used for modeling frequency (about 80%). Regarding severity distributions, this also highlights the utility of modeling the tail of the distribution with a GPD and confirms that the method for selecting severity and frequency distributions seems appropriate.

7.2 Non-pooled treaties

For non-pooled treaties, the objective is the same, i.e., to compare market premiums with computed technical premiums. For this purpose, 57 treaties were selected using the same process, covering over 13 cedents and approximately 20 different risk classifications. A linear regression was then conducted using the technical premium for the most appropriate severity and frequency distributions according to the ranking methodology, with 50 000 simulations conducted for each treaty. The figure below displays the results of the linear regression for these treaties.

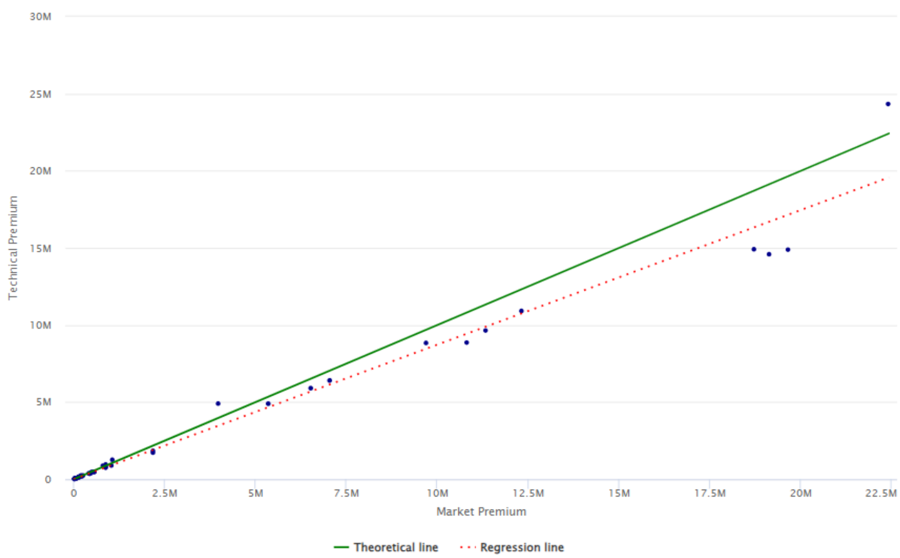


Figure 78: Comparison with market premium - non pooled treaty

Regarding the severity distributions, the three most frequent ones are the Gaussian kernel, the ME GPD, and the GPD (MLE) (approximately 60%). For frequency distributions, the negative binomial distribution remains the most used (around 60%).

For non-pooled treaties, the estimated technical premium is lower than the market premium. The coefficient of the linear regression is about 0.87, indicating that, on average, the technical premium is around 13% lower than the market premium. However, the R-squared value of the regression is close to 1, and the coefficient is statistically significant, which reflects the strong correlation between the two amounts. The model thus leads to an underestimation of the market premium for non-pooled treaties, and to a greater extent compared to pooled treaties.

Non-pooled treaties are less accurately modeled than pooled treaties. This could stem from several factors, including the different method for calculating safety loadings between these two types of treaties. Generally, it is necessary to analyze each treaty individually to understand the source of this underestimation by inspecting historical claims. However, in our case, this provides an internal estimate of the premium amount for non-modeled treaties which is quite satisfactory. Additionally, treaties with premiums consistently lower than the market premium may be flagged by AXA during discussions with external reinsurers.

8. Limits and perspectives

The methodology developed as part of this tool's construction has, however, several limitations.

To start with, the methodology is currently applicable only to traditional reinsurance treaties offering 1-year coverage, meaning it does not cover multi-year treaties or capital management treaties. Extending the methodology to include these types of treaties represents a significant area for improvement, with the goal of broadening the scope of treaties that can be priced. Additionally, the impact of underlying treaties applicable before the considered treaty is not fully accounted for across all types of treaties. Instead, an approximation is used, involving an inuring rate that can represent the prior impact of a QS or a SP. For underlying XL or AGG treaties, their modeling and subsequent impact on the considered treaty become more complex, and this functionality has not been considered. As a result, historical claims amounts are modeled net of all inuring treaties that may apply. This simplification may lead to an overestimation of some indicators like the technical premium, the loss ratio or the exceedance probabilities, as the influence of these preceding treaties is not explicitly assessed in the calculations.

Furthermore, another limitation of the tool concerns the modeling of CAT treaties, which are not currently modeled. These treaties are more complicated to model, as they involve complex components and need specialized models such as RMS, AIR, and ICM. Attempting to evaluate a reinsurance treaty without using these advanced models would be a challenging task. However, an alternative approach could involve integrating the outputs of these models, which are already computed by other AXA teams, such as ELT (Event Loss Table) or YELT (Year Event Loss Table). This could allow CAT and hybrid treaties to be modeled within the pricing tool. While this approach offers operational benefits, it would provide less flexibility in terms of underlying models and internal vision compared to the approach developed in this actuarial work, except perhaps with an increase in the simulation capacity.

Regarding SP treaties, a methodology has been implemented for the property LoB. By applying the exposure methodology as the one implemented for property XL treaties, indicators can be produced for surplus treaties. This methodology is relevant for surplus treaties as it allows for the modeling of sums insured, which are critical in surplus treaty calculations. For other LoBs, however, SP treaty modeling becomes more complex. This complexity arises due to the less direct relationship between sums insured and claim amounts. Effective modeling in these cases would require access to the underlying distribution of sums insured associated with each historical claim. Additionally, both claims and sums insured would need to be modeled jointly, potentially using multivariate distributions or copulas.

Additionally, the exposure methodology is currently mainly exclusive to the property LoB in this study. Extending the exposure methodology to other LoBs presents an opportunity for further development. Even if we proposed a methodology using exposure curves for liability treaties it would require estimating parameters that could not be determined during this study, such as the premium sensitivity to changes in limits (Mack & Fackler, 2003). Access to individual premiums and sums insured provided by cedents would enable the modeling of XL per-risk liability treaties through exposure curves. Furthermore, the current method does not explicitly account for deductibles and limits in underlying policies: they are only partially captured in the frequency-severity model via inuring rates.

The tool also does not currently manage life treaties, as these differ significantly in terms of methodology. However, there have been previous attempts by the team to model these treaties, which could potentially be integrated into the tool.

Regarding the modeling of treaties based on loss ratios, such as AGG and QS treaties, the developed methodology is straightforward. It involves fitting a single distribution over the entire range of loss ratios without distinguishing between atypical claims, attritional and CAT claims. A more refined approach would involve using different distributions, allowing for separate modeling of these different types of claims. However, implementing such an approach is challenging due to the limited historical data available. One way to address this limitation is by grouping similar entities to expand the historical loss ratios available, enabling more robust modeling but less granular. Additionally, as mentioned previously, the model assumes that the "as-if" approach does not impact loss ratios significantly. This assumption - where historical loss ratios are used without adjustments for inflation in premiums or losses - could be revisited in the future.

Finally, from a computational perspective, the simulations required to derive treaty indicators can be time-consuming, particularly when calibrating different frequency-severity distributions or dealing with a high number of claims per year. Moreover, the number of simulations is sometimes insufficient to accurately estimate extreme loss indicators, such as VaR or exceedance probabilities. To address these limitations, variance reduction techniques using conditional Monte Carlo, control variables, or importance sampling could be conducted. Based on conditional Monte Carlo, the Asmussen-Kroese estimator might be particularly helpful for effectively measuring the exceedance probabilities and return periods of non-proportional treaties. However, these techniques are more complex to implement and require the estimation and selection of additional parameters, such as control variables. Nevertheless, these methods should not be forgotten to improve the speed and accuracy of estimators obtained through Monte Carlo simulations.

Subsequently, there are several ways to improve this pricing tool. For example, it could support the optimization of reinsurance treaties by integrating optimization problems, such as determining the optimal commission for QS treaties, while considering factors like ERD, profit, and cost of capital. Concerning XL or AGG treaties, the retention and limit could be chosen in order to minimize ceded premiums, ceded margins, and ceded RoC while considering other parameters such as the underlying portfolio's net result volatility and the extreme quantiles of potential losses, all while staying within the risk appetite limits set by AXA SA. Furthermore, the use of credibility models in the calculation of technical premiums for non-proportional treaties could provide a market-based perspective when setting premiums, as opposed to the current internal view. It would also be beneficial to enhance the modeling of XL treaties by employing multiple thresholds in the distribution fitting process, as opposed to the single-threshold approach used in this study. A combination of exposure-based and experience-based pricing could also be examined to strengthen the results. Similarly, different provisioning methods could be developed, allowing for more flexibility, and potentially yielding different results compared to the current chain-ladder model.

To conclude, while the methodology developed in this study still has many limitations and areas for improvement, it provides a foundation for centralizing reinsurance pricing and evaluation. This represents a first step for the Analytics and Pricing team in expanding their approach to pricing and improving their evaluation of reinsurance treaties.

9. Pricing tool structure

The methodology developed throughout this endeavor addressed part of our problematic, which was to propose an approach based on internal data enabling the Analytics and Pricing team to have full control over the modeling and pricing process while increasing the number of treaties that can be priced. However, to ensure direct usability, facilitate transparency, and allow for easy modifications and refinement of methodologies, the development of a pricing tool represents the culmination of the methodology and the operational added value of our response to the problem at hand. Indeed, building this tool provides time savings, enables rapid control of the modeling process, and centralizes all the developments outlined in the actuarial thesis. Its construction involved design considerations, both in terms of the tool's visual rendering and the underlying code structure.

The reinsurance pricing tool was developed using the R programming language. To ensure flexibility and support future modifications, the tool was designed using the object-oriented programming (OOP) paradigm. OOP focuses on organizing software design around "objects" - entities with unique attributes and behaviors - rather than just functions or logic. This approach is particularly suitable for building tools that need to be maintainable and extensible, aligning with the tool's intended purpose. This is due to two properties of the OOP:

- **Encapsulation**: Objects are categorized into specific "classes," restricting access and modification to objects outside their class. Multiple classes were defined to address the specific requirements of the tool.
- **Inheritance**: This allows relationships and hierarchies between objects, enabling shared logic while preserving unique structures. This makes the approach highly reusable, efficient, and adaptable for handling future improvements by users unfamiliar with the codebase.

Now, let us describe the architecture and organization of the tool. The tool's core functionality is built around the following classes:

- **“Processing classes”:** This type of class encompasses three primary classes: one dedicated to handling operations for XL treaties, one for SP treaties and another designed for AGG, and QS treaties. Each class includes functions for operations such as payment pattern calculations, IBNR estimations, and other related tasks. Overall, these classes centralize the operations detailed in sections 3 and 4.
- **“Modeling classes”:** This type of class includes a general class that manages the fitting, ranking, and statistical testing of loss distributions. Subsequently, four classes are defined, each one adapted to a specific treaty type in order to perform Monte Carlo simulations and compute recoveries or losses. Each of these specialized classes can inherit the properties of a fitting class, which fits and ranks distribution on loss ratios/individual claims, or an exposure class for exposure modeling. Overall, these classes perform the operations described in part 5.
- **“Pricing classes”:** This class is composed of two independent subclasses: one for non-proportional treaties and the other for proportional treaties. Each of them provides the indicators described in section 6.
- **“Treaty classes”:** The treaty-specific classes inherit functionality from the previously mentioned classes, allowing users to independently analyze or extend their studies on a treaty. There is one subclass for each type of treaty.

The following figure summarizes the architecture of the reinsurance pricing tool:

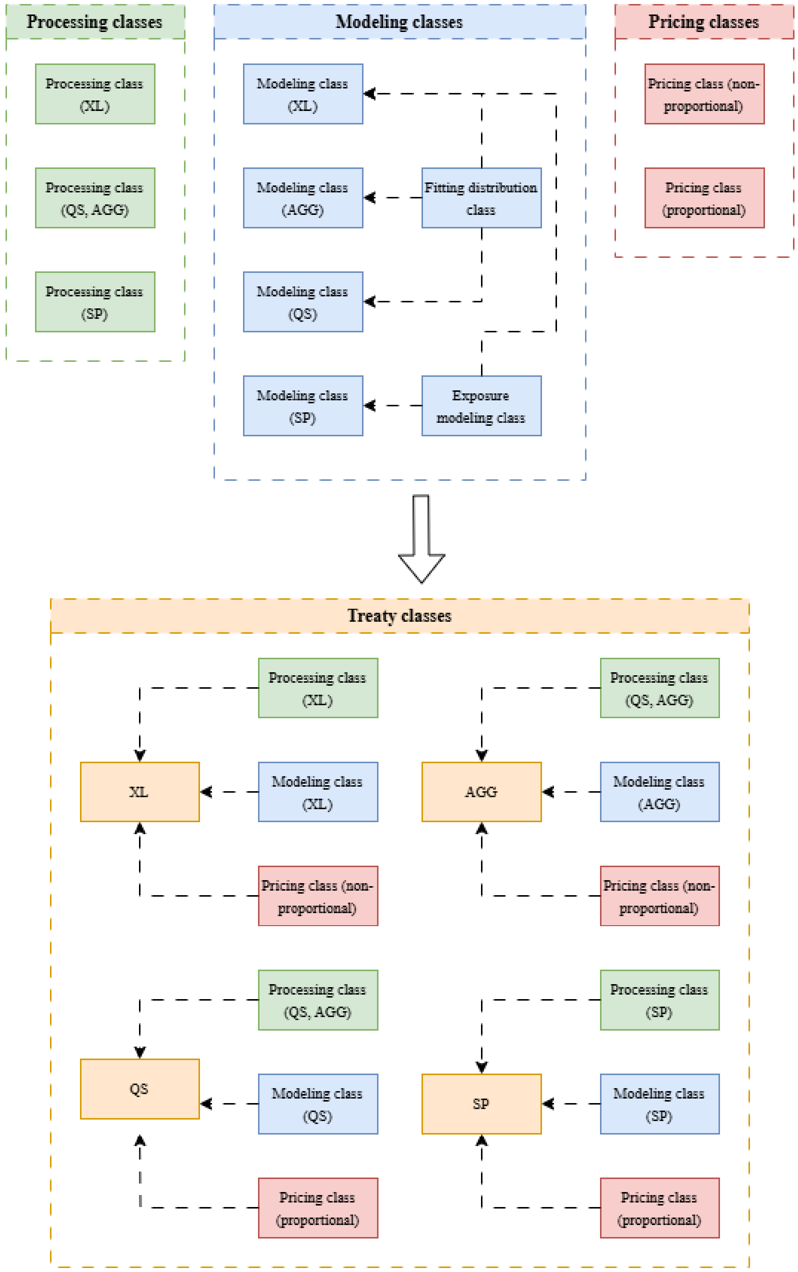
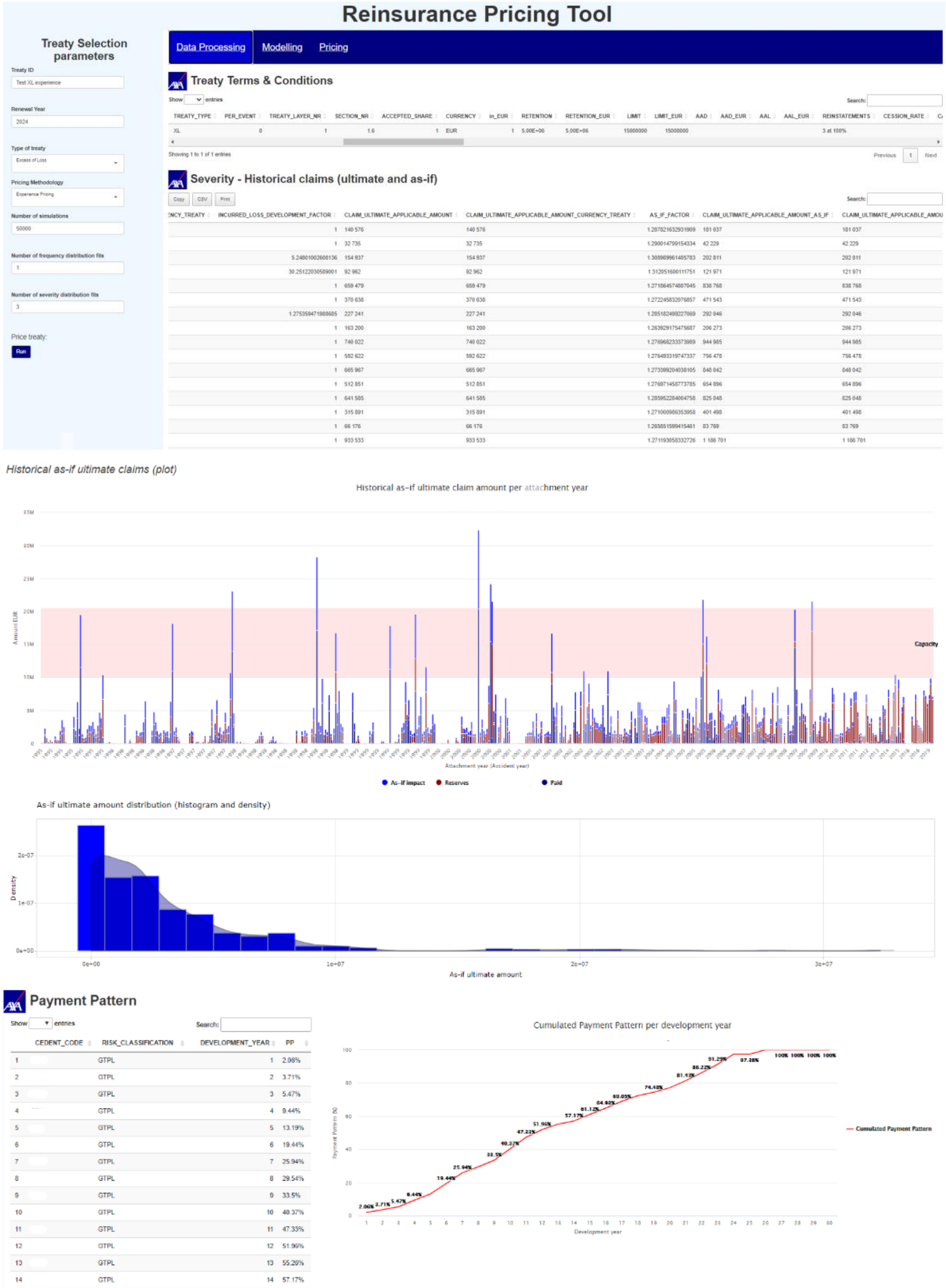


Figure 79: Class architecture

To simplify user interaction, a dashboard built with R Shiny has also been developed. Initially, information about the treaty must be provided: treaty ID, the number of desired simulations, and the number of statistical models (among the "best" fits) to consider for reinsurance modeling and pricing purpose as well as for risk and profitability indicators production. All graphs within the tool are interactive and were developed using the *highcharter* and *plotply* packages available on R, enabling the user to explore results effectively.

Once the calculations are initiated and completed, the user can access three panels:

1. **Data Processing Panel**: This panel displays historical data and outputs from the data processing part. In this panel, one can visualize and download different data tables such as raw historical (individual and aggregated) claims, historical exposure amounts and their evolution, as well as development triangles, claim counts and their ultimate as-if amounts. The treaty's TC are also visible in this panel (see Databases description and Data processing). The following figure provides an overview the processing panel for different treaties.


Reinsurance Pricing Tool
Treaty Selection parameters
Treaty ID
Test XL experience
Renewal Year
2024
Type of treaty
Excess of Loss
Pricing Methodology
Experience Pricing
Number of simulations
50000
Number of frequency distribution fits
1
Number of severity distribution fits
3
Price treaty:
Run
Data Processing
Modelling
Pricing
Treaty Terms & Conditions
Severity - Historical claims (ultimate and as-if)
Historical as-if ultimate claims (plot)
Historical as-if ultimate claim amount per attachment year
Capacity
Attachment year (Accident year)
As-if impact
Reserves
Paid
As-if ultimate amount distribution (histogram and density)
Density
As-if ultimate amount
Payment Pattern
Show
entries
Search:
CEDENT_CODE
RISK_CLASSIFICATION
DEVELOPMENT_YEAR
PP
1 GTPL 1 2.06%
2 GTPL 2 3.71%
3 GTPL 3 5.47%
4 GTPL 4 9.44%
5 GTPL 5 13.19%
6 GTPL 6 19.44%
7 GTPL 7 25.94%
8 GTPL 8 29.54%
9 GTPL 9 33.5%
10 GTPL 10 40.37%
11 GTPL 11 47.33%
12 GTPL 12 51.96%
13 GTPL 13 55.26%
14 GTPL 14 57.17%
Cumulated Payment Pattern per development year
Cumulated Payment Pattern
Development year

Frequency

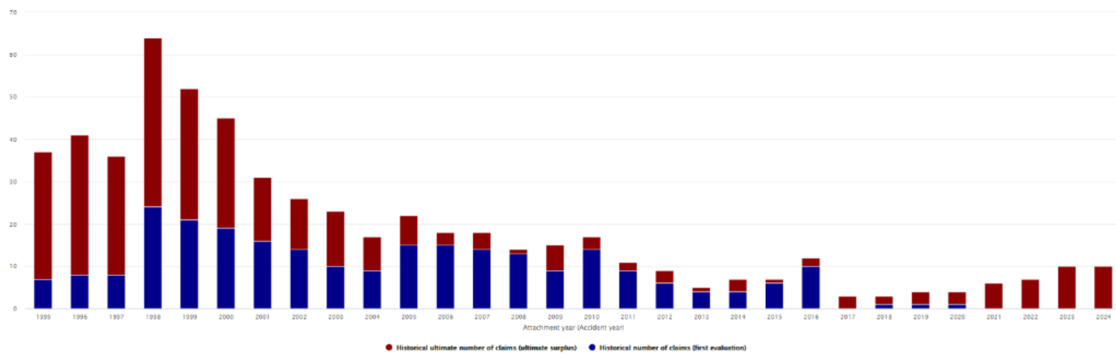
Frequency distribution moments

Show 10 entries　　Search:

	CEDENT_CODE	RISK_CLASSIFICATION	LAMBDA	MEAN	VAR
1		GTPL	2.64806788169461e-8	18.4325405782949	263.930652425277

Showing 1 to 1 of 1 entries　　Previous 1 Next

Historical claims number (Plot)



Triangle not developped

Show 10 entries　　Search:

	ATTACHMENT_YEAR	ATTACHMENT_YEAR_TYPE	CEDENT_CODE	RISK_CLASSIFICATION	NB_CLAIMS_LAST_EVALUATION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1995	Accident Year		GTPL	37	7	8	11	15	19	20	25	29	33	36	36	37	37	37	37	37	37	37	37	37	37
2	1996	Accident Year		GTPL	41	8	13	21	24	26	35	35	36	40	40	40	40	40	40	41	41	41	41	41	41	41
3	1997	Accident Year		GTPL	36	8	16	20	22	28	33	34	34	34	35	36	36	36	36	36	36	36	36	36	36	36
4	1998	Accident Year		GTPL	64	24	45	50	55	59	61	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
5	1999	Accident Year		GTPL	52	21	33	39	42	47	51	51	51	51	51	52	52	52	52	52	52	52	52	52	52	52
6	2000	Accident Year		GTPL	45	19	33	36	40	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
7	2001	Accident Year		GTPL	31	16	21	25	29	29	30	30	30	30	30	31	31	31	31	31	31	31	31	31	31	31
8	2002	Accident Year		GTPL	26	14	20	23	23	23	24	24	24	24	24	24	24	24	24	24	24	24	25	25	25	25
9	2003	Accident Year		GTPL	23	10	20	21	21	21	21	21	21	21	21	21	21	21	22	22	22	22	23	23	23	23
10	2004	Accident Year		GTPL	17	9	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	17

Showing 1 to 10 of 30 entries　　Previous 1 2 3 Next

Exposure and development factors

Show 10 entries　　Search:

	ATTACHMENT_YEAR	CEDENT_CODE	RISK_CLASSIFICATION	VALUE	VALUE_TYPE
1	1995		GTPL		GEP
2	1996		GTPL		GEP
3	1997		GTPL		GEP
4	1998		GTPL		GEP
5	1999		GTPL		GEP
6	2000		GTPL		GEP
7	2001		GTPL		GEP
8	2002		GTPL		GEP
9	2003		GTPL		GEP
10	2004		GTPL		GEP

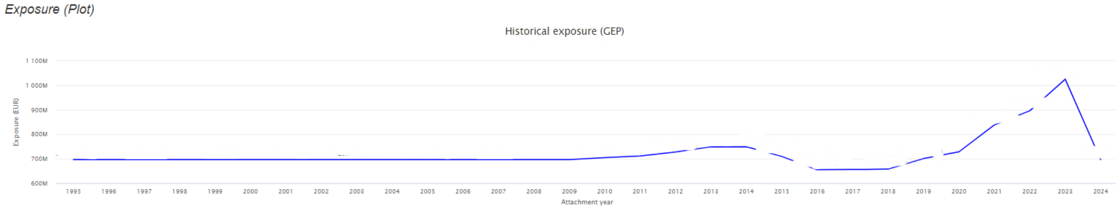
Showing 1 to 10 of 30 entries　　Previous 1 2 3 Next

Show 10 entries　　Search:

	CEDENT_CODE	RISK_CLASSIFICATION	DEVELOPMENT_YEAR	LAMBDA
1		GTPL	1	1.19207584686806e-8
2		GTPL	2	5.01268852618512e-9
3		GTPL	3	1.95764271786458e-9
4		GTPL	4	1.47167403352845e-9
5		GTPL	5	1.5944748127711e-9
6		GTPL	6	1.31732007702306e-9
7		GTPL	7	5.37017266614212e-10
8		GTPL	8	3.72630387334971e-10
9		GTPL	9	5.17942767730699e-10
10		GTPL	10	3.38040000339442e-10

Showing 1 to 10 of 30 entries　　Previous 1 2 3 Next

Exposure (Plot)



Triangle developped

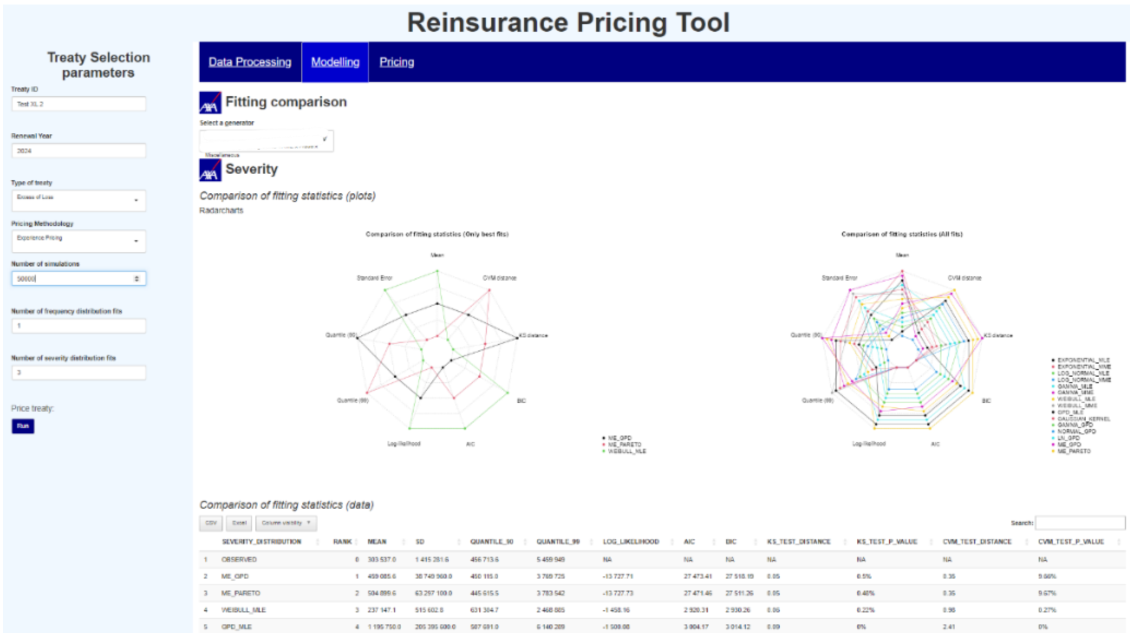
Show 10 entries　　Search:

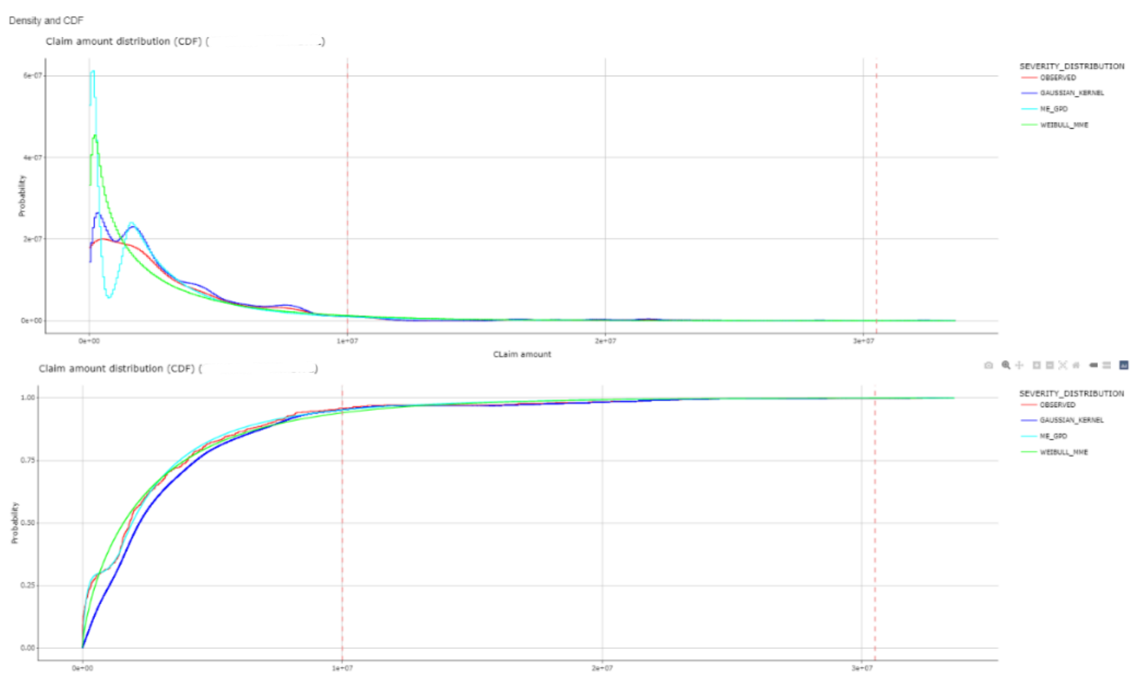
	ATTACHMENT_YEAR	ATTACHMENT_YEAR_TYPE	CEDENT_CODE	RISK_CLASSIFICATION	NB_CLAIMS_LAST_EVALUATION	NB_CLAIMS_ULTIMATE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	1995	Accident Year		GTPL	37	37	7	8	11	15	19	20	25	29	33	36	36	37	37	37	37	37	37
2	1996	Accident Year		GTPL	41	41	8	13	21	24	26	35	35	36	40	40	40	40	40	40	41	41	41
3	1997	Accident Year		GTPL	36	36	8	16	20	22	28	33	34	34	34	35	36	36	36	36	36	36	36
4	1998	Accident Year		GTPL	64	64	24	45	50	55	59	61	64	64	64	64	64	64	64	64	64	64	64
5	1999	Accident Year		GTPL	52	52	21	33	39	42	47	51	51	51	51	51	52	52	52	52	52	52	52
6	2000	Accident Year		GTPL	45	45	19	33	36	40	45	45	45	45	45	45	45	45	45	45	45	45	45
7	2001	Accident Year		GTPL	31	31	16	21	25	29	29	30	30	30	30	30	31	31	31	31	31	31	31
8	2002	Accident Year		GTPL	26	26	14	20	23	23	23	24	24	24	24	24	24	24	24	24	24	24	24
9	2003	Accident Year		GTPL	23	23	10	20	21	21	21	21	21	21	21	21	21	21	21	22	22	22	22
10	2004	Accident Year		GTPL	17	17.1111111111111	9	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

Showing 1 to 10 of 30 entries　　Previous 1 2 3 Next

Figure 80: Reinsurance pricing tool – Processing panel

2. **Modeling Panel**: It provides outputs from the loss modeling part. In this part, one can visualize every fitted distribution parameters and their corresponding rank, with the metrics used to elaborate this ranking. A visual representation of these metrics rank is displayed with spider web graphs. Furthermore, one can also explore various graphs such as the CDF and density plots, QQ plot (general and the one used for extreme value theory), simulated recoveries and loss ratios, exposure curves, and other relevant metrics/plots (see Modeling losses). The data tables can also be downloaded by the tool's user.





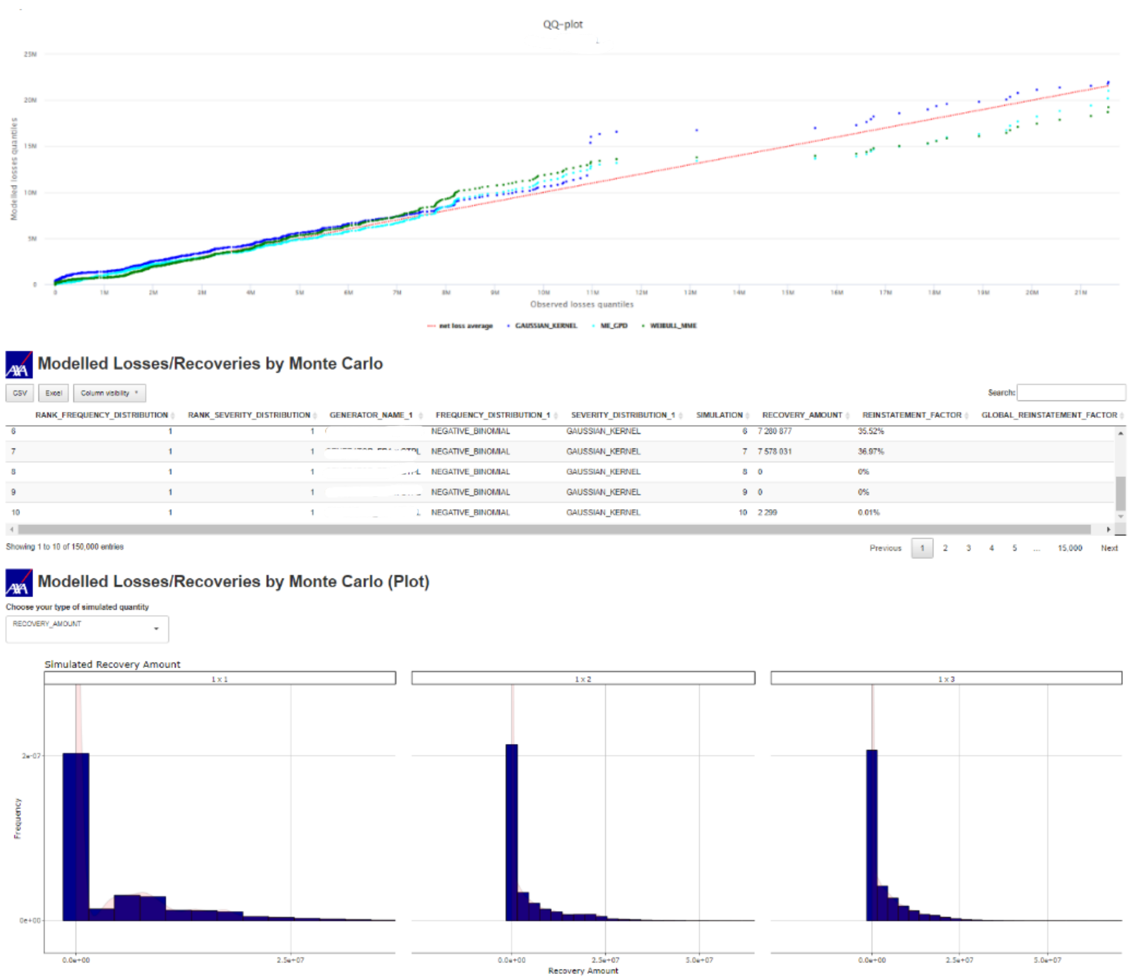
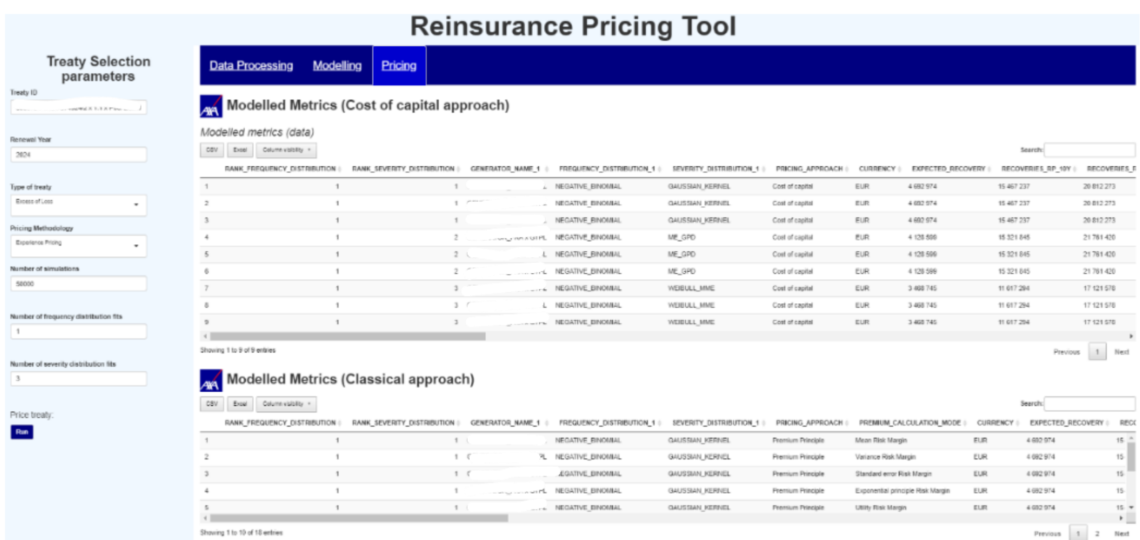


Figure 81: Reinsurance pricing tool – Modeling panel

3. **Pricing Panel**: The final panel presents the different indicators calculated, such as the technical premium, the treaty's cost of capital, loss ratios, commissions, etc. These indicators are presented through a global downloadable data table, with options for the user to interactively select and plot specific indicators (see Pricing indicators).



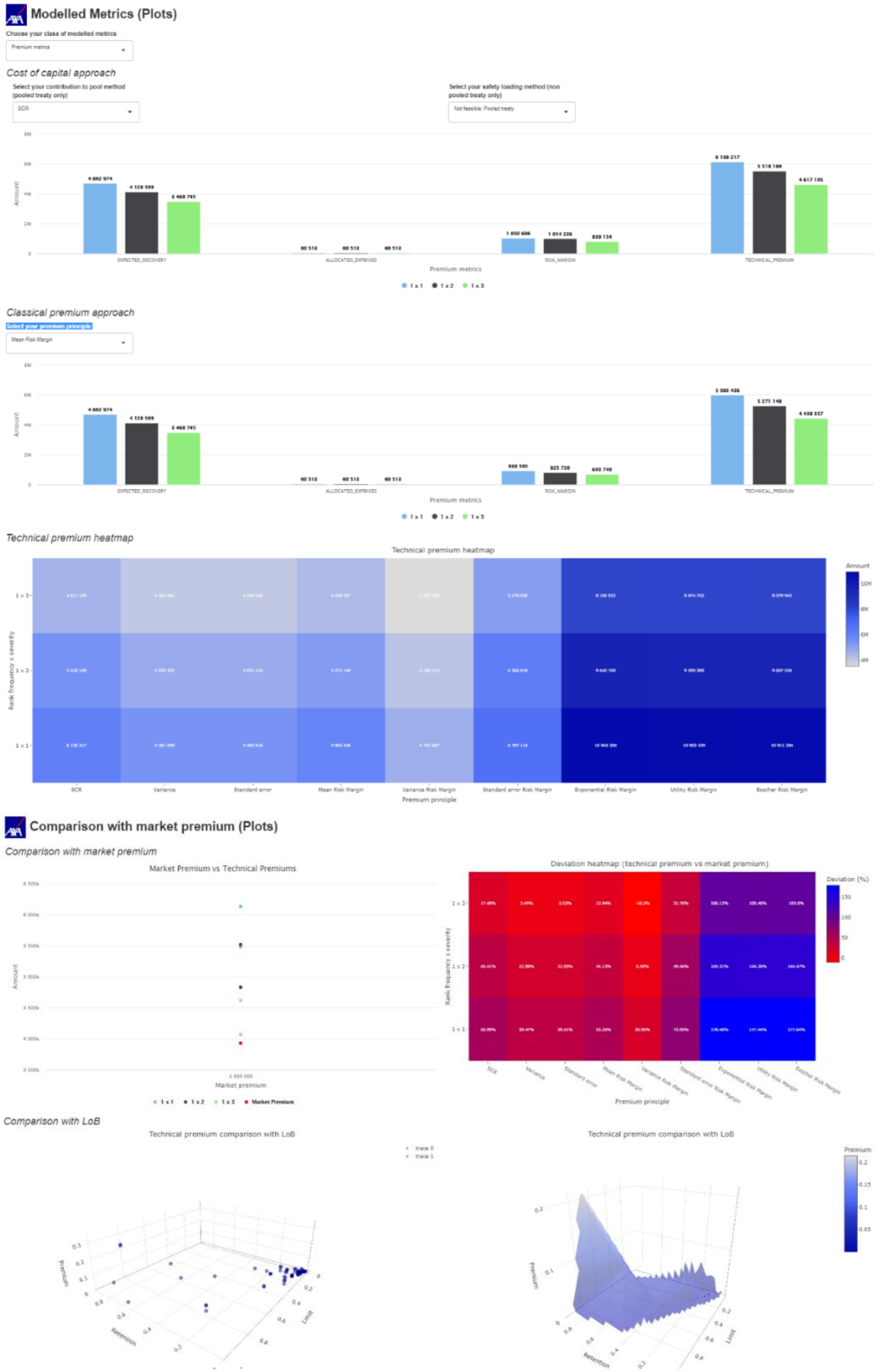


Figure 82: Reinsurance pricing tool – Pricing panel

Conclusion

The main purpose of this actuarial thesis was to establish a pricing methodology based on AGCRe's internal data, independent from AXA Group guidelines, to evaluate the risk related to most underwritten and renewed reinsurance treaties characteristics. According to the formulated problematic, this process also needed to be standardized and automated in order to appraise a wide range of reinsurance treaties, including those not currently modeled with the internal model. The methodology should also be capable of assessing both existing and newly underwritten treaties, enabling AGCRe to fully control their modeling process and therefore facilitating development prospects.

Throughout this actuarial project, we focused primarily on pricing treaties that are partly modeled within the internal model such as XL per risk treaties or QS treaties to an even lesser extent, and those not modeled such as SP and AGG treaties. We developed a structured approach that started with data collection and standard formatting application in accordance with each modeling methodology This involved compiling the necessary data sets, such as historical claims, exposure data, pricing data, risk profiles, etc. For both individual and aggregated claims, an initial step in estimating ultimate loss amounts has been integrated within the tool. This opens the door for future improvements in collaboration with the reserving team, as estimating ultimate losses is usually not the responsibility of the Analytics and Pricing team. A Schnieper model, distinct from the classic Chain Ladder approach, was also implemented to estimate the ultimate number of claims, a key variable for modeling XL per risk treaties.

Due to its flexibility in considering the terms and conditions of the different treaties, the Monte Carlo method was applied to approximate the probability distributions of the underlying gross losses and corresponding reinsured amounts under any considered reinsurance treaties and derived from those simulated distributions a wide range of risk and profitability metrics. Concerning AGG and QS treaties, loss ratios were modeled using historical data, incorporating loss development based on the Chain Ladder model to estimate ultimate loss ratios and effectively assess the actual loss borne by the reinsurer through various continuous probability distributions. Regarding XL per risk treaties, a frequency-severity approach has been developed to simulate the distribution of claims. Based on the frequency distribution moments estimated using the Schnieper model and the ultimate as-if historical amounts, various frequency and severity distributions could be fitted. These distributions, commonly used in reinsurance for claims modeling, were taken into consideration because of their statistical properties, based on extreme value theory. Additionally, an exposure curve approach for XL and SP property treaties has also been implemented. It provides a point of comparison for XL treaties, and a second method in case where the frequency-severity approach fails to properly estimate losses. Using Monte Carlo simulations, cedent loss distributions were derived into reinsurer's losses by applying the treaty terms and conditions for each type of treaty.

These simulations were finally wielded to derive various indicators such as return periods, technical premiums, and treaty margins. Cost of capital considerations based on the Solvency II framework were also integrated with different methodologies to establish the safety loading for the technical premium of non-proportional treaties, and for information purposes regarding QS and SP treaties.

The methodology was finally tested on XL per risk treaties by comparing the computed technical premium with market premiums. In the analyzed sample, the technical premium is underestimated by an average of around 13% for non-pooled treaties, while its calculation seems more reliable for pooled treaties.

Ultimately, our solution to the problem addressed in this actuarial thesis resulted in the development of a pricing tool for AGCRe's Analytics & Pricing team, which encapsulates and articulates automatically in a consistent way, all the data processing, modeling methodologies, and pricing techniques developed throughout this document. The tool improves the team's ability to price a broader spectrum of treaties based on internal data, including previously unmodeled XL per risk treaties (essentially non pooled treaties), as well as QS, SP, and AGG treaties. Its structure allows for the addition of new data coming from ceding entities as well as the integration of future complementary models. Furthermore, the inclusion of interactive graphical outputs within the tool's R Shiny user interface, along with the display of distributions parameters and intermediate data frames, improves transparency and auditability together with the user experience throughout the reinsurance risks assessment process.

Despite certain limitations arising from methodological choices or data insufficiencies, which still restrict the number of treaties that can be modeled, the tool has increased the scope of priced treaties and established a foundation for an internal tool that aspires to be refined over time.

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Appendix

Appendix A: Frequency distributions

Frequency distribution	Properties
Poisson distribution	**Probability distribution**: For all $k \in \mathbb{N}$, $\mathbb{P}(N = k) = e^{-\lambda}\frac{\lambda^k}{k!}$ **Mean**: $\mathbb{E}(N) = \lambda$ **Variance**: $Var(N) = \lambda$
Negative Binomial distribution	**Probability distribution**: For all $k \in \mathbb{N}$, $\mathbb{P}(N = k) = \frac{\Gamma(k+n)}{k!\Gamma(n)}p^n(1-p)^k$ where $\Gamma(n) = (n-1)!$ denotes the Euler function. **Mean**: $\mathbb{E}(N) = \frac{n(1-p)}{p}$ **Variance**: $Var(N) = \frac{n(1-p)}{p^2}$
Binomial distribution	**Probability distribution**: For all $k \in [0; n]$, $\mathbb{P}(N = k) = \binom{n}{k}p^k(1-p)^{n-k}$. **Mean**: $\mathbb{E}(N) = np$ **Variance**: $Var(N) = np(1-p)$

Appendix B: Continuous distributions

Exponential distribution

Let $\lambda > 0$. We list below the various properties of the exponential distribution with parameter λ noted X:

- Probability density: For all $t \in \mathbb{R}^+$, $f(t) = 1_{t\geq 0}\lambda \exp(-\lambda t)$
- Mean: $\mathbb{E}(X) = \frac{1}{\lambda}$
- Variance: $Var(X) = \frac{1}{\lambda^2}$
- Cumulative Distribution Function (CDF): $F(t) = \begin{cases} 1 - e^{-\lambda t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

Log-Normal distribution

The Log-Normal distribution is a continuous probability distribution with strictly positive real values. A random variable X follows a Log-Normal distribution with parameters μ and σ^2 if the random variable $\ln(X)$ follows a Normal distribution with parameters μ and σ^2.

Let $\mu \in \mathbb{R}$ and $\sigma^2 > 0$. We list below the various properties of the random variable X which we assume to follow a Log-Normal distribution with parameters μ and σ^2:

- Probability density: For all $t \in \mathbb{R}^{+*}$, $f(t) = \frac{1}{t\sigma\sqrt{2\pi}}\exp(-\frac{(\ln(t)-\mu)^2}{2\sigma^2})$
- Mean: $\mathbb{E}(X) = \exp(\mu + \frac{\sigma^2}{2})$
- Variance: $Var(X) = (\exp(\sigma^2) - 1) * \exp(2\mu + \sigma^2)$

Gamma distribution

Gamma distribution is a type of probability distribution for positive real random variables. A Gamma distribution is characterized by two parameters k and θ which are respectively designed by the shape and scale of the gamma distribution.

Let $k > 0$ and $\theta > 0$. We list below the various properties of the random variable X which we assume to follow a gamma distribution with parameters k and θ:

- Probability density: For all $t \in \mathbb{R}^{+*}$, $f(t) = \frac{t^{k-1}\exp(-\frac{t}{\theta})}{\Gamma(k)\theta^k}$
- Mean: $\mathbb{E}(X) = k\theta$
- Variance: $Var(X) = k\theta^2$

Generalized Pareto Distribution

The Generalized Pareto Distribution (GPD) is a family of continuous probability distributions. It can be specified by three parameters: location μ, scale σ and shape γ.

Let $\sigma > 0$ and $\gamma \in \mathbb{R}$ We list below the various properties of the random variable X which we assume to follow a GPD of parameters σ and γ:

- Density function: For $t \geq 0$ when $\gamma \geq 0$ and for $0 \leq t \leq -\frac{1}{\gamma}$ when $\gamma < 0$

$$f(t) = \begin{cases} \frac{1}{\sigma}\left(1+\gamma(\frac{t}{\sigma})\right)^{(-\frac{1}{\gamma}-1)}, & \gamma \neq 0 \\ \frac{1}{\sigma}\exp(-\frac{t}{\sigma}), & \gamma = 0 \end{cases}$$

- Mean: If $\gamma < 1$, $\mathbb{E}(X) = \frac{\sigma}{1-\gamma}$
- Variance: If $\gamma < 0.5$, $Var(X) = \frac{\sigma^2}{(1-\gamma)^2(1-2\gamma)}$
- Cumulative Distribution Function (CDF): For $t \geq 0$ when $\gamma \geq 0$ and for $0 \leq t \leq -\frac{\sigma}{\gamma}$ when $\gamma < 0$

$$F(t) = GPD_{\sigma,\gamma}(t) = \begin{cases} 1-\left(1+\gamma(\frac{t}{\sigma})\right)^{-\frac{1}{\gamma}}, & \gamma \neq 0 \\ 1-\exp(-(\frac{t}{\sigma})), & \gamma = 0 \end{cases}$$

Weibull distribution

The Weibull distribution is a family of continuous probability distributions. It is defined by one parameter α.

Consider $\alpha > 0$ and $\lambda > 0$. We list below the various properties of the random variable X which we assume to follow a Weibull distribution of parameter α:

- Density function: For $t \geq 0$, $f(t) = \begin{cases} \alpha\lambda t^{\alpha-1}\exp(-\lambda t^\alpha), & t \geq 0 \\ 0, & t < 0 \end{cases}$
- Mean: $\mathbb{E}(X) = \frac{1}{\lambda} * \Gamma(1+\frac{1}{\alpha})$
- Variance: $Var(X) = \frac{1}{\lambda^2} * \Gamma\left(1+\frac{2}{\alpha}\right) - \frac{1}{\lambda^2} * \Gamma\left(1+\frac{1}{\alpha}\right)^2$
- Cumulative Distribution Function (CDF): For $t \in \mathbb{R}$

$$F(t) = \begin{cases} 1-\exp(-\lambda t^\alpha), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Fréchet distribution

The Fréchet distribution is specified by one parameter α.

Consider $\alpha > 0$. We list below the various properties of the random variable X which we assume to follow a Fréchet distribution of parameter α:

- Density function: For $t \geq 0$ $f(t) = \begin{cases} \alpha t^{-\alpha-1} \exp(-t^{-\alpha}), & t \geq 0 \\ 0, & t < 0 \end{cases}$
- Mean: $\mathbb{E}(X) = \Gamma\left(1 - \frac{1}{\alpha}\right)$ if $\alpha > 0$, ∞ otherwise
- Variance: $Var(X) = \Gamma\left(1 - \frac{2}{\alpha}\right) - \Gamma\left(1 - \frac{1}{\alpha}\right)^2$ if $\alpha > 2$, ∞ otherwise
- Cumulative Distribution Function (CDF): For $t \in \mathbb{R}$

$$F(t) = \begin{cases} \exp(-t^{-\alpha}), & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Gaussian Kernel

Unlike previous distributions, kernel estimation is a non-parametric method used for estimating the density of a random variable. Taking a vector of observations $(x_i)_{1 \leq i \leq n}$ assumed to be an iid sample of a random variable, the non-parametric estimator of the density function of the random variable is:

$$\hat{f}(x) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)$$

Where:

- h is a real number corresponding to a smoothing parameter
- K is a kernel, equal in our case to the density of a standard normal distribution

Pareto distribution

The Pareto distribution is specified by one parameter γ and a threshold u.

Consider $\gamma > 0$ and $u > 0$. We list below the various properties of the random variable X which we assume to follow a Pareto distribution:

- Density function: For $t \geq 0$ $f(t) = \begin{cases} \frac{1}{\gamma u}\left(\frac{t}{u}\right)^{-\frac{1}{\gamma}-1}, & t \geq u \\ 0, & t < u \end{cases}$
- Mean: $\mathbb{E}(X) = \frac{u}{1-\gamma}$ if $\gamma < 1$, ∞ otherwise
- Variance: $Var(X) = \frac{\frac{u^2}{\gamma}}{\left(1-\frac{1}{\gamma}\right)^2\left(\frac{1}{\gamma}-2\right)}$ if $\gamma < \frac{1}{2}$, ∞ otherwise
- Cumulative Distribution Function (CDF): For $t \in \mathbb{R}$

$$F(t) = \begin{cases} 1 - \left(\frac{t}{u}\right)^{-\frac{1}{\gamma}}, & t \geq u \\ 0, & t < u \end{cases}$$

Appendix C: Maximum Likelihood Estimation

Maximum Likelihood Estimation (MLE) is a statistical method used to estimate the parameters of a statistical model from observed data. The likelihood of the parameter vector θ given the observed data $(X_i)_{1 \le i \le n}$ is defined by the function $L(.)$:

$$L(\theta) = \prod_{i=1}^{n} f(X_i; \theta)$$

The log-likelihood is often preferred over the likelihood. This function is defined by $l(.)$ function:

$$l(\theta) = \log\big(L(\theta)\big) = \sum_{i=1}^{n} \log(f(X_i; \theta))$$

The maximum likelihood estimator (MLE) of θ which is denoted as $\hat{\theta}$ is the solution of the following equation:

$$\sup_{\theta} \; l(\theta) = l(\hat{\theta})$$

For the considered probability distributions, the function $l(.)$ is differentiable. Thus, if the first derivative of $l(\theta)$ equals zero at $\theta = \hat{\theta}$ and the second derivative is strictly negative at $\theta = \hat{\theta}$, then $\hat{\theta}$ is a local maximum of $l(.)$. If $\hat{\theta}$ satisfies these two conditions, $\hat{\theta}$ is a maximum likelihood estimator.

Regarding its properties, the estimator obtained by MLE is:

- Consistent: MLE estimators converge in probability to the true parameter θ
- Asymptotically normal: The MLE estimator is asymptotically distributed according to a normal distribution. This property of the estimator makes it possible to construct confidence intervals C_n with probability $1 - \alpha$ with $C_n = (\hat{\theta} - \phi^{-1}\left(1 - \frac{\alpha}{2}\right) * \widehat{\sigma_{\hat{\theta}}}, \hat{\theta} + \phi^{-1}\left(1 - \frac{\alpha}{2}\right) * \widehat{\sigma_{\hat{\theta}}})$ where $\phi^{-1}\left(1 - \frac{\alpha}{2}\right)$ denotes the quantile of order $1 - \frac{\alpha}{2}$ of the Normal distribution and $\widehat{\sigma_{\hat{\theta}}}$ the estimated standard deviation of $\hat{\theta}$.
- Asymptotically efficient: MLEs reach the Cramer-Rao bound, which means they have the lowest variance among all possible unbiased estimators, for large sample sizes.

Let us take an example to illustrate the application of maximum likelihood estimation. Suppose that the realizations $(X_i)_{1 \le i \le n}$ are distributed according to an exponential distribution with parameters λ. In our case, the log likelihood is equal to $l(\lambda) = -n * \ln(\lambda) - \lambda \sum_{i=1}^{n} \ln(X_i)$. This function is differentiable, and its derivative is $l'(\lambda) = \frac{n}{\lambda} - \sum_{i=1}^{n} \ln(X_i)$. We can easily check that the second derivative is negative at any point. Consequently, the maximum likelihood estimator is $\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} \ln(X_i)$.

Appendix D: Distribution parameters of XL treaty number 1

Frequency distribution parameters:

LAMBDA	double [1]	2.550414e-07
MEAN	double [1]	13.87425
VAR	double [1]	58.35193

POISSON	list [4] (S3: fitdist)	List of length 4
estimate	double [1]	13.87425
lambda	double [1]	13.87425
method	character [1]	'mme'
data	character [1]	'NUMBER_OF_CLAIMS'
distname	character [1]	'pois'
NEGATIVE_BI...	list [4] (S3: fitdist)	List of length 4
estimate	double [2]	4.328 0.238
size	double [1]	4.327898
prob	double [1]	0.2377685
method	character [1]	'mme'
data	character [1]	'NUMBER_OF_CLAIMS'
distname	character [1]	'nbinom'

Gamma distribution

GAMMA_MME	list [17] (S3: fitdist)	List of length 17
estimate	double [2]	1.75 5.15
shape	double [1]	1.750855
rate	double [1]	5.151245
method	character [1]	'mme'

ME – GPD distribution

ME_GPD	list [9] (S3: SpliceFit)	List of length 9
const	double [1]	0.9486607
pi	double [2]	0.9487 0.0513
trunclower	double [1]	0
t	double [1]	838379.7
type	character [2]	'ME' 'GPD'
MEfit	list [5] (S3: MEfit)	List of length 5
p	double [2]	0.094 0.906
shape	double [2]	1 4
theta	double [1]	81295.86
M	integer [1]	2
M_initial	double [1]	3
EVTfit	list [3] (S3: EVTfit)	List of length 3
gamma	double [1]	0.09829782
sigma	double [1]	257909.7
endpoint	double [1]	Inf
loglikelihood	double [1]	-6085.509

ME – Pareto distribution:

ME_PARETO	list [9] (S3: SpliceFit)	List of length 9
const	double [1]	0.9486607
pi	double [2]	0.9487 0.0513
trunclower	double [1]	0
t	double [1]	838379.7
type	character [2]	'ME' 'Pa'
MEfit	list [5] (S3: MEfit)	List of length 5
p	double [2]	0.094 0.906
shape	double [2]	1 4
theta	double [1]	81295.86
M	integer [1]	2
M_initial	double [1]	3
EVTfit	list [2] (S3: EVTfit)	List of length 2
gamma	double [1]	0.2633786
endpoint	double [1]	Inf
loglikelihood	double [1]	-6085.716

Appendix E: Distribution ranking of XL treaty number 1

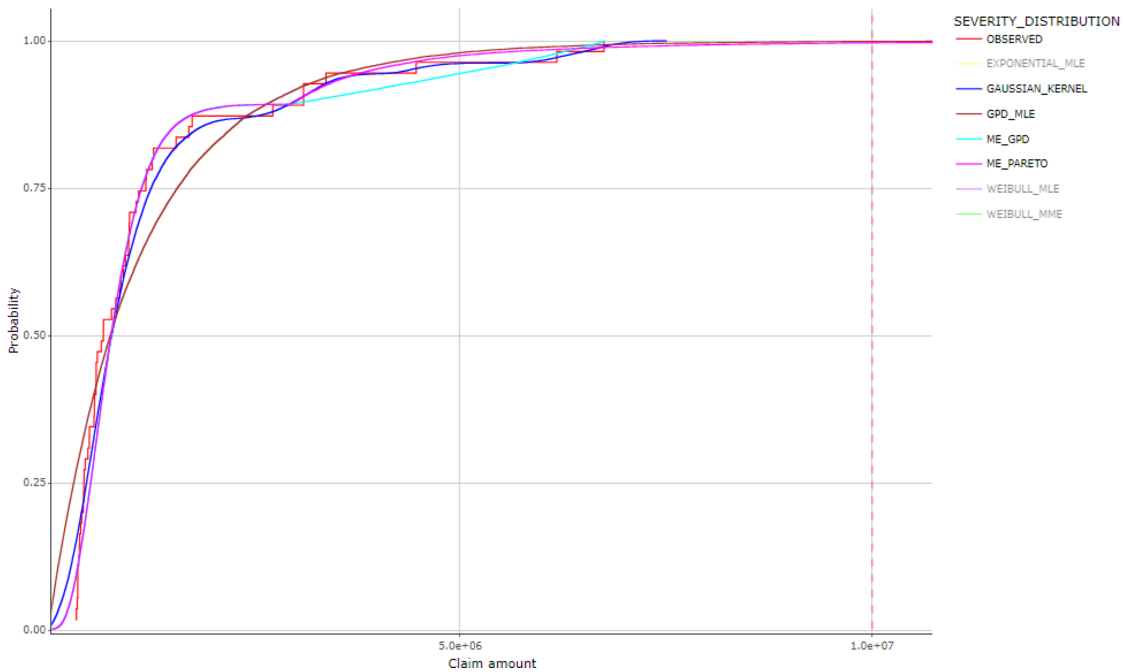
Distribution	*Global rank index*	Rank
GAUSSIAN_KERNEL	5,80	1
ME_GPD	11,17	2
ME_PARETO	15,17	3
GAMMA_MME	21,60	4
WEIBULL_MME	32,40	5
GAMMA_MLE	42,33	6
WEIBULL_MLE	43,67	7
GAMMA_GPD	78,83	8
NORMAL_GPD	95,83	9
GPD_MLE	96,67	10
KERNEL_GPD	104,40	11
WEIBULL_GPD	116,17	12
EXPONENTIAL_MLE	152,83	13
EXPONENTIAL_MME	166,40	14
LN_GPD	200,67	15
LOG_NORMAL_MLE	234,50	16
LOG_NORMAL_MME	250,20	17
FRECHET_MLE	294,00	18
FRECHET_MME	361,00	19

Appendix F: Distribution parameters for exposure curve comparison

Fitting statistics obtained with frequency-severity approach:

Distributions	Mean	Standard deviation	Quantile (90)	Quantile (99)	Log likelihood	KS distance	KS p-value	CVM distance	CVM p-value
GAUSSIAN_KERNEL	1 172 644 €	1 355 177 €	2 964 660 €	6 721 673 €	/	0,16	0,11	0,16	0,36
ME_GPD	1 194 298 €	1 638 288 €	3 048 522 €	7 123 707 €	-80,75	0,14	0,25	0,18	0,31
ME_PARETO	1 227 042 €	1 413 615 €	3 312 736 €	6 511 733 €	-80,43	0,14	0,25	0,18	0,31
GPD (MLE)	1 171 567 €	1 302 444 €	2 729 108 €	6 146 294 €	-63,45	0,26	0	0,6	0,02

Severity CDF:



Appendix G: R packages

data.table, dplyr, tidyverse, lubridate, stringr, here, ggplot2, highcharter, shiny, DT, ggpubr, plotly, RColorBrewer, fitdistrplus, goftest, univariateML, VGAM, gamlss, ReIns, actuar, mev, evmix, ks, mbbefd, moments, zoo, akima, mco, fmsb

Appendix H: Market premium comparison

Linear regression result with quantile safety loading approach:

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
MARKET_PREMIUM  1.05059    0.02477   42.41   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 330900 on 29 degrees of freedom
Multiple R-squared:  0.9841,	Adjusted R-squared:  0.9836 
F-statistic:  1798 on 1 and 29 DF,  p-value: < 2.2e-16
```

Linear regression result with standard deviation safety loading approach:

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
MARKET_PREMIUM  0.94923    0.02019   47.02   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 269600 on 29 degrees of freedom
Multiple R-squared:  0.9871,    Adjusted R-squared:  0.9866 
F-statistic:  2211 on 1 and 29 DF,  p-value: < 2.2e-16
```

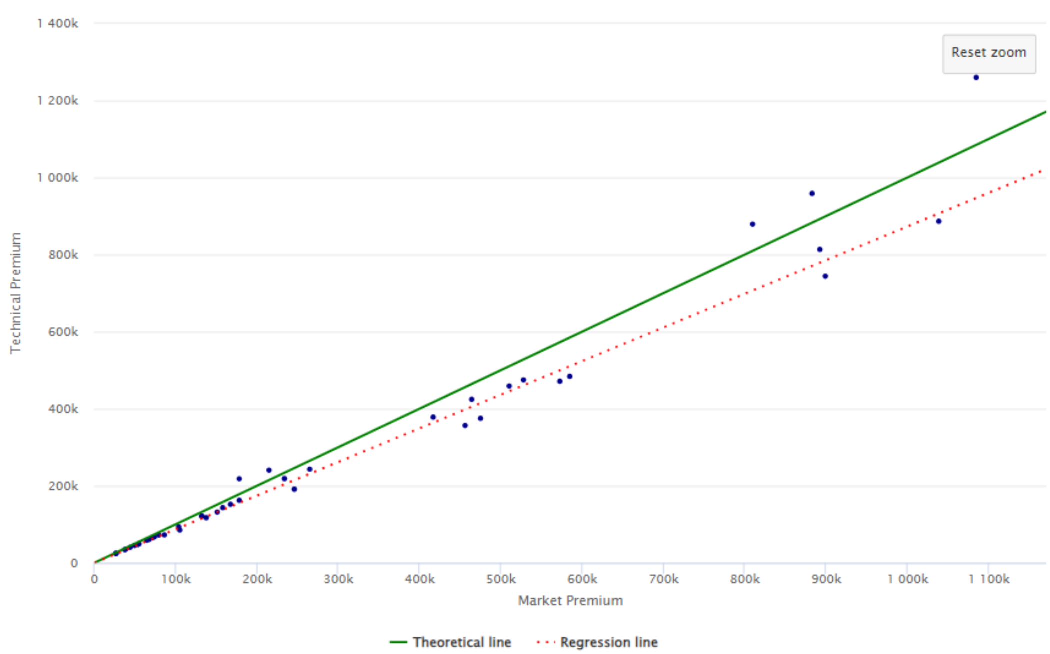
Linear regression result with variance safety loading approach:

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
MARKET_PREMIUM  0.94145    0.02013   46.78   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 268800 on 29 degrees of freedom
Multiple R-squared:  0.9869,    Adjusted R-squared:  0.9865 
F-statistic:  2188 on 1 and 29 DF,  p-value: < 2.2e-16
```

Linear regression result for non-pooled treaties:

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
MARKET_PREMIUM  0.87291    0.01722   50.69   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 817500 on 56 degrees of freedom
Multiple R-squared:  0.9787,    Adjusted R-squared:  0.9783 
F-statistic:  2569 on 1 and 56 DF,  p-value: < 2.2e-16
```

Zoom on linear regression for non-pooled treaties:



Appendix I: Risk measure definition

P is a risk measure if it satisfies the following conditions:

- Positive loading: $\mathbb{E}(R) \leq P(R)$
- No-ripoff: $P(Y) \leq Q_R(1) = \inf\{ x : F_R(x) = 1\}$
- No unjustified risk loading: $\mathbb{P}(R = b) = 1$ implies that $P(R) = b$
- Positive homogeneity: $P(aR) = aP(R)$ for any real constant $a > 0$
- Translation invariance: $P(R + b) = P(R) + b$ for any real constant b.
- Monotonicity: For two distributions R_1 and R_2, $P(R_1) \leq P(R_2)$ if $F_{R_1}(x) \geq F_{R_2}(x)$ for all x
- Subadditivity: $P(R_1 + R_2) \leq P(R_1) + P(R_2)$
- Additivity: $P(R_1 + R_2) = P(R_1) + P(R_2)$ if R_1 and R_2 are independent.
- Convexity: $P(pR_1 + (1 - p)R_2) \leq pP(R_1) + (1 - p)\,P(R_1)$ for any $0 < p < 1$.