SECTIONS VIRTUAL COLLOQUIUM | 2020

Pro-Cyclicality Beyond Business Cycles: The Case of Risk Measures

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About the speaker

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- CREAR – **C**enter of **R**esearch in **E**cono-finance and **A**ctuarial science on **R**isk
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Abstract: Adopting the point of view of risk management and regulation, there is an accepted idea that risk measurements are pro-cyclical: in times of crisis, they overestimate the future risk, while they underestimate it in quiet times. We examine two questions: How to quantify the pro-cyclicality in the way financial institutions measure risk? How to explain it? Using a methodology novel to empirical finance, we evaluate the error made on risk measurement itself, considering the market state. We introduce a new indicator conditioned to the realized volatility, which quantifies the difference between the historically predicted risk and the estimated realized future risk. We identify, empirically and theoretically, two main factors characterizing this pro-cyclical effect: the clustering and return-to-the-mean of volatility, as expected but not quantified until now, and, more surprisingly, the very way risk is measured, independently of the first factor. It means that pro-cyclicality of risk measurements exists independently of business cycles!

This is a joint work with Marcel Bräutigam (ESSEC CREAR & Sorbonne Université, LPSM) and Michel Dacorogna (Prime Re Solutions, Zug)

Introduction

- Since the introduction of **risk-based solvency regulation**, *pro-cyclicality* has been a **subject of concerns** from all market participants.

- Accepted idea: *risk measurements* made with 'regulatory' risk measures, are *pro-cyclical*:
  - in times of crisis, they overestimate the future risk
  - they underestimate it in quiet times

- Quoting Gilles Moec, Chief Economist of AXA, '*The big mistake of 2010 was to impose austerity at the worst time*' (Le Monde, 2020/01/21. Translated).

- Two questions:
  1. How to *quantify the pro-cyclicality* in the way financial institutions measure risk?
  2. How to *explain* it?

1 – How to quantify pro-cyclicality?

1.1 – Traditional risk measurement - VaR

- In financial markets, most popular risk measure: *Value-at-Risk* (VaR) defined, for a loss rv L (with cdf F), at level $\alpha \in (0,1)$, by

$$\mathrm{VaR}(\alpha) = \inf\{x \in \mathbb{R} : \mathbb{P}(L \le x) \ge \alpha\}$$

- Practically, VaR estimated on a $(L_1, \cdots, L_n)$ historical sample as an empirical quantile:

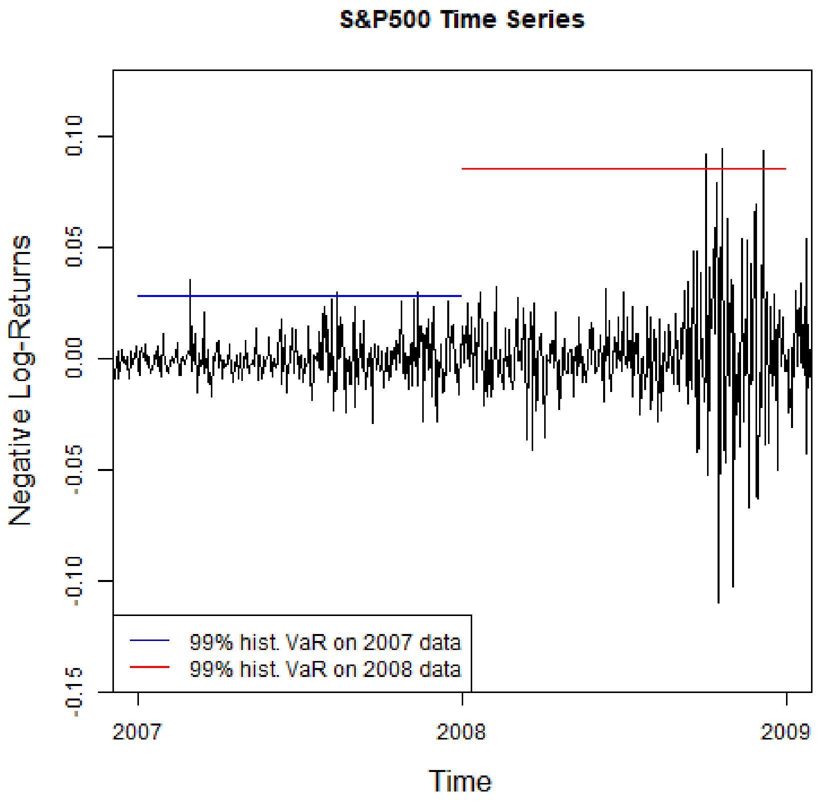
$$\widehat{\mathrm{VaR}}(L) = F_{n;L}^{-1}(\alpha) = \inf\left\{x : \frac{1}{n}\sum_{i=1}^{n} \mathbb{I}_{(L_i \le x)} \ge \alpha\right\} = L_{[n\alpha]}$$

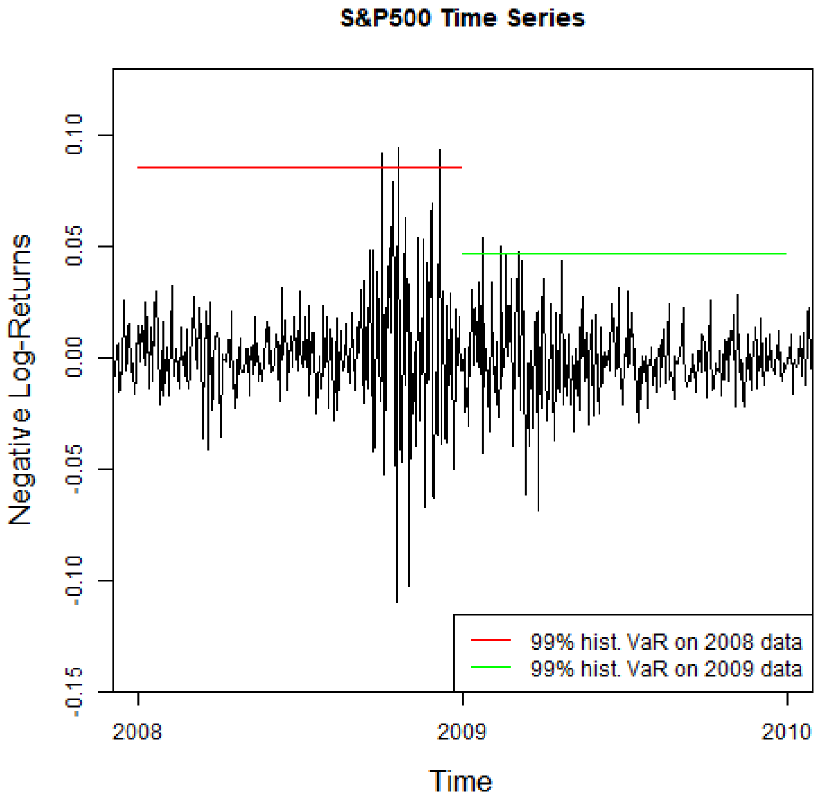
- Known pro-cyclicality of risk estimation

1 – How to quantify pro-cyclicality?

1.1 – Traditional risk measurement - VaR

- Known pro-cyclicality of risk estimation





1 – How to quantify pro-cyclicality?

1.2 – Sample Quantile Process (SQP): dynamic extension of VaR

Consider the measurement itself as a stochastic process:

- Sample Quantile Process (SQP) (Miura (92), Akahori (95), Embrechts & Samorodnitsky (95)). For $L = (L_t, t \geq 0)$, $\alpha \in (0,1)$, a fixed time frame T, and a random measure μ on $\mathbb{R}^+$ at time t:

$$Q_{T,\alpha,t}(L) = \inf\left\{x : \frac{1}{\int_{t-T}^{t} \mu(s)ds} \int_{t-T}^{t} \mathbb{I}_{(L_s \leq x)}\, d\mu(s) \geq \alpha\right\}$$

- Ex: $\mu =$ Lebesgue measure: the VaR process $Q_{T,\alpha,t}(L)$ becomes:

$$Q_{T,\alpha,t}(L) = \inf\left\{x : \frac{1}{T} \int_{t-T}^{t} \mathbb{I}_{(L_s \leq x)}\, ds \geq \alpha\right\}$$

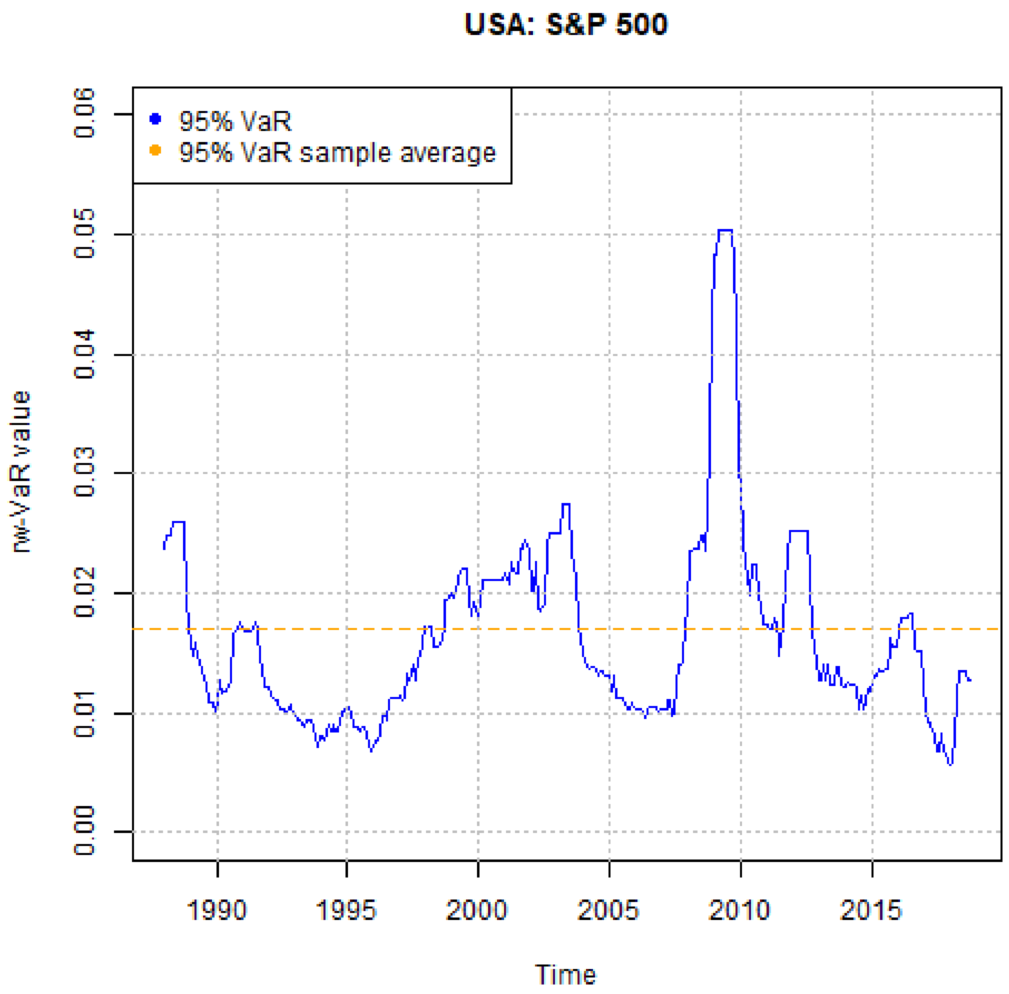
- Empirical estimator of Q: `rolling-window' empirical VaR

$$\hat{Q}_{T,\,\alpha,\,t}(L) = \widehat{\mathrm{VaR}}_{T,\,\alpha,\,t}(L) = \inf\left\{x : \frac{1}{T} \sum_{i \in (t-T,T)} \mathbb{I}_{(L_i \leq x)} \geq \alpha\right\}$$

1 – How to quantify pro-cyclicality?

Empirical study done over 11 stock indices of the major developed economies

Example of the S&P 500
T=1 year
Monthly rolling-window VaR



1 – How to quantify pro-cyclicality?

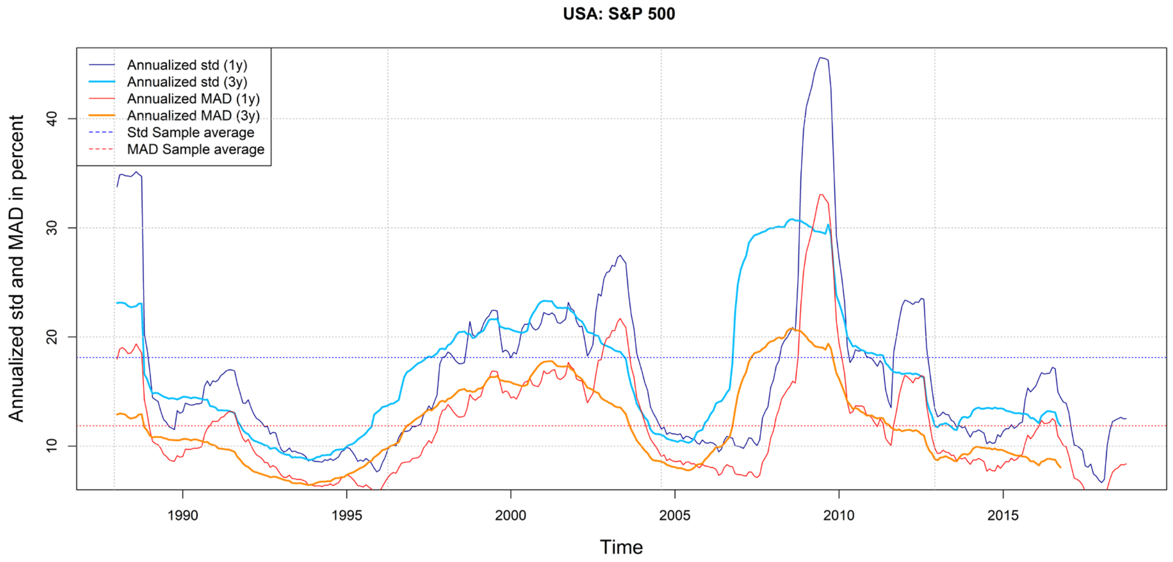
1.3 – A proxy for market states: annualized realized volatility

Estimated via:

$$v_{k,n}(t-1) := \sqrt{252} \times \left\{ \frac{1}{n-1} \sum_{i=t-n}^{t-1} \left| X_i - \frac{1}{n} \sum_{i=t-n}^{t-1} X_i \right|^k \right\}^{1/k},$$

$k = 2$: $v_{2,n} = \hat{\sigma}(t)$ is the empirical standard deviation

$k = 1$: $v_{1,n} = \hat{\theta}(t)$ is the empirical mean absolute deviation (MAD)



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1 – How to quantify pro-cyclicality?

1.3 – A look-forward ratio of SQP, conditioned on realized volatility

To quantify the predictive power of the SQP, according to the volatility state:

$$R_{T,\alpha,t} = \frac{\widehat{\mathrm{Q}}_{1,\,\alpha,\,t\,+\,1y}(L)}{\widehat{\mathrm{Q}}_{T,\,\alpha,\,t}(L)}$$

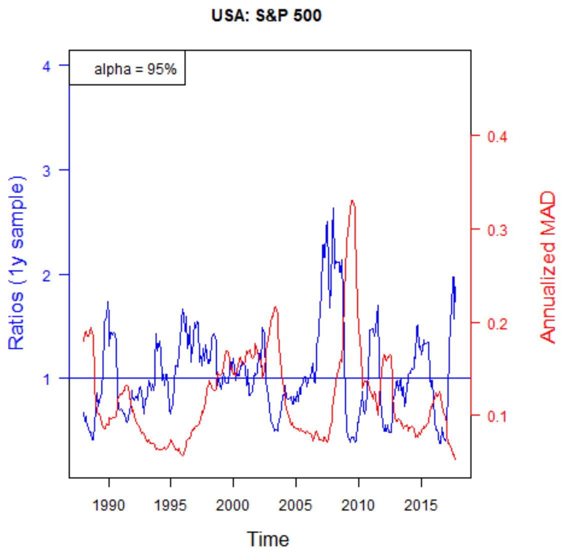
with: $\widehat{\mathrm{Q}}_{1,\,\alpha,\,t\,+\,1y}(L)$: estimated *realized* risk at time $(t+1y)$ *(a posteriori)* (empirical VaR on 1 year, as asked by regulators)

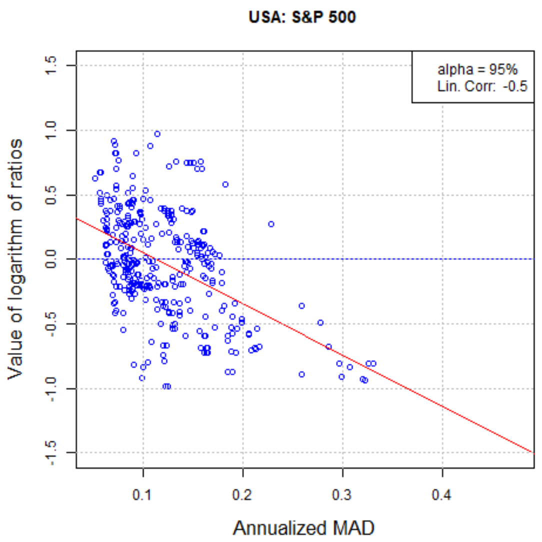
$\widehat{\mathrm{Q}}_{T,\,\alpha,\,t}(L)$ *used as a predictor* of the risk 1 year later $(t+1y)$,

- $R_{T,\alpha,t} \approx 1$: correctly assess the `future risk'
- $R_{T,\alpha,t} > 1$: under-estimation of the `future risk'
- $R_{T,\alpha,t} < 1$: over-estimation of the `future risk'

1 – How to quantify pro-cyclicality?

1.3 – Relation between Volatility and log-Ratio



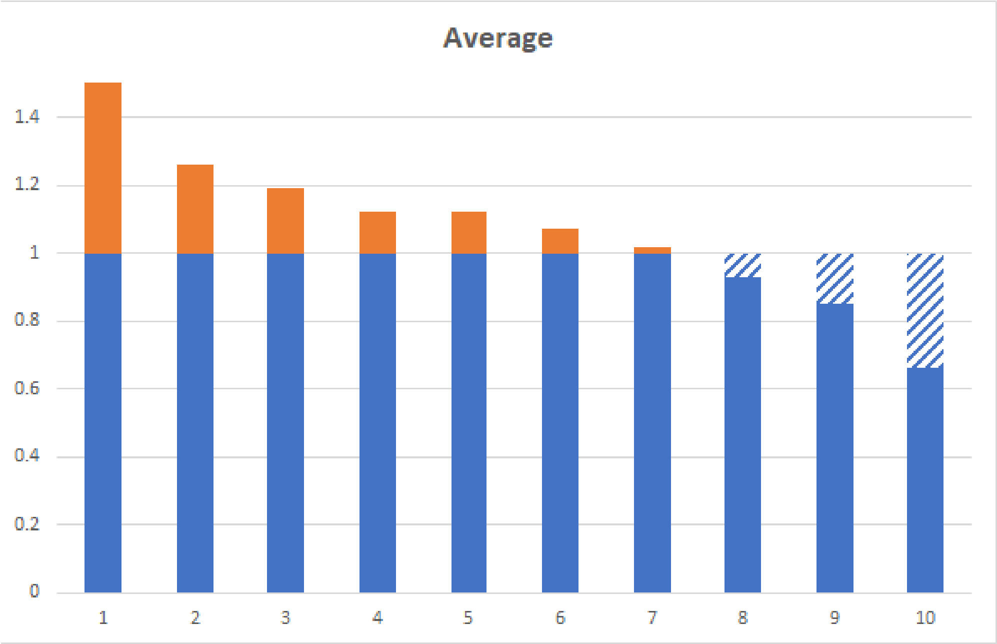


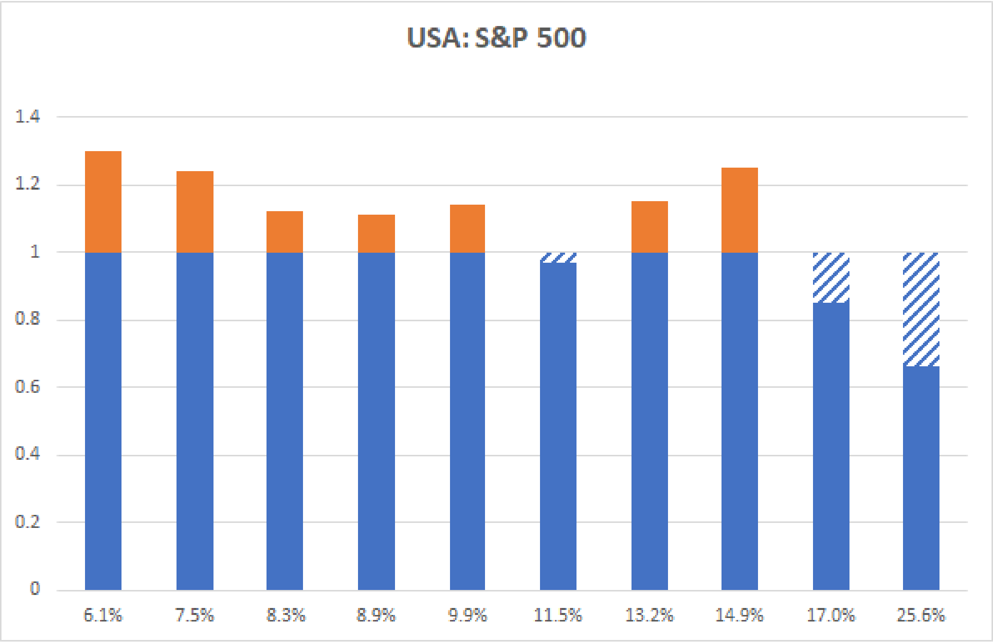
$\log(R_{T,\alpha,t})$ *negatively correlated* with annualized realized volatility:

Volatility year t	Log of SQP ratio	Meaning
Low Volatility	High Ratio (> 0)	Underestimation of Risk
High Volatility	Low Ratio (< 0)	Overestimation of Risk

1 – How to quantify pro-cyclicality?

1.4 – Quantification via volatility binning





The average SQP-ratios $R_{T,\alpha}$, with $T = 1y$ and $\alpha = 99\%$, within 10 uniform bins of volatility (in ascending order) over the whole historical sample, for the average over all 11 indices (left plot) and for the S&P 500 index (right plot).

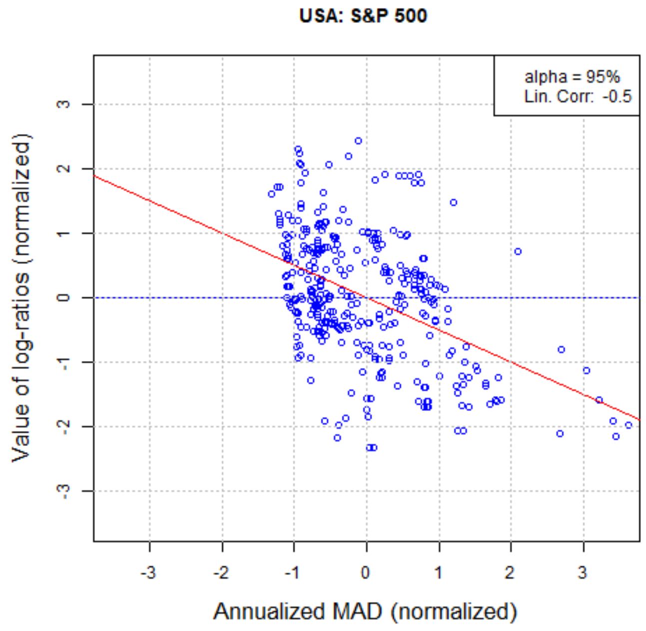
2 – How to explain pro-cyclicality?

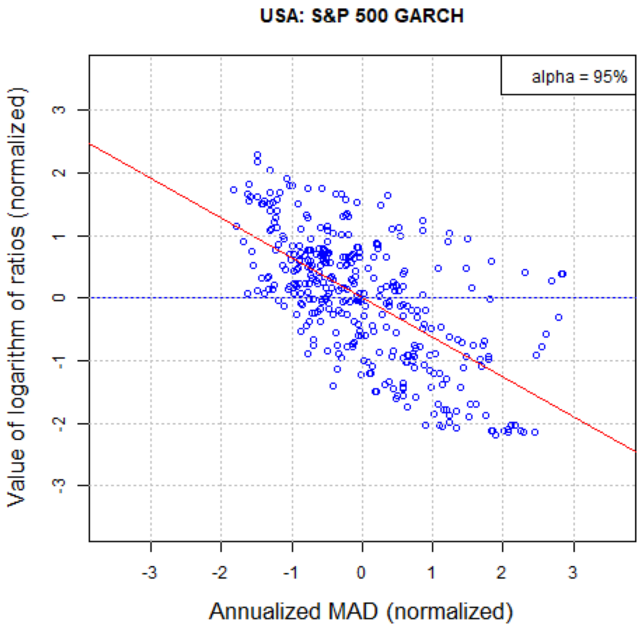
2.1 – Two factors

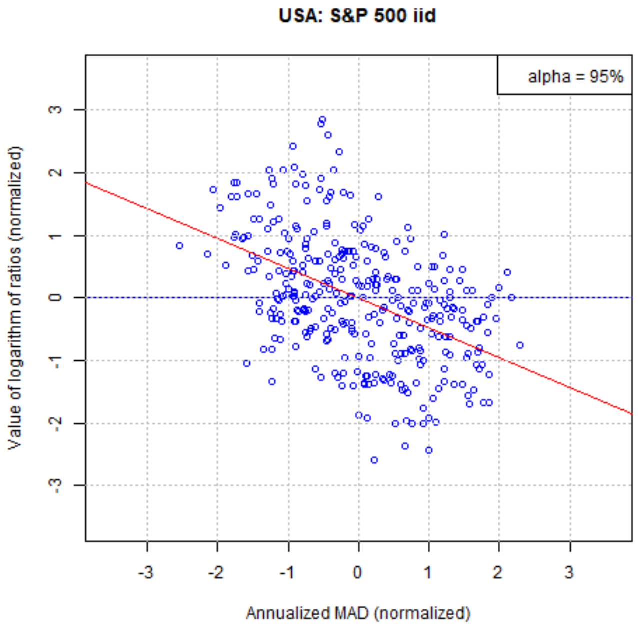
- Use models able to isolate effects:
  - a **GARCH**(1,1) model (with normal and Student innovations) : to explore if the *volatility clustering* present in the data is fast enough to produce pro-cyclicality
  - an **iid** model: to check if the *very way we measure risk*, **independently of volatility** models, hence of business cycles, creates negative dependence between the log-ratio and the realized volatility.
- Compute: $\mathbf{Cor}\left(\log R_{n,\alpha,t}\,, \widehat{\theta}_t\right)$

2 – How to explain pro-cyclicality?

2.2 – Empirical Results







	Data (S&P 500)	GARCH	iid
$\mathrm{Cor}\left(\log R_{n,\alpha,t}\,,\hat{\theta}_t\right)$	- 50%	- 63 %	- 34% (N) / -35% (t3)

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2 – How to explain pro-cyclicality?

2.3 – Theoretical results

- Bivariate CLT between functions of the estimators of the SQP and of the dispersion measure, $m(X,r)$, for iid model X, under some conditions ($\mathbb{E}[X^2] < \infty$ for MAD (r=1)):

$$\sqrt{n}\begin{pmatrix} h_1\big(q_n(\alpha)\big) - h_1\big(q_X(\alpha)\big) \\ h_2\big(\widehat{m}(X,n,r) - h_2(m(X,r))\big) \end{pmatrix} \underset{n\to\infty}{\overset{d}{\longrightarrow}} \mathcal{N}\big(0, \Gamma^{(r)}\big)$$

(h_i are continuous differentiable real functions)

In particular, $\lim\limits_{n\to\infty} \mathrm{Cor}\left(\log\left|\frac{q_{n,t+1y}(\alpha)}{q_{n,t}(\alpha)}\right|, \hat{\theta}_n\right) = -\frac{1}{\sqrt{2}} \frac{\left|\Gamma_{12}^{(1)}\right|}{\sqrt{\Gamma_{11}^{(1)}\Gamma_{22}^{(1)}}}$

- Bivariate FCLT for the GARCH: Under some conditions, the **asymptotic distribution** of the **logarithm of the look-forward ratio** of the risk measure estimator with the r-th absolute central sample moment (in part. MAD) is *bivariate normal* too, with a *negative correlation*, which structure is as in the iid case.

- Extension of the results to the main popular risk measures, as Expected Shortfall, Expectile

- Hence: **no matter** the choice of **risk measure** (VaR, ES, expectile) or **dispersion measure** (r-th absolute central sample moment), one has **pro-cyclicality** !

Conclusion

- **Pro-cyclicality of the *SQP***, a **dynamic** generalization of **VaR**, confirmed and **quantified** (by conditioning to realized volatility)

- Identification of *2 factors explaining pro-cyclicality* of risk measurement,
  1. *clustering effect of the volatility* (via GARCH models)
  2. *the way risk is measured*, **independently of business cycles** (via iid model)

  with a negative dependence between the realized volatility and the log SQP-ratios shown *empirically* and confirmed *theoretically*

- Regulation should, in fact, enhance the capital requirements in quiet times and relax them during the crises. It means introducing *anti-cyclical risk management rules*

- Regulators are aware of it and address it **indirectly** (Basel III, economic measures; Solvency 2, transitional measures). Our methodology should allow to address it **directly**

Our ongoing work: **design of a SQP** with **proper dynamical behavior** as a good basis for *anti-cyclical regulation*

Main references - preprints available on ssrn and arXiv

- *Empirical study*:
  - M. Bräutigam, M. Dacorogna, M. Kratz (2018-2019). Pro-Cyclicality of Traditional Risk Measurements: Quantifying and Highlighting Factors at its Source

- *Theoretical study*:
  - M. Bräutigam and M. Kratz (2019). On the Dependence between Quantile and Dispersion Estimators
  - M. Bräutigam and M. Kratz (2019). Bivariate FCLT for the Sample Quantile and Measures of Dispersion for Augmented GARCH(p,q) processes
  - M. Bräutigam and M. Kratz (2019-20). The Impact of the Choice of Risk and Dispersion Measure on Procyclicality

Thank you for your attention

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